Isoscaling in Statistical Sequential Decay Model *

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A sequential decay model is used to study isoscaling, i.e. the factorization of the isotope ratios from sources of different isospins and sizes over a broad range of excitation energies, into fugacity terms of proton and neutron number, $R_{21}(N,Z) = Y_2(N,Z)/Y_1(N,Z) = C\exp(\alpha N + \beta Z)$. *It is found that the isoscaling parameters* α *and* β *have a strong dependence on the isospin difference of equilibrated source and excitation energy, no significant influence of the source size on* α *and* β *has been observed. It is found that* α *and* β *decrease with the excitation energy and are linear functions of* $1/T$ *and* $\Delta(Z/A)^2$ *or* $\Delta(N/A)^2$ *of the sources. Symmetry energy coefficient* $C_{\rm sym}$ *is constrained from the relationship of* α *and source* $\Delta(Z/A)^2$, β *and source* $\Delta(N/A)^2$.

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The isotopic composition of the nuclear reaction products[1] contains important information of the role of the isospin in the reaction process. N/Z degree of freedom and its equilibration, as well as the isospin asymmetry dependent term of the nuclear equation of state (EOS),[2−7] has motivated detailed measurements of the isotopic distributions of reaction products. Ideally, primary fragments should be detected right after emission in order to extract information about the collisions. However, most particles decay to stable isotopes in their ground states before detected. Recently ratio of isotope yields from two different reactions 1 and 2, $R_{21}(N,Z) = Y_2(N,Z)/Y_1(N,Z)$, is found to exhibit an exponential relationship as a function of the isotope neutron number N, and proton number Z,[8,10]

$$R_{21}(N,Z) = Y_2(N,Z)/Y_1(N,Z) = C\exp(\alpha N + \beta Z), \quad (1)$$

where α and β are the two scaling parameters and C is an overall normalization constant. This behaviour is called the isoscaling.[10] Isoscaling has been obtained in different reactions[8,9,12−16,18,19] and theoretical Calculations,[11,20−27] but no agreement have been reached on whether the secondary decay effect can be bypassed or not.[11,16,17,26] In our recent calculation by the isospin-dependent quantum molecular dynamics (IQMD) model followed by the sequential decay model GEMINI, no significant sequential decay effect was found for the primary products, but sequential decays in that work were made on the whole primary products which do not have the same state such as excitation energy, isospin asymmetry, source size etc., thus it is necessary and interesting in the frame of sequential decay model frame (GEMINI) to learn the sensitivity of the isoscaling parameters to the observable characterizing the source state. We consider the compound sources with different sizes whose nucleon numbers vary from 150 to 200, excitation energies from 2 to 6 MeV/nucleon, and isospin asymmetries are $N/Z = 1.0$, 1.24, 1.48, and 1.72, respectively.

The GEMINI model[28] calculates the decay of compound nuclei by sequential binary decays. All possible binary divisions from light-particle emission to symmetric division are considered. The model employs a Monte Carlo technique to follow the decay chains of individual compound nuclei through sequential binary decays until the resulting products are unable to undergo further decay.

The decay width for the evaporation of fragments with $Z \leq 2$ is calculated by using the Hauser–Feshbach formalism.[30] For the emission of a light particle (Z_1, A_1) of spin J_1 from a system (Z_0, A_0) of excitation energy E^* and spin J_0, leaving the residual system Z_2, A_2 with spin J_2, the decay width is given by

$$\Gamma(Z_1,A_1,Z_2,A_2) = \frac{2J_1+1}{2\pi\rho_0} \cdot \sum_{l=|J_0-J_2|}^{J_0+J_2} \int_0^{E^*-B-E_{\rm rot}(J_2)} T_l(\varepsilon)\rho_2(U_2,J_2)d\varepsilon, \quad (2)$$

where l and ε are the orbital angular momentum and

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kinetic energy of the emitted particle, $\rho_2(U_2, J_2)$ is the level density of the residual system with thermal excitation energy

$$U_2 = E^* - B - E_{\rm rot}(J_2) - \varepsilon, \qquad (3)$$

where B is the binding energy, $E_{\rm rot}(J_2)$ is the rotation plus deformation energy of the residual system, ρ_0 is level density of the initial system and T_l is the transmission coefficients.

