Teleportation of Mixture of Diagonal Bell States via Bound Entangled States *

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We propose a protocol for teleportation of arbitrary mixture of diagonal Bell states, it is shown that the channel can be constructed with either pure maximally entangled states or mixed bound entangled states. We also present protocols to realize the controlled teleportation of mixture of diagonal Bell states via multi-particle mixed states. Our results show that bound entangled states are also important and useful resources in quantum communication tasks.

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Since the pioneering work of Bennett *et al.*,[1] there have been wide investigations on the field of quantum teleportation both in theories[1−5] and in experiments.[6−11] During the teleportation process, an unknown quantum state is destroyed at a sending place while its perfect replica state appears at a remote place via quantum and classical channels. Generally, to be better than a classical channel, it is believed that the channel should be constructed with an entangled state either pure or mixed. Perfect teleportation is possible only when maximally entangled states are available.

In the usual teleportation protocols, the state to be teleported is a pure state. Under this condition the pure state channel might be utilized to perform the teleportation tasks better than the mixed state channel. However, it will be a different situation if we consider the problem of teleportation of mixed states. In this Letter, we show that mixture of diagonal Bell states can be teleported via mixed bound entangled states faithful and deterministic. Linden and Popescu[12] have investigated the problem of teleportation with some special bound entangled states. They show that at least in some cases teleportation can be achieved with better than classical fidelity. Here, we consider a different case, the channel is composed of a Smolin state,[13] i.e. four-particle bounded entangle state. It is shown that we can achieve the same fidelity versus pure maximally entangled channels when we use the Smolin state. In the following we present our protocol in detail.

We denote the four Bell states as

$$|\Phi^1\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle),$$
$$|\Phi^2\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle),$$
$$|\Phi^3\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle),$$
$$|\Phi^4\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle). \quad (1)$$

Without loss of generality, the mixture of diagonal Bell states to be teleported can be written as

$$\rho_{12} = F[\Phi^1]_{12} + (1-F)[\Phi^3]_{12}, \quad (2)$$

where we have used the notation $[\cdot]$ for pure state projector $|\cdot\rangle\langle\cdot|$. We first consider the case that channels are composed of two maximally entangled pure states. Let Alice and Bob share two Bell states $|\Phi^3\rangle_{34}$ and $|\Phi^3\rangle_{56}$. Alice owns particles 1, 2, 3 and 5, the other on Bob's side. Now, to achieve the teleportation process, Alice only needs to perform two Bell state measurements on particles 1 and 3, 2 and 5 respectively. Corresponding to Alice's different measurements, Bob could perform different unitary operation on his particle, then he will obtain the faithful replica state of the original two-particle mixed states on his side. Next, we show that if the two maximally entangled channels are substituted by four-particle bound entangled states, it will also works the same well. We suppose that Alice and Bob share the following four-particle bound entangled state:

$$\rho_{3456} = \frac{1}{4}\sum_{i=1}^{4}[\Phi^i]_{35} \otimes [\Phi^i]_{46}, \quad (3)$$

where $|\Phi^i\rangle$ denotes the Bell state, and it is a four-party activable bound entangled state, i.e. the states that are not distillable by local operations and classical communication (LOCC) when the four-party is separated from every other but becomes distillable, if certain parties decide to come together. At first

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glance, it seems that this state is not suitable for teleportation, because it is separable across the three bipartite cuts 35:46, 34:56, and 36:54.[13] However, by analysis we find that it is not the case. In fact, we only need to repeat the standard teleportation protocol, then Bob could obtain the perfect replica state of the two-particle mixed states on his side with a corresponding unitary operation on his particle. The unitary transformations corresponding to the state of particles 4 and 6 are given in Table 1. Furthermore, one can verifies that the other mixture of diagonal Bell states can also be teleported in our scheme.

Table 1. The relation between Alice's different measurement results and Bob's relevant unitary transformations. We only list the four cases, the other cases are similar. MR denotes the measurement performed on particles 1, 2, 3 and 5. O denotes unitary operations corresponding to the state of receiver.

