Feasible Scheme for Teleportation of an Arbitrary N-Atom State with Thermal Cavity *

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We present a scheme for teleportation of an arbitrary N-atom state without Bell state measurement in thermal cavity QED, and show the feasibility in experiment. Our scheme is also insensitive to both cavity decay and thermal field, and the fidelity of teleportation is only slightly affected by the experimental errors. In addition, the success probability reaches 1.0.

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Quantum teleportation has attracted much attention since the first scheme was proposed by Bennet *et al.*[1] The seminal research concerned has wildly started and greatly developed theoretically[2−6] and experimentally.[7,14] Usually, to realize the teleportation, a joint measurement must be operated on the particles the sender possesses, and then the initial particles can be properly reconstructed by the receiver according to the measurement results informed from the receiver. Most schemes are considered with certain entangled resource as their quantum channels, such as Einstein–Podolsky–Rosen (EPR) pair or GHZ triplet as well as combination of EPR pair and GHZ triplet.[2,3] Yan and Ding[4] put forward a scheme for probabilistic teleportation of entangled states with pure entangled state. Cao and Song[5] proposed to teleport an unknown two-particle entangled state via W class state. In Ref. [6], the authors suggested a new way of teleportation with a three-Bell-state analyser, which was soon successfully realized in experiment.[7] Recent years, cavity QED technology has been widely used in the quantum information, such as generation of entangled states[8,9] and teleportation.[10−18] Almeida *et al.*[10] presented a scheme for teleporting a superposition of coherent states from a high-quality cavity to another with a successful probability 0.5. Cao *et al.*[11] presented a scheme for probabilistic teleportation and entanglement purification based on cavity QED. Zheng Guo[12] suggested to realize the logic gates and teleportation by adding a extra classical field, and in this paper the requirement on quality factor of cavities is greatly loosened. Cao and Yang[13] proposed a physical scheme of probabilistic teleportation by using W states as its quantum channel. Zheng[14] showed a method of approximate conditional teleportation of an unknown state without Bell measurement by using cavity as the memory, but the probability of success is only 0.25. Xue *et al.*[15] proposed an experimentally feasible scheme for teleportation of entangled states by entanglement swapping. More recently, Ye and Guo[16] put forward a protocol of teleportation of an unknown atomic state within a vacuum cavity, the joint measurement is also avoided and the probability reaches 0.5. However, it was proven that the scheme of Ye and Guo is sensitive to thermal field.[17] The scheme of Jin *et al.*[18] is insensitive to cavity decay, with a probability 1.0 by using a GHZ state as its quantum channel, but is just limited to teleport a two-atom entangled state.

Inspired by teleportation[1−7] and quantum information processing in cavity QED,[8−18] in this Letter we propose a scheme for teleporting an arbitrary N-atom state without Bell state measurement, and the probability reaches unity. The cavity is only virtually excited and no transfer of quantum information will appear between the atoms and the cavity. During the passage of atoms through the cavity field, a strong classical field is accompanied, thus this scheme is insensitive to both cavity decay and thermal field. By the atom–cavity field interaction, we bring a small amount of ground state atoms into entanglement as the quantum channel, with which an arbitrary N-atom state can be perfectly transmitted. In the following, we show a simpler and more straightforward way for state transfer, in which even the preparation of quantum channels is not needed. In addition, to consider the experimental errors in practice, we show that the fidelity of teleportation is just slightly affected under a loose experimental requirement.

Suppose that Alice wants to transmit a certain two level N-atom state to the receiver Bob. Without loss of generality, it can be written as

$$|\psi\rangle_{1\sim N} = (a_1|gg\ldots gg\rangle + a_2|gg\ldots ge\rangle + a_3|gg\ldots eg\rangle + \cdots + a_{2^N}|ee\ldots ee\rangle)_{1\sim N}, \quad (1)$$

where $a_1, a_2, a_3, \ldots, a_{2^N}$ are unknown coefficients, satisfying $|a_1|^2 + |a_2|^2 + \cdots + |a_{2^N}|^2 = 1$; $|g\rangle$ and $|e\rangle$ are the ground and excited states of the atom, respectively. In addition, there exists $2N$ atoms, which are all initialed in the ground state, namely the quantum state of the whole system reads

$$|\psi\rangle_{1\sim N} \otimes |ggg\ldots g\rangle_{1A\sim NA} \otimes |ggg\ldots g\rangle_{1B\sim NB}. \quad (2)$$

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Now, we send atoms $1A$ and $1B$ into a single-mode cavity.