For binary divisions corresponding to the emission of heavier fragment, the decay width is calculated using the transition state formalism of Moretto[31]

$$\Gamma(Z_1, A_1, Z_2, A_2) = \frac{1}{2\pi\rho_0}\int_0^{E^*-E_{\rm sad}(J_0)} \rho_{\rm sad}(U_{\rm sad}, J_0)d\varepsilon, \qquad (4)$$

where $U_{\rm sad}$ and $\rho_{\rm sad}$ are the thermal energy and level density of the conditional saddle-point configuration, respectively,

$$U_{\rm sad} = E^* - E_{\rm sad}(J_0) - \varepsilon, \qquad (5)$$

where $E_{\rm sad}$ is the deformation plus rotation energy of the saddle-point configuration and ε is the kinetic energy of the translational degree of freedom.

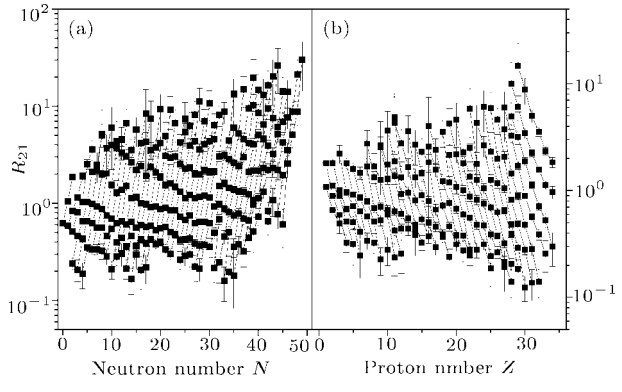

Fig. 1. Isotope yield ratios (a) and isotone yield ratios (b) for a pair of equilibrated sources ($Z = 75$, $A = 150$ and $A = 168$) with excitation energy $E_{ex} = 5$ MeV/nucleon, from left to right represents the isotope sequence from $Z = 1$ to $Z = 41$ (a) and isotone sequence from $N = 1$ to $N = 37$ (b), the error bar comes from only statistics.

The symmetry energy term due to the neutron-proton excess is represented in calculating the binding energy for heavy systems ($Z > 12$), the masses of the initial and residual systems were obtained from the Yukawa-plus-exponential model of Krappe, Nix and Sierk[34] without the shell and pairing correction terms. The parameters for this model were taken from the fit to experimental masses of Möller and Nix.[35] For very light systems ($A \leq 12$), binding energies were calculated from the experimental masses.

In this work a systematic study was made on heavy source pairs. Pairs of equilibrated sources with atomic number $Z = 75$ and mass number $A = 150$, 168, 186 and 204, corresponding to isospin asymmetries $N/Z = 1$, 1.24, 1.48 and 1.72. Excitation energies of sources are chosen from 2 to 6 Mev/nucleon. To avoid possible effects of different magnitudes of Coulomb interaction on isotopic distributions, considered pairs of sources have the same proton number and different mass numbers. In the present calculation all asymmetric divisions are considered for decay model of fission and intermediate mass fragment (IMF).

Equation (1) can be expressed as a function of only atomic number Z when the neutron number N is fixed