MR	ρ_{46}	O
$\|\Phi^3\rangle_{13}\|\Phi^3\rangle_{25}$	$F[\Phi^1]_{46}+(1-F)[\Phi^3]_{46}$	I
$\|\Phi^3\rangle_{13}\|\Phi^4\rangle_{25}$	$F[\Phi^2]_{46}+(1-F)[\Phi^4]_{46}$	σ_z
$\|\Phi^3\rangle_{13}\|\Phi^1\rangle_{25}$	$F[\Phi^3]_{46}+(1-F)[\Phi^1]_{46}$	σ_x
$\|\Phi^3\rangle_{13}\|\Phi^2\rangle_{25}$	$F[\Phi^4]_{46}+(1-F)[\Phi^2]_{46}$	σ_y

Since bound entangled states can be used to teleport the mixed states, a question naturally arises: can pure entangled states be also teleported? Without loss of generality, we consider that it is possible to teleport the two particle entangled state $|\psi\rangle_{12}=\alpha|00\rangle+\beta|11\rangle$ with the channel (3). After performing the standard teleportation procedure, the output state on Bob's side will becomes $\rho_{46}=\frac{1}{2}([\psi]_{46}+[\varphi]_{46})$, where $|\varphi\rangle=\beta|00\rangle+\alpha|11\rangle$. Using the Peres–Horodecki criterion,[14] one can easily check that this density matrix is always a separable mixed state except $|\alpha|=|\beta|=\frac{1}{\sqrt{2}}$. Thus we conclude that except for the four Bell states, the other two-particle pure states cannot be teleported with the four-particle Smolin state.

Next we discuss how to realize controlled teleportation[15] of mixture of diagonal Bell states. Suppose Alice want to send the mixed state ρ_{12} which in the form of Eq. (2) to one of the agents Bob and Charlie randomly, and the receiver can obtain the replica of the original state only when he or she obtains the help of the other agent, i.e. Bob reconstructs it with the control of Charlie's, or vice versa. Of course, we could implement the controlled teleportation of mixture of diagonal Bell states with two GHZ states. Now we are interested in how to realize the controlled teleportation via multi-particle mixed states. In the following, we explore two approaches to accomplish this task.

Firstly we consider how to use multi-particle distillable mixed states as channels, let Alice prepare the following six-particle mixed states locally:

$$\begin{aligned}\rho_{345678}=&\frac{1}{4}[([\Phi^1]_{34}\otimes[\Phi^1]_{56}+[\Phi^3]_{34}\otimes[\Phi^3]_{56})\\&\otimes[\Phi^3]_{78}+([\Phi^2]_{34}\otimes[\Phi^2]_{56}\\&+[\Phi^4]_{34}\otimes[\Phi^4]_{56})\otimes[\Phi^4]_{78}],\end{aligned}\tag{4}$$

then she sends particles 5 and 7 to Bob, 6 and 8 to Charlie, respectively. Alice performs Bell state measurements on the particles 13 and 24, respectively. At this time the state of particles 5–8 will collapse into