Here we consider two identical two level atoms simultaneously interacting with a single mode cavity field, and at the same time driven by a classical field. The interaction between the atoms and the cavity can be described by the Hamiltonian[19]

$$H = \omega_0 \sum_{j=1}^{2} S_{z,j} + \omega_a a^\dagger a + \sum_{j=1}^{2} [g(a^\dagger S_j^- + a S_j^\dagger) + \Omega(S_j^\dagger e^{-i\omega t} + S_j^- e^{i\omega t})], \quad (3)$$

where $S_{z,j} = \frac{1}{2}(|e_j\rangle\langle e_j| - |g_j\rangle\langle g_j|)$, $S_j^\dagger = |e_j\rangle\langle g_j|$; $S_j^- = |g\rangle\langle e_j|$; $a^\dagger$ and a are the creation and annihilation operators for the cavity mode; g is the atom–cavity coupling strength; Ω is the Rabi frequency; ω_0 is the atomic transition frequency ($g \leftrightarrow e$); ω_a is the cavity frequency; and ω is the frequency of the classical field. Considering the case of $\omega = \omega_0$, in the large detuning $\delta \gg g/2$ and the strong driving regime $\Omega/2 \gg \delta, g$ (δ is the detuning between the atomic transition frequency and the cavity field frequency), there will exist no energy exchange between the atomic system and the cavity. In the interaction picture, the evolution operator of the system is[20]

$$U(t) = e^{-iH_0 t} e^{-iH_e t}, \quad (4)$$

where $H_0 = \Sigma_{j=1}^{2} \Omega(S_j^\dagger + S_j^-)$, and H_e is the effective Hamiltonian, which is independent of the cavity field state, in the form

$$H_e = \frac{\chi}{2}\Big[\sum_{j=1}^{2}(|e_j\rangle\langle e_j| + |g_j\rangle\langle g_j|) + \sum_{j,k=1,j\neq k}^{2}(S_j^\dagger S_k^\dagger + S_j^\dagger S_k^- + \text{H.c.})\Big] \quad (5)$$

where $\chi = g^2/2\delta$.

After an interaction time t, the two ground state atoms $1A$ and $1B$ evolve into

$$\begin{aligned}|g\rangle_{1A}|g\rangle_{1B} \to e^{-i\chi t}[&\cos\chi t(\cos\Omega t|g\rangle_{1A} \\ &- i\sin\Omega t|e\rangle_{1A})(\cos\Omega t|g\rangle_{1B} \\ &- i\sin\Omega t|e\rangle_{1B}) - i\sin\chi t(\cos\Omega t|e\rangle_{1A} \\ &- i\sin\Omega t|g\rangle_{1A})(\cos\Omega t|e\rangle_{1B} \\ &- i\sin\Omega t|g\rangle_{1B})]. \end{aligned} \quad (6)$$

We choose the interaction time and Rabi frequency appropriately so that $\Omega t = \pi$ and $\chi t = \pi/4$, then we obtain a maximally two-atom entangled state, the Bell state,

$$\frac{1}{\sqrt{2}}(|g\rangle_{1A}|g\rangle_{1B} - i|e\rangle_{1A}|e\rangle_{1B}). \quad (7)$$

Continuing to send atoms $(jA, jB)_{j=1,2,3,\ldots,N}$ into the single mode cavity simultaneously, and at the same time the atoms driven by a classical field also with the choices of $\Omega t = \pi$ and $\chi t = \pi/4$, we can bring these $2N$ ground state atoms into N EPR pairs. Denote the EPR pair generated from particles (jA, jB) by

$$|\phi\rangle_j = \frac{1}{\sqrt{2}}(|g\rangle_{jA}|g\rangle_{jB} - i|e\rangle_{jA}|e\rangle_{jB}). \quad (8)$$