$$R_{21}(N) = Y_2(N, Z)/Y_1(N, Z) = C^{'} \exp(\alpha N), \qquad (6)$$

or a function of neutron number N when the atomic number Z is fixed

$$R_{21}(Z) = Y_2(N, Z)/Y_1(N, Z) = C'' \exp(\beta Z). \qquad (7)$$

Here 2 denotes the more neutron-rich compound source and 1 the more neutron-deficient one. For all considered cases we obtain the isoscaling in the sense that $\ln[R_{21}(Z, N)]$ shows an almost perfect linear dependence as a function of N and Z. Figure 1 shows the isotopic and isotonic yield ratios as functions of N and Z for the pair of equilibrated sources ($Z = 75, A = 168$) and ($Z = 75, A = 150$) at excitation energy of 5 Mev/nucleon, the linear behaviour of $\ln[R_{21}]$ as functions of N and Z is excellent, from light isotope (isotone) sequences up to very heavy isotope (isotone) sequences. In present study, if not specified in a different manner, all charged particles produced in GEMINI calculation can be accumulated to perform the ratios, α and β are calculated by linear fitting $\ln[R_{21}(N)]$ as a function of N and $\ln[R_{21}(Z)]$ as a function of Z by Eqs. (6) and (7).

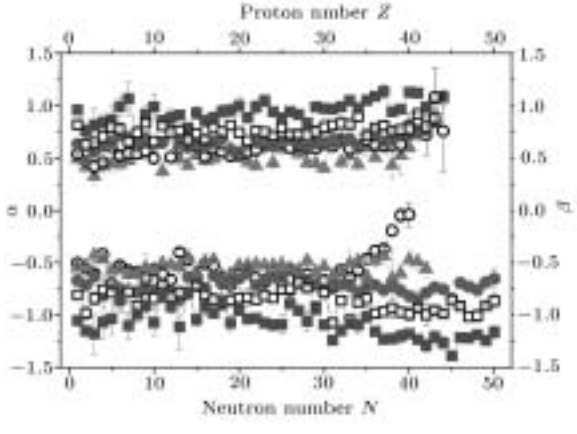

Fig. 2. Isoscaling parameters α (positive) and β (negative) as functions of the atomic number Z and neutron number N for sources ($Z = 75$, $A = 150$) and ($Z = 75$, $A = 168$) at excitation energies $E_{ex} = 2$ (solid square), 3 (open square), 4 (solid circle), 5 (open circle), 6 (solid up-triangle) MeV/nucleon.

Figure 2 shows the extracted α and β versus Z or N for the equilibrated sources ($Z = 75$, $A = 150$) and ($Z = 75$, $A = 168$) at different excitation energies. One can see that both α and β keep the same value in the wide Z or N range though fluctuation can be seen clearly, the exception only presents for β at $E_{ex} = 5\,\mathrm{MeV/nucleon}$ for high N values, the same α and β values over wide Z and N range can make us calculate their average value to study the dependence of α and β on excitation energy. This average value is calculated over produced particles $Z \leq 40$ and $N \leq 42$, which guarantees the averaging calculation on the same fragments range at different excitation energies, except for β at excitation energy 5 MeV/nucleon, in this case, only covers the flat range with $N \leq 32$.

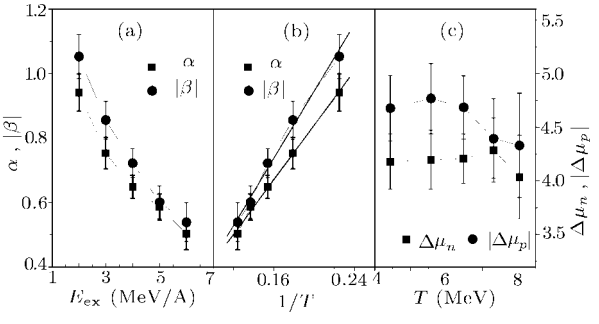

Fig. 3. (a) Isoscaling parameters α and $|\beta|$ as a function of excitation energy from the source yield ratios for source pairs ($Z = 75$, $A = 150$ and $A = 168$). (b) Isoscaling parameters α and $|\beta|$ versus $1/T$. The solid line is the linear fitting of the scattering point. (c) Dependence of neutron or proton chemical potential difference ($\Delta\mu_n$ and $|\Delta\mu_p|$) on temperature T.

