Stochastic Resonance and Ratchets: Two Closely Related Phenomena in Brownian Motors *

QIAN Min(钱敏)[1], WANG Yan(王彦)[1], ZHANG Xue-Juan(张雪娟)[1,2**]

[1]*Key Laboratory of Pure and Applied Mathematics, School of Mathematical Sciences, Peking University, Beijing 100871*
[2]*Department of Mathematics, Shaoxing College of Arts and Sciences, Shaoxing 312000*

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Ratchet effect as well as the phenomenon of stochastic resonance (SR) is investigated in a simple flashing ratchet model. Numerical simulations show that both the power spectrum amplitude and the mean mobility of the system vary non-monotonically in similar manners with increasing the noise intensity and both reach their highest point nearly at the same noise level. We point out that the consistency of these two phenomena are just due to the existence of circular flux in non-equilibrium state.

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Noise in systems is usually thought to play destructive roles. However, it was discovered that under a certain nonlinear condition, the interplay of white noise and a weak periodic signal can result in resonance-like behaviour of the system response as a function of the noise level.[1,2] Such a counterintuitive phenomenon was termed as the stochastic resonance (SR). For the past two decades, the SR has been extensively studied both theoretically and experimentally,[3−6] to quote but a few. Physical mechanisms of the SR in some typical nonlinear stochastic systems have been clearly elaborated.[7−9] For a recent review of the SR, we can see Ref. [10].

Another striking role of thermal noise played constructively on systems is its inducing unidirectional transport in a periodic but asymmetric potential even in the absence of any macroscope force or mean potential gradient.[17,18] Such a noise-driven transport is termed as ratchet effect (or Brownian ratchet, molecular ratchet). It may well explain the mechanism of a sort of important biological phenomena such as cell locomotion, membrane protein transport and muscle contraction, and so forth. As an interesting nonlinear phenomenon with many future applications, many important works concerning Brownian motors were contributed in the literature,[11−15] for the latest review, we can see Refs. [16,17].

Physically, the SR and ratchets reflect the best matching effect between noise and the transport property of the system from different aspects. Dynamically, they are both directly related to the synchronized behaviour of the system induced by noise. From such points of view, it is natural to conclude that the ratchet effect and SR are closely related to each other (see Astumian[18] and Reimann[16]). However, what are the concrete manifestations of their relationship. So far as we know, no evidence based on numerical data or theoretical study in concrete models has been presented to confirm the above conclusion.

In this Letter, we present a numerical demonstration of the intimate connection between the SR and the ratchet effect in a simple flashing ratchet model. By numerically showing the non-monotonically increasing and decreasing property of the power spectrum amplitude as well as the particle mobility with the noise level, we show that in some systems, the conventional SR measured usually via the power spectrum peak is in agreement with the mobility-measured SR (a phenomenon that the mean mobility of the system exists a maximum with increasing the noise strength). In fact, the SR and ratchet effect are just two facets of the same origin—the non-equilibrium circulation, as suggested and discussed separately in Ref. [24] and [26].

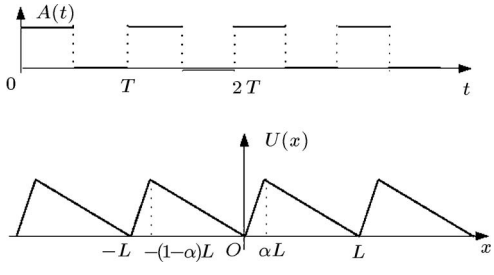


Fig. 1. Profile of an anisotropic potential $U(t)$, it is switched on and off periodically by a dichotomous signal $A(t)$ with period T.

The model of molecular motor here we focus on is a simple "flashing ratchet," whose equation can be expressed as

$$\dot{x}(t) = -U'(x)A(t) + D\xi(t), \qquad (1)$$

where the term $U(x)A(t)$ represents an anisotropic

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** Corresponding author.

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saw-tooth potential $U(x)$ (with periodic L and anisotropic parameter α) being switched on and off periodically in time by a dichotomous signal $A(t)$ (see Fig. 1), $\xi(t)$ is a Gaussian white noise satisfying: $\langle \xi(t) \rangle = 0$, $\langle \xi(t)\xi(s) \rangle = \delta(t-s)$.