Then the whole system shown in Eq. (2) will become

$$|\varphi\rangle = |\psi\rangle_{1\sim N} \otimes (\otimes_{j=1}^{N}|\phi\rangle_j) \quad (9)$$

which can also be written in the form

$$\begin{aligned}|\varphi\rangle = &\frac{1}{\sqrt{2}}(|g\rangle_j \otimes |\alpha\rangle + |e\rangle_j \otimes |\beta\rangle) \\ &\otimes (|g\rangle_{1A}|g\rangle_{jA} - i|e\rangle_{1A}|e\rangle_{jA}) \\ &\otimes (\otimes_{k=1}^{j-1}|\phi\rangle_k) \otimes (\otimes_{k=j+1}^{N}|\phi\rangle_k) \end{aligned} \quad (10)$$

with the notation

$$\begin{aligned}|\alpha\rangle = &(a_1|gg\ldots gg\rangle + a_2|gg\ldots ge\rangle + \cdots \\ &+ a_{2^N-2^{N-j}}|ee\ldots ee\rangle)_{1\sim j-1,j+1\sim N}, \\ |\beta\rangle = &(a_{2^N-j+1}|gg\ldots gg\rangle + a_{2^N-j+2}|gg\ldots ge\rangle + \cdots \\ &+ a_{2^N}|ee\ldots ee\rangle)_{1\sim j-1,j+1\sim N}. \end{aligned} \quad (11)$$

If atoms $(1, 2, 3, \ldots, N)$ and $(1A, 2A, 3A, \ldots, NA)$ belong to Alice, and the other atoms $(1B, 2B, 3B, \ldots, NB)$ belong to the receiver Bob. Then Alice let atom j and atom jA simultaneously interact with the single mode thermal cavity and be also driven by a classical field at the same time. After an interaction time τ, the evolution of the whole system shown in Eq. (10) is given by

$$\begin{aligned}|\varphi\rangle \to \frac{1}{\sqrt{2}}e^{-i\chi\tau}\{&|\alpha\rangle[\cos\chi\tau(\cos\Omega\tau|g\rangle_j - i\sin\Omega\tau|e\rangle_j) \\ &\cdot(\cos\Omega\tau|g\rangle_{jA} - i\sin\Omega\tau|e\rangle_{jA}) \\ &- i\sin\chi\tau(\cos\Omega\tau|e\rangle_j - i\sin\Omega\tau|g\rangle_j) \\ &\cdot(\cos\Omega\tau|e\rangle_{jA} - i\sin\Omega\tau|g\rangle_{jA})]|g\rangle_{jB} \\ &- i|\alpha\rangle[\cos\chi\tau(\cos\Omega\tau|g\rangle_j - i\sin\Omega\tau|e\rangle_j) \\ &\cdot(\cos\Omega\tau|e\rangle_{jA} - i\sin\Omega\tau|g\rangle_{jA}) \\ &- i\sin\chi\tau(\cos\Omega\tau|e\rangle_j - i\sin\Omega\tau|g\rangle_j) \\ &\cdot(\cos\Omega\tau|g\rangle_{jA} - i\sin\Omega\tau|e\rangle_{jA})]|e\rangle_{jB} \\ &+ |\beta\rangle[\cos\chi\tau(\cos\Omega\tau|e\rangle_j - i\sin\Omega\tau|g\rangle_j) \\ &\cdot(\cos\Omega\tau|g\rangle_{jA} - i\sin\Omega\tau|e\rangle_{jA}) \\ &- i\sin\chi\tau(\cos\Omega\tau|g\rangle_j - i\sin\Omega\tau|e\rangle_j) \\ &\cdot(\cos\Omega\tau|e\rangle_{jA} - i\sin\Omega\tau|g\rangle_{jA})]|g\rangle_{jB} \\ &- i|\beta\rangle[\cos\chi\tau(\cos\Omega\tau|e\rangle_j - i\sin\Omega\tau|g\rangle_j) \\ &\cdot(\cos\Omega\tau|e\rangle_{jA} - i\sin\Omega\tau|g\rangle_{jA}) \\ &- i\sin\chi\tau(\cos\Omega\tau|g\rangle_j - i\sin\Omega\tau|e\rangle_j) \\ &\cdot(\cos\Omega\tau|g\rangle_{jA} - i\sin\Omega\tau|e\rangle_{jA})]|e\rangle_{jB}\} \\ &\otimes(\otimes_{k=1}^{j-1}|\phi\rangle_k) \otimes (\otimes_{k=j+1}^{N}|\phi\rangle_k). \end{aligned} \quad (12)$$