GEMINI model predictions for the excitation energy dependence of α and $|\beta|$ from source pairs ($Z = 75$, $A = 150$) and ($Z = 75$, $A = 168$) are shown in Fig. 3(a), α and $|\beta|$ decrease with the increasing excitation energy of the source, which indicates that the isospin dependence of the fragment yields become weaker with the increasing excitation energy. Fermi gas expression $U = a_T T^2$ is used to calculate the system temperature, where U is thermal energy at the saddle-point configuration, which is the effective excitation energy before binary decay.[28] Here a_T is the temperature-dependent level density parameter, which is related to the level spacing at the Fermi surface and so it is also related to the kinetic part of the symmetry energy.[32] Table 1 lists the calculated temperature-dependent level density parameter a_T and temperatures at different excitation energies and systems. Figure 3(b) shows the relationship between α or $|\beta|$ and $1/T$, where α and $|\beta|$ present linear dependences on $1/T$. The slopes between α or $|\beta|$ and $1/T$ are defined as the free neutron or proton chemical potential difference $\alpha = \Delta\mu_n/T$ and $\beta = \Delta\mu_p/T$ in the grand canonical ensemble and some reasonable assumption.[11] The slopes in Fig. 3(b) equal to free neutron or proton chemical potential difference $\Delta\mu_n$ and $|\Delta\mu_p|$ of the equilibrated source pair ($Z = 75$, $A = 150$) and ($Z = 75$, $A = 168$) respectively, which seems to keep constant for different values. To insight into the free neutron or proton chemical potential difference, $\Delta\mu_n$ and $|\Delta\mu_p|$ are calculated as shown in Fig. 3(c), $\Delta\mu_n$ and $|\Delta\mu_p|$ show no significant dependence on temperature T inside the error bar.

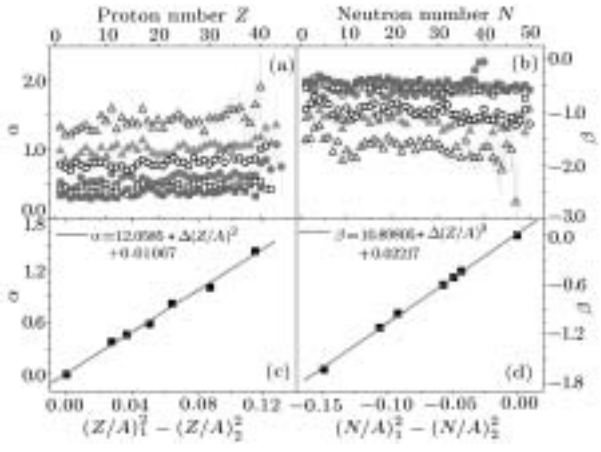

Fig. 4. Characteristics of α (a) and β (b) as functions of products Z and N at excitation energy $E_{ex} = 5\,\mathrm{MeV/nucleon}$ for different pairs of equilibrated source. The source pairs are ($A = 204$, 186) (solid squares), ($A = 186$, 168) (open squares), ($A = 168, 150$) (solid circles), ($A = 204, 168$) (open circles), ($A = 186, 150$) (solid triangles), ($A = 204, 150$) (open triangles). Behaviour of α (c) and β (d) as functions of $\Delta(Z/A)^2$ or $\Delta(N/A)^2$ of the equilibrated sources, the solid lines show the linear fitting of the calculated results.

If the difference of the symmetry energy in two systems gives dominant contribution to the proton and neutron separation energies of the two system Δs_n and Δs_p, then linear relationship between isoscaling parameters α and $\Delta(Z/A)^2$, β and $\Delta(N/A)^2$ can be deduced as[11]

