Generation of Ultrashort Light Bullets in Dispersive Kerr Media Using the Fourth-Order Dispersion-Dependent Spatiotemporal Instability *

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A new method for generation of a train of ultrashort pulses or a sequence of ultrashort light bullets is proposed. This method is based on the fourth-order dispersion-dependent spatiotemporal instability in dispersive Kerr media. The repetition-rate of the generated bullets can be made quite large by increasing the corresponding spatial modulation frequency locating in the new instability region resulted from fourth-order dispersion.

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During the last few years, the race toward the generation of extremely short light pulses has become very exciting with important achievements.[1,2] Nowadays, light pulses with durations comparable to the carrier oscillation cycle have become available. For the generation of such light pulses in the few-cycle regime, pulse compression based on the interplay between self-phase-modulation induced by the Kerr effect and anomalous group-velocity dispersion (GVD) is the key concept. Currently it is most efficiently implemented in mirror-dispersion-controlled Kerr-lens mode-locked Ti:sapphire laser oscillators and hollow-fibre chirped-mirror compressors. In contrast to these techniques for ultrashort light pulse generation, here we examine the modulation-instability(MI)-dependent method.

It is well known that modulation instability is an effective physical mechanism in optics[3] and fluids[4] for breakup of continuous modes into solitary waves and may act as a precursor for the formation of bright soliton. For example, the temporal instability in optical fibres has been used for pulse-train generation.[5,6] The spatial instability, usually called the small-scale self-focusing,[3,7−9] is often observed to break up a uniform laser beam into numerous filaments with very high intensity in high power laser systems. The spatiotemporal instability occurring as a result of interplay among diffraction, dispersion and self-phase modulation has also been shown its possibility for converting a cw beam or ground mode into a train of ultrashort pulses or a periodic sequence of light bullets.[10−14] For ultrashort light pulse generation and propagation, higher-order dispersions become important and must be taken into consideration. In Ref. [12], we have disclosed the generic role played by arbitrary higher-order dispersions in modulation instability. In this Letter, we further study the fourth-order dispersion(FOD)-dependent spatiotemporal instability. Special effort is focused on exploring the possibility of generation of ultrashort pulse trains or light bullets using the new instability region resulted from FOD.

Let us consider a propagation of the optical field, $E(x,y,z,t) = A(x,y,z,t)\exp(\mathrm{i}\beta_0 z - \mathrm{i}\omega_0 t) + \text{c.c.}$, where $A(x,y,z,t)$ is the complex envelope of the field, β_0 is the propagation constant at the central frequency ω_0, and c.c. represents the complex conjugate, in a dispersive Kerr medium with a Kerr refractive index, $n(\omega,|A|^2) = n_0(\omega) + n_2|A|^2$, with n_0 and n_2 being, respectively, the refractive index of the medium at the central frequency and the nonlinear refractive coefficient of the medium. Substituting the above expressions for $E(x,y,z,t)$ and n into the wave equation and using the Taylor expansion of the propagation constant that describes chromatic dispersion about ω_0,

$$\beta(\omega) = \beta_0 + \sum_{m=1}^{\infty} \frac{\beta_m}{m!}(\omega-\omega_0)^m, \tag{1}$$

where $\beta_m = (\partial^m\beta/\partial\omega^m)_{\omega_0}$, $m = 1, 2, \cdots$. Keeping the dispersion terms up to fourth-order, we obtain the following propagation equation for the complex envelope amplitude:[2]

$$\partial_\xi A = \frac{\mathrm{i}}{2\beta_0}\left(1+\frac{\mathrm{i}}{\omega_0}\partial_\tau\right)^{-1}\nabla_\perp^2 A - \mathrm{i}\frac{\beta_2}{2}\partial_\tau^2 A + \frac{\beta_3}{6}\partial_\tau^3 A + \mathrm{i}\frac{\beta_4}{24}\partial_\tau^4 A + \mathrm{i}\beta_0\frac{n_2}{n_0}\left(1+\frac{\mathrm{i}}{\omega_0}\partial_\tau\right)(|A|^2 A), \tag{2}$$

where we have introduced the moving reference frame, $\tau = t - \beta_1 z$, $\xi = z$. In Eq. (2) ∂_j denotes the partial derivative with respect to j, $\nabla_\perp^2 = \partial_x^2 + \partial_y^2$ is the transverse Laplacian. The presence of the correction operator $\left(1+\frac{\mathrm{i}}{\omega_0}\partial_\tau\right)$ in the terms accounting for diffraction and nonlinear effects gives rise to space–time focusing and self-steepening effects, respectively.