We realize the condition $\Omega\tau = \pi$ and $\chi\tau = \pi/4$, Eq. (12) will become

$$\begin{aligned}|\varphi\rangle \to \frac{1}{2}[&|\alpha\rangle(|g\rangle_j|g\rangle_{jA} - i|e\rangle_j|e\rangle_{jA})|g\rangle_{jB} \\ &- i|\alpha\rangle(|g\rangle_j|e\rangle_{jA} - i|e\rangle_j|g\rangle_{jA})|e\rangle_{jB} \\ &+ |\beta\rangle(|e\rangle_j|g\rangle_{jA} - i|g\rangle_j|e\rangle_{jA})|g\rangle_{jB} \end{aligned}$$

$$
\begin{aligned}
&- i|\beta\rangle(|e\rangle_j|e\rangle_{jA} - i|g\rangle_j|g\rangle_{jA})|e\rangle_{jB}] \\
&\otimes(\otimes_{k=1}^{j-1}|\phi\rangle_k)\otimes(\otimes_{k=j+1}^{N}|\phi\rangle_k) \\
=&\frac{1}{2}[|g\rangle_j|g\rangle_{jA}(|g\rangle_{jB}|\alpha\rangle - |e\rangle_{jB}|\beta\rangle) \\
&- i|g\rangle_j|e\rangle_{jA}(|e\rangle_{jB}|\alpha\rangle + |g\rangle_{jB}|\beta\rangle) \\
&- |e\rangle_j|g\rangle_{jA}(|e\rangle_{jB}|\alpha\rangle - |g\rangle_{jB}|\beta\rangle) \\
&- i|e\rangle_j|e\rangle_{jA}(|g\rangle_{jB}|\alpha\rangle + |e\rangle_{jB}|\beta\rangle)] \\
&\otimes(\otimes_{k=1}^{j-1}|\phi\rangle_k)\otimes(\otimes_{k=j+1}^{N}|\phi\rangle_k). \qquad (13)
\end{aligned}
$$

Then if Alice detect the atoms j and jA, we can see from Eq. (13) that the remaining particles of the system would collapse into a corresponding quantum state. For the results $|g\rangle_j|g\rangle_{jA}$, $|g\rangle_j|e\rangle_{jA}$ and $|e\rangle_j|g\rangle_{jA}$, up to a simple local unitary transformation σ_z, σ_x and σ_y on the atom jB respectively, we can obtain $|g\rangle_{jB}\otimes|\alpha\rangle + |e\rangle_{jB}\otimes|\beta\rangle$, and for the result $|e\rangle_j|e\rangle_{jA}$, Bob needs to do nothing, the remaining particles is just projected into $|g\rangle_{jB}\otimes|\alpha\rangle + |e\rangle_{jB}\otimes|\beta\rangle$, which is equivalent to the original N-atom state $|\psi\rangle_{1\sim N}$, that is to say, the atom j can be replaced by the atom jB essentially after operation on the atom jB.

