Generation of Multipartite Ionic Entangled States only by Single-Qubit Measurements via Linear Optical Elements *

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We propose a scheme for generation of multipartite ionic Greenberger–Horne–Zeilinger (GHZ) states only by single-qubit measurements. Our scheme not only does not need joint measurements but also avoids the difficulty of synchronizing the arrival time of the two scattered photons, which is faced by the previous schemes. Therefore our entanglement generation scheme can be implemented more easily than the schemes based on atomic interference in experiment.

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Entanglement is a unique phenomena in quantum mechanics because of its nonlocality. Due to its great advantage, entangled states have been regarded as a universal resource in quantum communication and quantum computation. Then some interesting applications of entanglement have been proposed, such as quantum teleportation,[1] quantum dense coding,[2] and quantum cryptography.[3] Bipartite entangled states and tripartite entangled states (W state and GHZ state) have been known well by scientists. Some of particular advantages of them are shown in practice.[4−6] Meanwhile many theoretical and experimental schemes for generation of them have been proposed in cavity QED,[7,8] ion trap systems,[9] and NMR systems.[10,11] Up to the recent date research outcome, the study of multipartite entangled states is important and necessary for in-depth understanding and extending quantum mechanics, realizing quantum communication and quantum computation, because the multipartite entanglement can violate local realism more intensively than bipartite entanglement,[12] can store more information and can improve the noise limit of tolerant-error in quantum cryptography.[13,14] Thus the multipartite entangled states and their applications attract many eyes in scientific community.

Recently some schemes for generation of multipartite entangled states have been proposed, such as cluster states,[15−20] W states,[21−23] and GHZ states.[24,25] The GHZ states prepared in previous papers are photonic state or atomic state. In this Letter, we propose a physical scheme for generating the multipartite ionic entangled GHZ states only by single-qubit measurements based on linear optical elements. For general atomic schemes based on atomic interference, we must detect the two photons after Bell state analyser (BSA) or polarization beam splitter (PBS), two special separated atoms will be left in entangled state after the photon detection. The schemes of this kind can entangle the spatially separated atoms. However, there is still a serious difficulty in practice, i.e. the two photons must reach the BSA or PBS simultaneously in these schemes. In order to overcome the difficulty, we adopt a Mach–Zehnder interferometer (MZI) in our generation setup. The MZI is made of two ordinary identical nonpolarizing 50/50 beam splitters (BS) and two holophotes. The setup can eliminate the relative phase problem inherent in the previous schemes. One incident circular polarized photon will be used in our scheme, so the photon detection will be easier than the scattered one used in the previous schemes. It is not easy to make the two photons from two different ions interfere in the previous schemes. In our scheme, the photon wave function of the incident circular polarized photon has been split into two parts in MZI, and the coherent condition is satisfied naturally. Furthermore, we can enhance the scattering rate of photon and the successful probability of entanglement generation by using an optical cavity[26,27] surrounding one MZI.

We first investigate the generation of tripartite ionic GHZ state using MZIs and other optical elements. Multilevel identical ions are used, whose level configuration is shown in Fig. 1. Here $|m_+\rangle$, $|m_-\rangle$ are the two metastable states, which are used to store quantum information. State of ions $|m_+\rangle$ ($|m_-\rangle$) can be excited into the excited state $|e\rangle$ by absorbing one σ^+ (σ^-) circular polarized photon with unit efficiency, and then they decay to the ground state $|g\rangle$ and scatter a photon $|s\rangle$ rapidly. Here we assume that the de-

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cay process is so rapid that the probability of excited emission of the upper state can be neglected. Thus the process can be expressed as $a^{\pm}|0\rangle|m_{\pm}\rangle \longrightarrow |s\rangle|g\rangle$, where $\pm$ denotes the direction of polarization. The setup for generation of a tripartite ionic GHZ state is shown in Fig. 2, which is composed of two MZIs and four photon detectors.