Introducing the normalized variables $T = \tau/\tau_p$, $X = x/l_\perp$, $Y = y/l_\perp$, $Z = \xi/l_d$, and $U = A/A_0$, where τ_p is the width of the input pulse, $l_d = \tau_p^2/|\beta_2|$ is the dispersion length, $l_\perp = \sqrt{l_d/\beta_0}$ is the characteristic transverse length introduced for normalization,

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and A_0 is the amplitude of the input field, we can transform Eq. (2) into the following form:

$$\partial_Z U = \frac{\mathrm{i}}{2}(1+\mathrm{i}\varepsilon)^{-1}\bar{\nabla}_\perp^2 U - \frac{\mathrm{i}}{2}s\partial_T^2 U + \delta\partial_T^3 U + \mathrm{i}\sigma\partial_T^4 U + \mathrm{i}N(1+\mathrm{i}\varepsilon)(|U|^2 U), \tag{3}$$

where $\varepsilon = 1/(\omega_0\tau_p)$, $s = \mathrm{sgn}(\beta_2)$, $\delta = l_d\beta_3/(6\tau_p^3)$, $\sigma = l_d\beta_4/(24\tau_p^4)$, $N = \beta_0|n_2|A_0^2 l_d/n_0$, and now the transverse Laplacian $\bar{\nabla}_\perp^2 = \partial_X^2 + \partial_Y^2$.

Performing a standard linear stability analysis on Eq. (3), we find that there exist spatiotemporal instabilities in the case of anomalous GVD and positive FOD, with the instability gain

$$g = \sqrt{-G(G - \Omega_c^2/2) - \varepsilon^2\Omega_c^4\Omega^2/16}, \tag{4}$$

where $G = (1-\varepsilon^2\Omega^2)^{-1}K^2/2 + \Omega^2/2 - \sigma\Omega^4$, with $K = \sqrt{K_X^2 + K_Y^2}$ and Ω being, respectively, spatial and temporal frequencies of the modulation, and $\Omega_c = 2\sqrt{N}U_0$ with U_0 being the normalized amplitude of the input field.

For instability to occur, g must be real and positive. Thus the instability regions are those enclosed by the contours of zero gain, $g = 0$, which can be written as

$$(\Omega^2 - \Omega_1^2)(\Omega^2 - \Omega_2^2)(\Omega^2 - \Omega_3^2)(\Omega^2 - \Omega_4^2) = 0, \tag{5}$$

where $\Omega_1, \Omega_2, \Omega_3$ and Ω_4 are the zero points of the gain g. Obviously, FOD exerts fundamental influence on spatiotemporal instabilities in dispersive Kerr media. Particularly, for positive FOD ($\sigma > 0$), $\Omega_1, \Omega_2, \Omega_3$ and Ω_4 may all be real, meaning that FOD may lead to the appearance of new instability regions. In the following analysis, we concentrate on this case.

When instability occurs, we are more interested in the fastest growing frequencies and the corresponding maximum gain which can be obtained from Eq. (4) by setting $\partial_K g = 0$ and $\partial_\Omega g = 0$:

$$(1-\varepsilon^2\Omega_m^2)K_m^2 + \Omega_m^2 - 2\sigma\Omega_m^4 = \Omega_c^2/2,$$
$$g_{\max} = \Omega_c^2\sqrt{1-\varepsilon^2\Omega_m^2}/4. \tag{6}$$

We see from Eq. (6) that FOD influences the fastest growing frequencies, but does not influence the maximum gain. To see the importance of the various effects considered we estimate their possible values for the parameters in Ref. [2]. For optical pulses with central wavelength $\lambda = 800\,\mathrm{nm}$ and peak intensity $A_0^2 \sim 10^{13}\,\mathrm{W/cm^2}$ propagating in fused silica, the product $\varepsilon\Omega_c^2$ can have a maximum value of 0.3. In addition, the FOD parameter σ is chosen such as $\sigma\Omega_c^2 \sim 0 - 0.15$.