Similarly, Alice repeat this procedure on all the atoms $(j, jA)_{j=1,2,3,\ldots,N}$ and inform Bob the measurement results, and then Bob needs to perform a corresponding Pauli operation on jB, the state of particles $1B\sim NB$ is reconstructed to

$$
\begin{aligned}
|\psi\rangle_{1B\sim NB} =&(a_1|gg\ldots gg\rangle + a_2|gg\ldots ge\rangle \\
&+ a_3|gg\ldots eg\rangle + \cdots \\
&+ a_{2^N}|ee\ldots ee\rangle)_{1B\sim NB}, \qquad (14)
\end{aligned}
$$

which is identical to $|\psi\rangle_{1\sim N}$. Now the information is thereby successfully teleported from the sender to the receiver, with a successful probability 1.0.

Particularly, as an example, we try to teleport a three-particle cat-like state supposed by $|\psi\rangle_{1\sim 3} = a|ggg\rangle_{1\sim 3} + b|eee\rangle_{1\sim 3}$, where a and b are unknown coefficients satisfying $|a|^2 + |b|^2 = 1$. Assume Alice and Bob to share three EPR pairs generated by the way discussed above. The whole system is given by

$$
|\varphi\rangle = |\psi\rangle_{1\sim 3}\otimes[\otimes_{j=1}^{3}\frac{1}{\sqrt{2}}(|gg\rangle - i|ee\rangle)_{jA,jB}]. \qquad (15)
$$

Then Alice has the atoms $(1, 1A), (2, 2A), (3, 3A)$ simultaneously cross a single-mode thermal cavity respectively, and at the same time these atoms are driven by a classical field with an appropriate interaction time such that $\chi\tau = \pi/4$ and $\Omega\tau = \pi$, the system evolves to

$$
\begin{aligned}
|\varphi\rangle \to \frac{1}{8}[&\Psi^{\pm}_{1,1A}\Psi^{\pm}_{2,2A}\Psi^{\pm}_{3,3A}(a|ggg\rangle \pm\pm\pm b|eee\rangle)_{1B\sim 3B} \\
&+ \Psi^{\pm}_{1,1A}\Psi^{\pm}_{2,2A}\Phi^{\pm}_{3,3A}(a|gge\rangle \pm\pm\pm b|eeg\rangle)_{1B\sim 3B} \\
&+ \Psi^{\pm}_{1,1A}\Phi^{\pm}_{2,2A}\Psi^{\pm}_{3,3A}(a|geg\rangle \pm\pm\pm b|ege\rangle)_{1B\sim 3B} \\
&+ \Psi^{\pm}_{1,1A}\Phi^{\pm}_{2,2A}\Phi^{\pm}_{3,3A}(a|gee\rangle \pm\pm\pm b|egg\rangle)_{1B\sim 3B} \\
&+ \Phi^{\pm}_{1,1A}\Psi^{\pm}_{2,2A}\Psi^{\pm}_{3,3A}(a|egg\rangle \pm\pm\pm b|gee\rangle)_{1B\sim 3B} \\
&+ \Phi^{\pm}_{1,1A}\Psi^{\pm}_{2,2A}\Phi^{\pm}_{3,3A}(a|ege\rangle \pm\pm\pm b|geg\rangle)_{1B\sim 3B} \\
&+ \Phi^{\pm}_{1,1A}\Phi^{\pm}_{2,2A}\Psi^{\pm}_{3,3A}(a|eeg\rangle \pm\pm\pm b|gge\rangle)_{1B\sim 3B} \\
&+ \Phi^{\pm}_{1,1A}\Phi^{\pm}_{2,2A}\Phi^{\pm}_{3,3A}(a|eee\rangle \pm\pm\pm b|ggg\rangle)_{1B\sim 3B}], \qquad (16)
\end{aligned}
$$

where $\Psi^{+}_{j,jA}$, $\Psi^{-}_{j,jA}$, $\Phi^{+}_{j,jA}$ and $\Phi^{-}_{j,jA}$ $(j = 1, 2, 3)$ symbolize the four possible measurement results of atoms j and jA, denoted by

$$
\begin{aligned}
&\Psi^{+}_{j,jA} = -i|ee\rangle_{j,jA}, \quad \Psi^{-}_{j,jA} = |gg\rangle_{j,jA}, \\
&\Phi^{+}_{j,jA} = -i|ge\rangle_{j,jA}, \quad \Phi^{-}_{j,jA} = -|eg\rangle_{j,jA}. \qquad (17)
\end{aligned}
$$