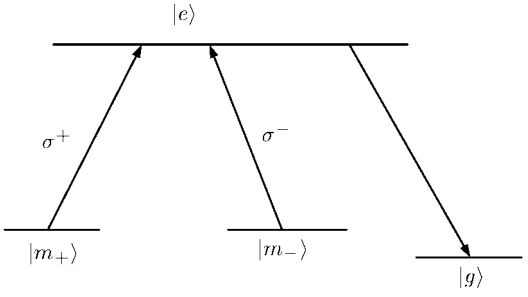


Fig. 1. Level configuration of the ions used in this study. Here $|g\rangle$ is the ground state, $|e\rangle$ is the excited state; $|m_+\rangle$, $|m_-\rangle$ are the two metastable states, which can be excited into the excited state $|e\rangle$ by absorbing one σ^+ and σ^- circular polarized photon, respectively, and then they will decay to the ground state $|g\rangle$ and scatter a photon rapidly.

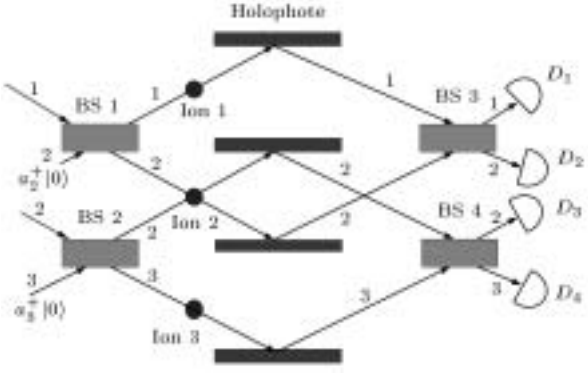


Fig. 2. The setup for generation of tripartite ionic GHZ state. BS denotes nonpolarizing 50/50 beam splitter. D_i $(i = 1, 2, 3, 4)$ are polarization sensitive single photon detectors. Here 1, 2, and 3 denote optical paths. Ions 1, 2, 3 locate on path 1, 2, 3, respectively. $a_{2,+}^+|0\rangle$ and $a_{3,+}^+|0\rangle$ denote two input photons.

In order to generate a tripartite ionic GHZ state, we first prepare three identical ions 1, 2 and 3 in the states

$$|\phi\rangle_i = \frac{1}{\sqrt{2}}(|m_+\rangle_i + |m_-\rangle_i), \quad (i = 1, 2, 3), \tag{1}$$

by a laser pulse focused on the ions, respectively. Then we adjust the MZIs such that the detections 1 and 3 register photons with certainty, and then place the three ions on the arms of the two MZIs as in Fig. 2 by trapping techniques.[28] We let σ^+ polarized photon enter BS1 from the left port. The BS 1 take no effect on the polarization of the input photons and reflect the wave function with $\pm\frac{\pi}{2}$ phase shift corresponding to the propagation direction of photons.[29] The process can be expressed as

$$a_{2,+}^+|0\rangle \longrightarrow \frac{1}{\sqrt{2}}(a_{1,+}^+ + ia_{2,+}^+)|0\rangle, \tag{2}$$

where $a_{2,+}^+|0\rangle$ denotes a input photon and takes two possible paths after BS 1. After σ^+ polarized photon interacting with atoms 1 and 2 and through the up MZI, the state of ions 1 and 2 will evolve as

$$\begin{aligned}
&\frac{1}{2}a_{2,+}^+|0\rangle(|m_+\rangle_1 + |m_-\rangle_1)(|m_+\rangle_2 + |m_-\rangle_2) \\
&\quad \longrightarrow -\frac{1}{4}(|m_+\rangle_1|m_-\rangle_2 - |m_-\rangle_1|m_+\rangle_2)a_{2,+}^+|0\rangle \\
&\quad + \frac{i}{4}(|m_+\rangle_1|m_-\rangle_2 + |m_-\rangle_1|m_+\rangle_2 \\
&\quad + 2|m_-\rangle_1|m_-\rangle_2)a_{1,+}^+|0\rangle + |S\rangle_1,
\end{aligned} \tag{3}$$