Figure 1 shows the spatiotemporal instability regions in the normalized spatial and temporal frequency plane $(K/\Omega_c, \Omega/\Omega_c)$ for $\sigma\Omega_c^2 = 0$ and 0.1, respectively, for $\varepsilon\Omega_c^2 = 0.1$. The solid curves enclosing the instability region in each case are the contours of zero gain, whereas the dotted curves indicate the contour of maximum gain. From Fig. 1, it can be concluded that FOD deforms the conventional instability region (with $\sigma = 0$) slightly, but it leads to new instability regions which can extend far away from the conventional instability region. Due to the FOD, temporal instability can develops for any spatial frequencies, and by increasing the spatial frequency K, the corresponding instability temporal frequency Ω can be easily increased.

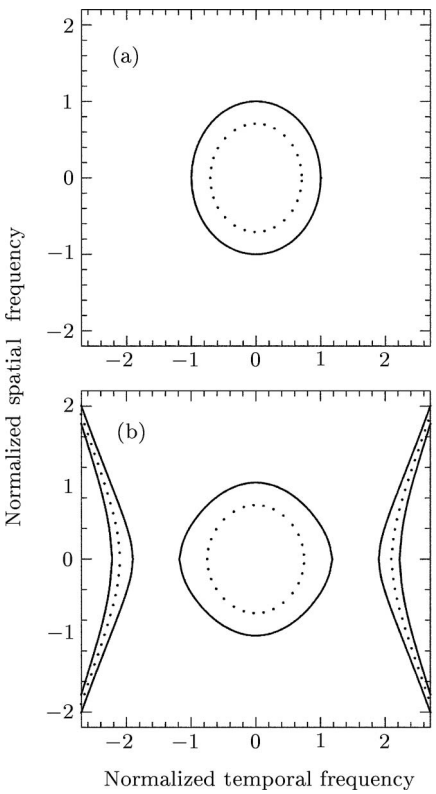


Fig. 1. Spatiotemporal instability regions in the (K, Ω) plane, (a) in the absence of FOD ($\sigma = 0$), (b) in the presence of FOD with $\sigma\Omega_c^2 = 0.1$ for $\varepsilon\Omega_c^2 = 0.1$. The solid curves enclosing the instability region are contours of zero gain. The dotted curves in each case indicates the contour of maximum gain.

As stated before, spatiotemporal instability can be used to convert a cw light into a train of ultrashort pulses or a sequence of light bullets. The properties of the new instability regions are most interesting since the repetition rate Ω of the pulses can be made quite large by increasing K. To demonstrate this, we numerically solve Eq. (3) using the standard split-step Fourier method in the (2+1)-dimensional case (one transverse spatial, one temporal, and one propagation dimension). The initial field distribution at the entrance to the nonlinear medium is a sinusoidally modulated plane wave,

$$U(X, T, Z = 0) = U_0[1 + u_1\sin(KX)][1 + u_2\sin(\Omega T)], \tag{7}$$

where U_0 is the initial amplitude of background wave which is set to be 1 in the following simulations; u_1 and u_2 are, respectively, the initial amplitudes of spatial and temporal modulation waves. In a laboratory experiment, noise in the system provides the seed from which instability frequencies begin to develop. How-

ever, for the numerical simulations, it is necessary to explicitly provide an initial seed at the spatial frequency K, temporal frequency Ω, or both to stimulate a response (induced modulation instability). In the following simulations, K and Ω are chosen that they are the fastest growing frequencies in the new instability region.

In the absence of any applied spatial modulation, temporal instability can develop at two normalized fastest temporal frequencies, $\Omega_{m1}/\Omega_c = 0.75$ in the conventional region and $\Omega_{m2}/\Omega_c = 2.1$ in the new region, as shown in Fig. 1(b). Thus the new instability frequency is larger than the conventional one by about 2 times. Firstly, we examine this spatially homogeneous case. Figure 2 shows the temporal and spectral distribution of the field intensity at $X = 0$ at different propagation distances into the medium. The linear stability theory predicts that the frequency Ω_{m1} should evolves simultaneously and at the same rate as Ω_{m2}. However, since no seed was provided at that frequency, only the frequency Ω_{m2} grows until it generates higher harmonics through nonlinear interactions within the medium as seen in Fig. 2. Since Ω_{m2} is the dominant spectral component, the spatiotemporal instability gives rise to a periodic modulation at a repetition rate of Ω_{m2} and breaks up the background plane wave into a train of ultrashort pulses.