If the results of measurement are $|gg\rangle_{1,1A}$, $|eg\rangle_{2,2A}$ and $|ee\rangle_{3,3A}$, we can know from Eq. (16), that the remaining particles would collapse into a certain state, to perform a corresponding transformation on the atoms $1B$, $2B$ and $3B$ respectively, then Bob will obtain

$$
\begin{aligned}
|\psi\rangle_{1B\sim 3B} =&\sigma_z^{(1B)}\sigma_y^{(2B)}\mathbf{1}^{(3B)}i(a|geg\rangle + b|ege\rangle)_{1B\sim 3B} \\
=&a|ggg\rangle_{1B,2B,3B} + b|eee\rangle_{1B,2B,3B}. \qquad (18)
\end{aligned}
$$

Thereby, the teleportation of an unknown three-particle cat-like state is successfully realized.

Stimulated by the approach of teleportation suggested above, here we put forward a more straightforward way for perfect-state transfer with thermal cavity, in which even the preparation of quantum channels is also avoided.

Suppose that an arbitrary two-level N-atom state $|\psi\rangle_{1\sim N}$ is required to be transferred to another N atoms $1' \sim N'$. Firstly, all the initial atoms $1' \sim N'$ are located in the ground state, then we let the atoms j and j' simultaneously interact with the cavity and the companied classical field. After an appropriate interaction time τ' that satisfies the condition $\Omega\tau' = \pi$ and $\chi\tau' = \pi/4$, the evolution of the whole system $|\varphi\rangle$ composed of the arbitrary state and N ground state atoms is given by

$$
\begin{aligned}
|\varphi\rangle =&(|g\rangle_j\otimes|\alpha\rangle + |e\rangle_j\otimes|\beta\rangle)\otimes|g\ldots gg\rangle_{1'\sim j'} \\
\to&[|g\rangle_j(|g\rangle_{j'}\otimes|\alpha\rangle - i|e\rangle_{j'}\otimes|\beta\rangle) \\
&- i|e\rangle_j(|e\rangle_{j'}\otimes|\alpha\rangle + i|g\rangle_{j'}\otimes|\beta\rangle)] \\
&\otimes|g\ldots gg\rangle_{1'\sim(j-1)',(j+1)'\sim N'}. \qquad (19)
\end{aligned}
$$

Obviously, only measuring the atom j and taking a corresponding transformation on the particle j', we can obtain $|g\rangle_{j'}\otimes|\alpha\rangle + |e\rangle_{j'}\otimes|\beta\rangle = |\psi\rangle_{1\sim N}$, which indicates that the atom j' can replace the atom j in this way. Continuing this procedure on all the atoms $(j, j')_{j=1,2,\ldots,N}$, then we can see that an arbitrary N-atom state is perfectly transferred from the particles $1 \sim N$ to the atoms $1' \sim N'$, and furthermore the steps of atom–cavity–field interaction and single-qubit measurements have been reduced by half compared with the way of teleportation suggested above.

Now we give a brief discussion on the experiment. For Rydberg atoms, the radiation time is about $T_r = 3\times 10^{-2}$s, and the coupling constant is $g = 2\pi\times 24$ kHz. With the choice of $\delta = 10g$, the required atom–thermal–cavity–field interaction time is

on the order of $T \approx 10^{-4}$ s. Then the time required to complete the whole procedure is much shorter than T_r. Meanwhile it is noted that the atomic state evolution in the cavity is independent of the cavity field state, thus the cavity decay will not affect the generation of EPR pairs and the teleportation of quantum state.