where $|S\rangle_1 = \frac{1}{2\sqrt{2}}[|g\rangle_1|s\rangle_1(|m_+\rangle_2 + |m_-\rangle_2) + i|g\rangle_2|s\rangle_2(|m_+\rangle_1 + |m_-\rangle_1)]$ denotes the state of the scatter photons. From the evolvement, we can know that if D_2 fires, we will obtain a bipartite maximally entangled state,i.e. $(|m_+\rangle_1|m_-\rangle_2 - |m_-\rangle_1|m_+\rangle_2)/\sqrt{2}$. The successful probability is 0.125. Otherwise, this step is failed and the state of ions 1 and 2 will be $(|m_+\rangle_1|m_-\rangle_2 + |m_-\rangle_1|m_+\rangle_2 + 2|m_-\rangle_1|m_-\rangle_2)/\sqrt{6}$. However, we can let another identical σ^+ polarized photon enter BS1, the remain state will evolve as

$$\begin{aligned}
&\frac{1}{\sqrt{6}}a_{2,+}^+|0\rangle(|m_+\rangle_1|m_-\rangle_2)(|m_-\rangle_1|m_+\rangle_2 + 2|m_-\rangle_1|m_-\rangle_2) \\
&\quad \longrightarrow -\frac{1}{2\sqrt{6}}(|m_+\rangle_1|m_-\rangle_2 - |m_-\rangle_1|m_+\rangle_2)a_{2,+}^+|0\rangle \\
&\quad + \frac{i}{2\sqrt{6}}(|m_+\rangle_1|m_-\rangle_2 + |m_-\rangle_1|m_+\rangle_2 \\
&\quad + 4|m_-\rangle_1|m_-\rangle_2)a_{1,+}^+|0\rangle + |S\rangle_a,
\end{aligned} \tag{4}$$

where $|S\rangle_a = \frac{1}{2\sqrt{3}}(|g\rangle_1|s\rangle_1|m_-\rangle_2 + i|g\rangle_2|s\rangle_2|m_-\rangle_1)$. If D_2 fires, this step will be successful with probability 0.083. Similarly, if it is failed, we can let another identical σ^+ polarized photon enter BS1 again and repeat the above process until D_2 fires. The total successful probability is $0.125 + 0.083 + 0.031 + \cdots \simeq 0.24$.

Then we let the second σ^+ polarized photon enter BS 2 from the left port. After the photon interacting with ions 2 and 3, and through the down MZI, the total state of ions 1, 2 and 3 becomes

$$\begin{aligned}
&\frac{1}{2}a_{3,+}^+|0\rangle(|m_+\rangle_1|m_-\rangle_2 - |m_-\rangle_1|m_+\rangle_2)(|m_+\rangle_3 + |m_-\rangle_3) \\
&\quad \longrightarrow \frac{1}{4}(|m_-\rangle_1|m_+\rangle_2|m_-\rangle_3 \\
&\quad + |m_+\rangle_1|m_-\rangle_2|m_+\rangle_3)a_{3,+}^+|0\rangle \\
&\quad + \frac{i}{4}(2|m_+\rangle_1|m_-\rangle_2|m_-\rangle_3 - |m_-\rangle_1|m_+\rangle_2|m_-\rangle_3 \\
&\quad + |m_+\rangle_1|m_-\rangle_2|m_+\rangle_3)a_{2,+}^+|0\rangle + |S\rangle_2,
\end{aligned} \tag{5}$$

where $|S\rangle_2 = \frac{1}{2\sqrt{2}}[-|g\rangle_2|s\rangle_2|m_-\rangle_1(|m_+\rangle_3 + |m_-\rangle_3) + i|g\rangle_3|s\rangle_3(|m_+\rangle_1|m_-\rangle_2 - |m_-\rangle_1|m_+\rangle_2)]$. Obviously, we can attain the tripartite entangled state of ions 1, 2 and 3

$$|\psi\rangle_{123} = \frac{1}{\sqrt{2}}(|m_-\rangle_1|m_+\rangle_2|m_-\rangle_3 + |m_+\rangle_1|m_-\rangle_2|m_+\rangle_3), \tag{6}$$

if photon detector D_4 fires. Otherwise, we should also repeat the second process as stated above. The successful probability of this step is also about 0.24. Finally, we perform a single-qubit rotation on ion 2 by using laser pulses on it, the standard tripartite ionic GHZ state can be generated, i.e.