$$\alpha = 4C_{\mathrm{sym}}[(Z/A)_1^2 - (Z/A)_2^2]/T, \quad (8)$$

$$\beta = 4C_{\mathrm{sym}}[(N/A)_2^2 - (N/A)_1^2]/T. \quad (9)$$

Although the freeze-out density and system temperature affect the nuclear symmetry energy, the derived symmetry energy coefficient can constrain both the freeze-out temperature and density of the fragments,[36] where GEMINI code deals with the equilibrated nuclear source at normal nuclear density $\rho_0 = 0.16\,\mathrm{fm}^{-3}$, and determined system temperature in the calculation, the connections between α and $\Delta(Z/A)^2$, β and $\Delta(N/A)^2$ could be used to constrain the symmetry energy coefficient C_{sym}. To explore the origin of the relationships between α and $\Delta(Z/A)^2$, β and $\Delta(N/A)^2$, and to examine the de-

pendence of the system isospin composition, we perform the calculations on four source systems with different N/Z values at fixed $Z = 75$, these sources are ($Z = 75$, $A = 150$, 168, 186, 204) at excitation energy $E_{ex} = 5$ MeV/nucleon. Figures 4(a) and 4(b) show α and β as functions of product Z and N for several pairs of source at excitation energy $E_{ex} = 5$ MeV/nucleon. Figures 4(c) and 4(d) show the performed results of α as a function of $(Z/A)_1^2 - (Z/A)_2^2$ of the source pairs ((a)), and β as a function of $(N/A)_1^2 - (N/A)_2^2$ of the source pairs ((b)), respectively. All system pairs with different $\Delta(Z/A)^2$ or $\Delta(N/A)^2$ lie along the linear line. The linear fits in the figure are $\alpha = 12.0585\Delta(Z/A)^2 + 0.01067$ and $\beta = 10.89805(N/A)^2 + 0.02217$, respectively. The offsets in the two linear functions ($\alpha_0 = 0.01067, \beta_0 = 0.02217$) are very small as compared with α and β, which can be neglected. From Eq. (8) and (9), the slope of the linear fitting in Figs. 4(a) and 4(b) equals to $4C_{\rm sym}/T$, i.e. $4C_{\rm sym}/T|_\alpha$=12.06 and $4C_{\rm sym}/T|_\beta$=10.90.

The calculated level density parameter a_T and temperatures T in the GEMINI simulation for different source systems at excitation energy $E_{ex} = 5$ MeV/nucleon are listed in Table 1. Taking the average temperature $T = 7.31$ MeV, then $4C_{\rm sym}/T|_\alpha = 12.06$ and $4C_{\rm sym}/T|_\beta = 10.90$, we can give a value of $C_{\rm sym}|_\alpha = 22.04$ MeV and $C_{\rm sym}|_\beta = 19.92$ MeV, comparable to the value in the liquid-drop model[33] and in the SMM calculation.[11] Since determination of the symmetry energy coefficient is temperature dependent, temperature determination of the system plays a domination role in the determination of the symmetry energy coefficient.

Table 1. Level density parameters a_T and corresponding temperatures T for different source systems and excitation energies. Here $Z = 75$ is taken for different sources and T (MeV) refers to the saddle-point configuration temperature.

Source	E_{ex} (MeV)	a_T	T (MeV)	$T_{\rm ave}$ (MeV)
$A = 150/168$	2.0	15.32/16.89	4.38/4.50	4.44
	3.0	14.64/16.12	5.53/5.62	5.58
	4.0	14.27/15.76	6.47/6.51	6.49
	5.0	14.09/15.60	7.30/7.34	7.32
	6.0	14.01/15.52	8.03/8.05	8.04
$A = 150$	5.0	14.09	7.30	7.31
$A = 168$	5.0	15.60	7.34	
$A = 186$	5.0	17.11	7.32	
$A = 204$	5.0	18.61	7.28	

In summary, we have calculated the isoscaling parameters α and β using statistical sequential decay model GEMINI, for several pairs of compound nuclei sources. The linear relationships between α or β and $1/T$, $\Delta(Z/A)^2$ and $\Delta(N/A)^2$ have been verified. Free neutron and proton chemical potential difference ($\Delta\mu_n$ and $|\Delta\mu_p|$) looks independent on temperature. Symmetry energy coefficients constrained from Eqs. (8) and (9) give a value comparable to that determined by the liquid-drop model and other models.

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