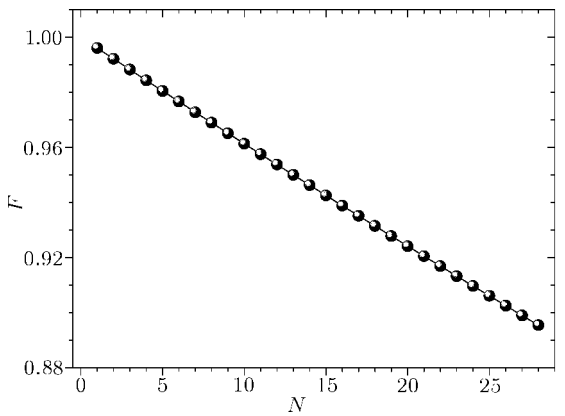


Fig. 1. Fidelity of teleportation F as a function of N ($N < 30$), with the choices of $\Omega t = \pi$ and $\chi t = \chi\tau = \pi/4$ for $\Delta t = 0.01t$, $\Delta\tau = 0.01\tau$ and $\Delta\Omega = 0.01\Omega$.

One of the difficulties for the present scheme is that two atoms are required to be simultaneously sent through the cavity, otherwise there will appear an error. In addition, the fluctuation of the Rabi frequency will also bring about an error. Considering these experimental errors in practice, here we give an estimation of the fidelity of teleporting an arbitrary N-atom state. When we send two ground-state atoms into the cavity to prepare EPR pairs, we can assume that one atom is earlier to enter into the cavity than the other with time Δt ($\Delta t \ll t$) on average. In the teleportation procedure, during the passage of two atoms through the cavity, we can also suppose that one atom is earlier to enter into the cavity than the other with time $\Delta\tau$ ($\Delta\tau \ll \tau$) on average, and the fluctuation of the Rabi frequency is assumed to be $\Delta\Omega$ ($\Delta\Omega \ll \Omega$). Then we find that the fidelity of teleportation is not only dependent on the interaction time delay Δt, $\Delta\tau$ and the fluctuation of Rabi frequency $\Delta\Omega$, but also is related to the number of particles to be teleported and even the measurement results. To consider the worst situation, if the measurement results all are $|ee\rangle_{j,jA}$ ($j = 1, 2, \ldots, N$), then the fidelity would be at a minimum described by such a relationship

$$\begin{aligned} F \gtrsim \Big\{ & 1 + [\tan^2(\Omega - \Delta\Omega)(t - \Delta t) \\ & + \tan^2(\Omega - \Delta\Omega)(\tau - \Delta\tau)] \\ & \cdot \left[\sin^2\chi(t - \Delta t)\cos^2\chi(\tau - \Delta\tau) \right. \\ & \left. + \cos^2\chi(t - \Delta t)\sin^2\chi(\tau - \Delta\tau)\right]^{-1} \Big\}^{-N}, \end{aligned} \tag{20}$$

where we assume $\Delta t = 0.01t$, $\Delta\tau = 0.01\tau$ and $\Delta\Omega = 0.01\Omega$ as an example, and show the fidelity F with different particle numbers N ($N < 30$) in Fig. 1, where it is shown that the fidelity decrease caused by these experimental errors is less than 0.1, which indicates that our scheme is slightly affected in these cases. For the way of state transfer proposed in this study, the affection caused by experimental errors is much less because of a great reduction of interaction with the cavity, thus its fidelity would be even higher. For simplicity, here we have omitted the errors caused by other experimental causes.

In summary, we have proposed a simple and feasible scheme to realize the teleportation of an arbitrary N-atom state with thermal cavity QED. This scheme dose not require any auxiliary levels or classical fields for implementing Control-NOT gate and Hadamard transformation to discriminate the Bell state, which can largely reduce the experimental steps and time. In addition, the insensitivity to the cavity decay and thermal field will also contribute to fidelity in practice. In addition, note that in this study, all the atom-field interactions are relatively independent, and the measurements are of no order as well, namely, it can be performed simultaneously, which will largely enhance the efficiency of teleportation. In the research of quantum communication and quantum computation based on a network, it is always demanded to transmit some multi-atom states, but not only limited to a two-atom state[5,18] or some especial entangled states, such as cat-like state. Therefore, an efficient teleportation of a multi-atom arbitrary state with a high fidelity is practically useful for the quantum communication and quantum computation.

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