$$\rho_{5678}=F[C]_{5678}+(1-F)[D]_{5678},\tag{5}$$

where $|C\rangle=|0001\rangle+|0010\rangle+|1101\rangle+|1110\rangle$, $|D\rangle=|0101\rangle+|0110\rangle+|1001\rangle+|1010\rangle$. If Bob wants to obtain the replica of ρ_{12}, Charlie does the Bell state measurements on his two particles, then Bob could reconstruct the original mixed state corresponding to Charlie's different measurements, or vice versa. The basic idea of this process is shown in Fig. 1. This protocol can be generalized to the multi-party agent case. We should make a little modification when the number of the agents are odd. For example, if there are three agents, Alice should prepare the eight-particle mixed state $\rho_{345678910}=\frac{1}{4}\sum_{i=1}^{4}[\Phi^i]_{34}\otimes[\Phi^i]_{56}\otimes[\Phi^i]_{78}\otimes[\Phi^i]_{910}$, then she distributes particles 57, 69 and 810 to the three agents, respectively. Now, one of the three agents could obtain the replica of the state ρ_{12} with unitary operations by collaboration with the other two controllers. We generalize this result as follows: suppose that there are $m+1$ agents in the controlled teleportation, the number of controllers will be m ($m\geq 1$). If m is odd, the mixed channel state should be in the form $\rho_{\rm odd}=\frac{1}{4}[([\Phi^1]^{\otimes(m+1)}+[\Phi^3]^{\otimes(m+1)}),[\Phi^3]+([\Phi^2]^{\otimes(m+1)}+[\Phi^4]^{\otimes(m+1)})[\Phi^4]]$; if m is even, the mixed channel state should be $\rho_{\rm even}=\frac{1}{4}\sum_{i=1}^{4}[\Phi^i]^{\otimes(m+2)}$. In Ref. [16], Chen *et al.* have shown the distillable entanglement of $\rho_{\rm even}$ is m ebits. That is to say, we need m ebits entangled to prepare $\rho_{\rm even}$ locally. For the mixed state $\rho_{\rm odd}$, we can calculate that it has $m+1$ ebits of distillable entanglement. Now we show how to explicitly calculate the amount of distillable entanglement for $\rho_{\rm odd}$. We know that the relative entanglement of mixed state ρ is defined as $E_r(\rho)=\min_{\sigma\in D}S(\rho\parallel\sigma)$, where D is the set of separable states, and $S(\rho\parallel\sigma)=\mathrm{Tr}\rho(\ln\rho-\ln\sigma)$ is the relative entropy. We denote $\rho^{(n)}=\frac{1}{4}\sum_{i=1}^{4}[\Phi^i]^{(n)}$. It is obvious that $\rho^{(2)\otimes\frac{m+1}{2}}\otimes\rho^{(1)}$ is separable since $\rho^{(2)}$ and $\rho^{(1)}$ are separable, where m is an odd no less

than 1, thus

$$S(\rho_{\rm odd} \parallel \rho^{(2)\otimes\frac{m+1}{2}} \otimes \rho^{(1)}) = {\rm Tr}\rho_{\rm odd}(\ln\rho_{\rm odd} - \ln\rho^{(2)\times\frac{m+1}{2}} \otimes \rho^{(1)}) = m+1 \quad (6)$$

Thus we have $E_d(\rho_{\rm odd}) \leq E_r(\rho_{\rm odd}) \leq S(\rho_{\rm odd} \parallel \rho^{(2)\otimes\frac{m+1}{2}} \otimes \rho^{(1)}) = m+1$, the first inequality is due to the fact that the relative entanglement is an upper bound of distillable entanglement. On the other hand, we could construct a distillation protocol for $\rho_{\rm odd}$, supposing that there are n pairs in the two-particle state in $\rho_{\rm odd}$. We choose the last pair as the target state, the other pairs as the controlled states, then we perform the bilateral controlled-NOT operation (BXOR) on each pair of the controlled state and the target state, if we denote $B(mn)$ as the BXOR operation performed on the mth pair and the nth pair, we can reach

$$\begin{aligned} B(23)B(13)\Phi^1\Phi^1\Phi^3 &= \Phi^1\Phi^1\Phi^3, \\ B(23)B(13)\Phi^2\Phi^2\Phi^4 &= \Phi^1\Phi^1\Phi^4, \\ B(23)B(13)\Phi^3\Phi^3\Phi^3 &= \Phi^3\Phi^3\Phi^3, \\ B(23)B(13)\Phi^4\Phi^4\Phi^4 &= \Phi^3\Phi^3\Phi^4. \end{aligned} \quad (7)$$

We can always discriminate[17] $|\Phi^3\rangle$ and $|\Phi^4\rangle$ by LOCC, so two ebits can be distilled when $n=3$, it is easy to show that $m+1$ ebits can be obtained for $n = m+2$. Now we have the inequality $E_d(\rho_{\rm odd}) \geq m+1$, together with Eq. (6), we know that the distillable entanglement for $\rho_{\rm odd}$ is $m+1$ ebits.

To implement our protocol, let Alice prepare the above multi-particle mixed state locally, then distributes $m+1$ pairs of particles to $m+1$ agents. Now there are m controllers in the $m+1$ agents. For example, Bob will obtain the replica of state ρ_{12} with unitary operations by collaboration with the other m controllers.