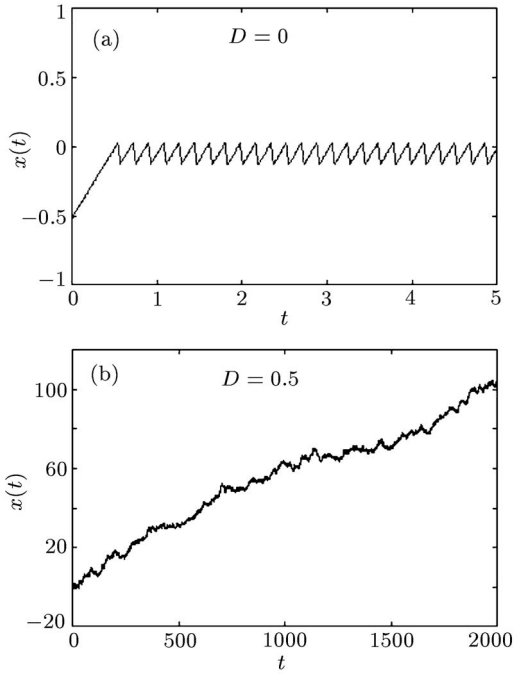


Fig. 2. Orbits of system (1) for (a) $D = 0.0$, (b) $D = 0.5$. One can clearly see the ratchet effect when noise is introduced.

The previous works concerning the noise-induced transport in this model can be found in Refs. [18–20], where the ratchet effect was well presented as a result of three necessary ingredients, i.e., the asymmetry of the potential $U(x)$, the on-off turning of the potential, and the thermal noise fluctuation. The mechanism can be briefly explained as follows (for details, see Ref. [18]). It is supposed that the potential is off at the beginning, then the particles diffuse symmetrically under the perturbation of noise. After half a period time of the switch, the anisotropic potential is on, then particles across the forward peak of the potential are more than those across the backward one. With the help of the potential force, more particles are in the forward bottom than those in the backward one before the potential is off again. Thus, with the time evolution, a unidirectional forward motion occurs. This feature can be directly seen from the orbits of the system (compared Fig. 2(b) with Fig. 2(a)); and correspondingly, the mean mobility of the system $\langle \dot{x} \rangle$ becomes positive. Here, for practical numerical simulations, time-averaged velocity $\lim_{t\to\infty} x(t)/t$ is calculated instead of the quantity $\langle \dot{x} \rangle$ due to their equivalence.[16] For illustration, we take the potential period $L = 1$, the asymmetry parameter $\alpha = 1/6$, and the period of the switch signal $T = 2$.

To show the variation of the ratchet effect with increasing thermal fluctuation, in Fig. 4(a), we plot the curve of the mean mobility $\langle \dot{x} \rangle$ versus the noise intensity D. One can see that properly increasing the noise level can accelerate the transport. At a certain noise intensity, the mobility reaches its maximum, which means that the best ratchet effect is reached. After that, however, it decreases with the further increase of the noise intensity. This is because too strong noise fluctuation makes the motion of the particle completely dominated by the noise which is not biased itself.

On the other hand, the appearance of nonzero mobility of the system must have its reflection in the power spectrum of the output. So far as we know, this kind of research has not been reported in the literature. To see this, we average the power spectra of the orbits of Eq. (1). Due to the transport motion under the noise perturbation, the values of $x(t)$ may be unbounded with the time evolution, here we take the time series of $\{\cos(2\pi x(t))\}_{t\geq 0}$ instead, since the motion of system (1) can also be regarded as on a circle with radius $1/(2\pi)$ because of the spatial periodicity of the potential. Numerical simulations show that when noise is present, the power spectrum of the output of system (1) is peaked at the frequency of potential switch [see Fig. 3(b)] and the height of the peak is much higher than the deterministic one (cf. Fig. 3 (b) with Fig. 3(a)]. This is because when the anisotropic potential is on during half a period of the switch, more particles pass across the potential peak and to slide down to the next bottom, by way of the noise perturbation. For a small amount of particles that miss a "good opportunity" to pass across the potential peak, they have to wait until the anisotropic potential is on again in the next period of the switch. Thus there is a time matching between the periodically switching of the anisotropic potential and the continuously skipping the potential maximum. This makes the spectrum peak centred just at the frequency of potential switch when noise is applied.