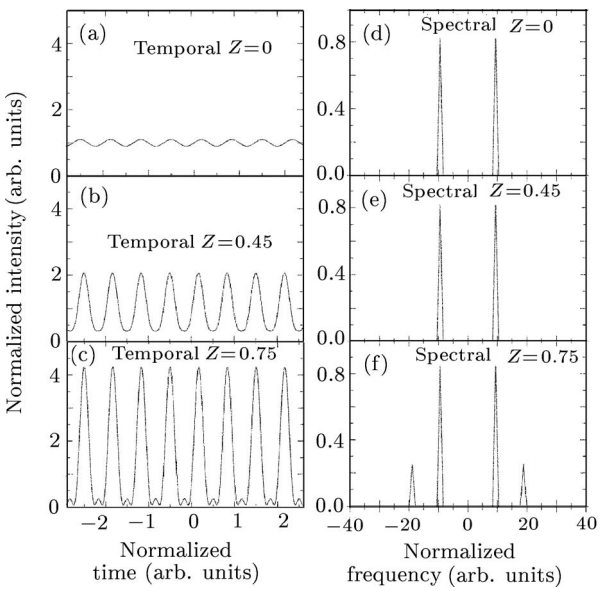


Fig. 2. Temporal and spectral distributions of the field intensity at different normalized propagation distances. The initial seeding is a purely temporal modulation with $u_2 = 0.05$, $\Omega = 9.42$.

Secondly, we consider the spatially inhomogeneous case in which temporal modulations are accompanied by spatial modulations. In this case, the temporal frequency can be set to be a much larger value by increasing the corresponding spatial frequency out of the conventional instability region. Figure 3 shows the intensity distribution as a function of both the spatial coordinate and time at a propagation distance $z = 1.6$ for $K = 8.9$ and $\Omega = 12.2$ which are in the new instability region far away from the conventional one. We see that a cw has broken up into a periodic sequence of light bullets with high peak intensity. The spatial and temporal repetitions of the bullets equal $2\pi/K$ and $2\pi/\Omega$, respectively. In principle, the temporal frequency Ω can be arbitrarily large by properly choosing the spatial frequency K, as shown in Fig. 1(b). Thus we can break up a cw plane wave into a pulse train with arbitrarily high repetition rate.

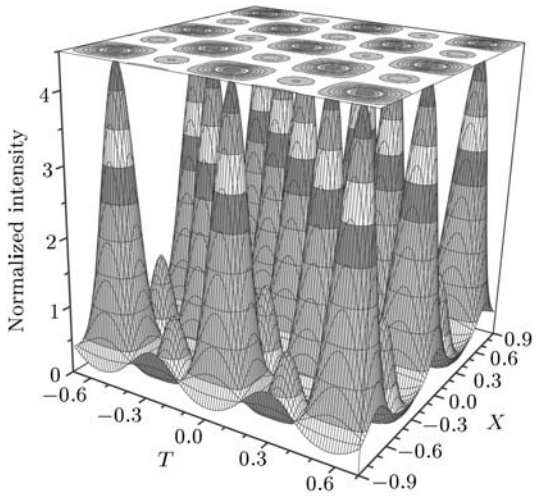


Fig. 3. Field intensity distribution as a function of both the normalized transverse coordinate X and time T at a normalized propagation distance $Z = 1.6$ with the initial seeding at $K = 8.9, \Omega = 12.2$. The initial amplitudes of the seeding are set to be $u_1 = 0.05$, $u_2 = 0.01$.

In summary, we have proposed a new method for the generation of a train of ultrashort pulses or a sequence of ultrashort bullets. Through the standard linear stability analysis, we have found that the fourth-order dispersion may lead to a new spatiotemporal instability region which can extend far away from the conventional region in dispersive Kerr media. By numerically solving the modified nonlinear Schrödinger equation, we have shown the possibility for the generation of ultrashort bullets with quite large repetition-rate using the fastest growing modulation frequencies located in the new instability region.

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