Interestingly the controlled teleportation of mixture of diagonal Bell states could also be implemented with multi-particle activable bound entangled states. Firstly we consider the case that there is only one controller. Alice only needs to prepare the following six-particle activable bound entangled states introduced in Ref.[18],

$$\begin{aligned} \rho_{345678} = \frac{1}{16}([&000000+111111]+[000011+111100] \\ &+[000101+111010]+[000110+111001] \\ &+[001001+110110]+[001010+110101] \\ &+[001100+110011]+[001111+110000] \\ &+[010001+101110]+[010010+101101] \\ &+[010100+101011]+[010111+101000] \\ &+[011000+100111]+[011011+100100] \\ &+[011101+100010]+[011110+100001]), \end{aligned} \quad (8)$$

then she sends particles 5 and 7 to Bob, 6 and 8 to Charlie, respectively. After Alice have performed two Bell state measurements on the particles 1,3 and 2,4, one of the agents of Bob and Charlie could obtain the perfect replica of the original two-particle mixed states with the collaboration of the other. For example, if the results of the Bell state measurements of Alice and Bob are both $|\Phi^3\rangle$, then the particles of Charlie will be in the state $\rho_{68} = F[\Phi^3]_{68} + (1-F)[\Phi^1]_{68}$. If he perform a σ_x operation on one of his particles then he will obtain the perfect replica of the original state of Alice's. Now we generalize this protocol to the m controllers case. Let Alice prepare the following $2(m+1)$-particle activable bound entangled state:[18]

$$\rho_{2(m+1)} = \frac{1}{2^{2m}} \sum_{i=1}^{2^{2m}} [\Phi^i], \quad (9)$$

where $|\Phi^i\rangle = \frac{1}{\sqrt{2}}(|p_i\rangle + |\bar{p}_i\rangle)$; $|p_i\rangle = |a_1^i a_2^i, \ldots, a_{2(m+1)}^i\rangle$, with $a_1^i = 0$, and $a_j^i \in \{0,1\}$; there is a even number of 0 in the string $a_1^i a_2^i, \ldots, a_{2(m+1)}^i$; $|\bar{p}_i\rangle$ is the states orthogonal to $|p_i\rangle$ defined as $|\bar{p}_i\rangle = |\bar{a}_1^i \bar{a}_2^i, \ldots, \bar{a}_{2(m+1)}^i\rangle$, where $\langle\bar{a}_j^i|a_j^i\rangle = 0$, $\forall j = 1, \ldots, 2(m+1)$, and $i = 1, \ldots, 2^{2m}$. Actually $\rho_{2(m+1)}$ can be regarded as a generalization of four-particle Smolin state when the particle number is even. With the help of this state, we can also realize the controlled teleportation via the control of the $m+1$ agents, and the procedure is similar to the two-agent case mentioned above.

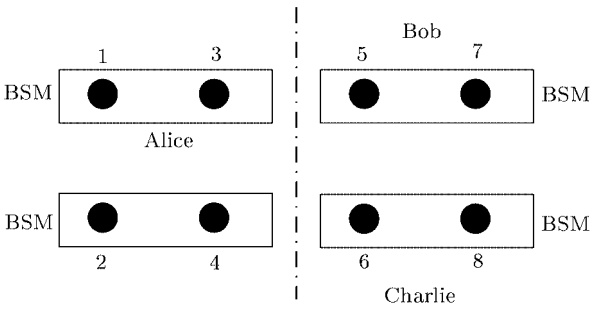


Fig. 1. Principle of controlled teleportation of mixture of diagonal Bell states in the case that there is one controller. BSM denotes the Bell state measurements.

In summary, we have shown that a four-particle bound entangled state can be used as channels for teleportation of mixture of diagonal Bell states directly. It has been shown that bound entangled states can be 'activated' under certain conditions.[19,20] That

is, with the help of some other entangled states they are able to carry out certain tasks that could not be performed by using these other entangled states and LOCC alone. Here, our results show that bound entangled states alone can make use of something, such as teleportation of a special kind of mixed states. We find that the mixed state channel is enough for accomplishing the teleportation of mixture of diagonal Bell states. We also investigate protocols to realize the controlled teleportation of mixture of diagonal Bell states via mixed state channels.

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