By changing the noise intensity from zero to a sufficiently large value, we can see that the height of the spectrum peak which is centred at the frequency of $A(t)$ first increases, reaches a maximum, and then decreases [see Figs. 3 and 4(b)]. This just manifests the occurrence of the SR in system (1). Furthermore, from Figs. 4(a) and 4(b), one can see that both the mean mobility $\langle \dot{x} \rangle$ and the height of the spectrum peak change non-monotonically with the noise level in a somehow resembled manner and they reach their highest point nearly at the same optimal noise intensity.

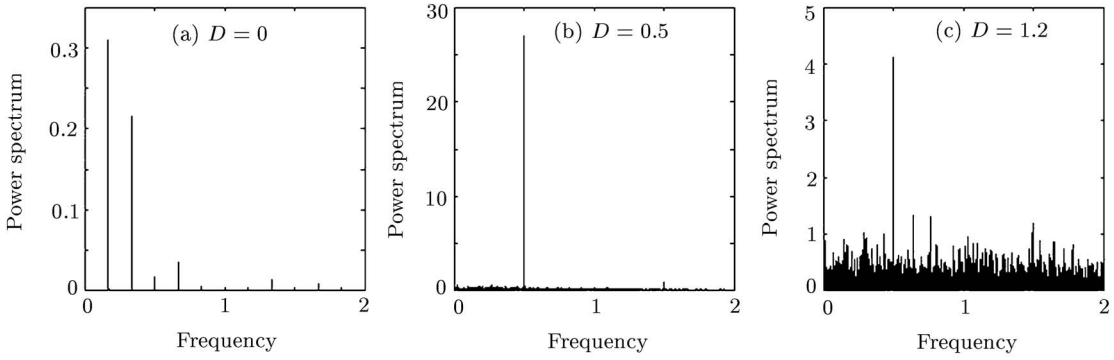


Fig. 3. Average power spectra of $\{\cos x(t)\}_{t\geq 0}$ for (a) $D = 0.0$, (b) $D = 0.5$, and (c) $D = 1.2$. One can see that certain amount of noise can greatly amplify the system output.

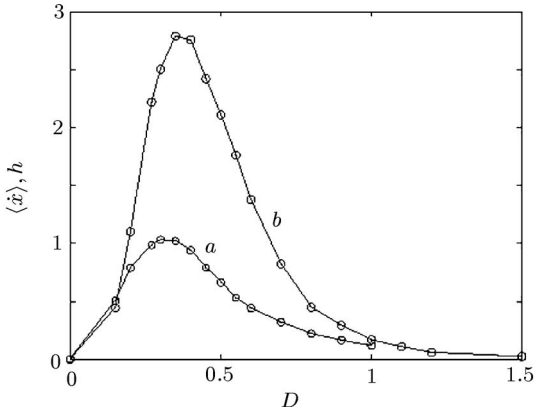


Fig. 4. (*a*) Mobility $\langle \dot{x} \rangle$ (here it is amplified 100 times) and (*b*) spectrum amplitude h of system (1) versus noise intensity D.

In conclusion, we have shown that the ratchet effect and the SR are indeed closely related in some systems, not only they both reflect the noise ability to extract useful work from fluctuating environment, but also there is a consistency of the behaviour in these two different designs. Dynamical source for such a consistency can be traced back to the noise-induced synchronized behaviour of the system. Then what is the physical essence underneath? As has been theoretically demonstrated in Ref. [16], under thermal equilibrium (i.e., detailed balance), a random system is impossible to exhibit unidirectional motion. In Ref. [24], the power spectrum is just Lorentzian. Thus a system being in a non-equilibrium state is a crucial condition for the occurrence of either the SR or the ratchet effect. Actually, the purely noise-induced SR is related to the non-equilibrium circulation, this has been elaborated in a cyclic reaction model with finite states in Ref. [24]. Non-equilibrium circulation in a thermal ratchet system is also discussed in Ref. [26]. Our discussion and results concerning system (1) with continuous states further confirm that both the SR and the ratchet effect are essentially due to the existence of circular flux which is a sufficient and necessary condition for non-equilibrium states.[25−27] However, making clear the way how the non-equilibrium circulation affects these phenomena depends on each specific concrete model and deserves further study.

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