$$|\psi\rangle_{123} = \frac{1}{\sqrt{2}}(|m_+\rangle_1|m_+\rangle_2|m_+\rangle_3 + |m_-\rangle_1|m_-\rangle_2|m_-\rangle_3). \tag{7}$$

Thus the total successful probability of generating the tripartite ionic GHZ state is about $0.24^2 = 0.0576$.

Similarly, we can generate the multipartite ($N \geq 3$) ionic GHZ states by the expanded setup of Fig. 2. The scheme requires $N-1$ MZIs and $2(N-1)$ detectors. Beam splitters 1 to $N-1$ place on the left, beam splitters N to $2N-2$ on the right. Detectors 1 to $2N-2$ place on the right from top to bottom. All of the ions (from 1 to N) place on the same location of left arms of MZIs from top to bottom, and they are all prepared in the states $|\phi\rangle_i = (|m_+\rangle_i + |m_-\rangle_i)/\sqrt{2}, (i = 1, 2, \ldots, N)$. Then we let one σ_1^+ polarized photon enter BS1 from the left port. After D_2 fired, we let another σ^+ polarized photon enter BS2 from the left port. After D_4 fired, we let another σ^+ polarized photon enter BS3 from the left port. Up to D_{2N-4} fired, we let the last σ^+ polarized photon enter BS N-1 from the left port. If the D_{2N-2} fires, we perform single-qubit rotations on ions, the standard N-partite ionic GHZ state can be generated, which can be expressed as

$$|\psi\rangle_{1,2,\ldots,N} = \frac{1}{\sqrt{2}}\Big(\prod_{i=1}^{N}|m_+\rangle_i + \prod_{i=1}^{N}|m_-\rangle_i\Big). \tag{8}$$

The total successful probability of generating the multi-ion entangled state is $(0.24)^{N-1}$.

We briefly discuss the feasibility of current scheme in the following. In quantum information experiment, we usually use the singly positively charged alkaline ions as trapped ions,[30] which have only one electron outside a closed shell. Here we choose $^{40}Ca^+$ as an example to discuss our generation scheme. The relevant levels of $^{40}Ca^+$ have been shown in Fig. 3, where e, S_1, S_2 and $S_{1/2}$ correspond to $|e\rangle$, $|m_-\rangle$, $|m_+\rangle$ and $|g\rangle$ in Fig. 1, respectively. That is to say, we take $S_{1/2}$ as the ground state, $|e\rangle$ as the excited state and S_1, S_2 as the two generate metastable states. The arbitrary superposition state of the two metastable states can be prepared by applying a laser pulse of appropriate length with a few microseconds.[32] The detection of the internal state of $^{40}Ca^+$ can be realized by using a cycling transition between $S_{1/2}$ and $P_{1/2}(397\,\mathrm{nm})$. Note that the level of ions used in our study is suitable for practical experiment.

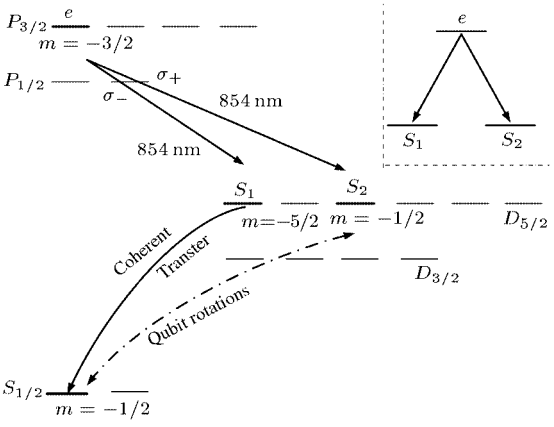


Fig. 3. The relevant levels of $^{40}Ca^+$. $D_{5/2}$ and $D_{3/2}$ are the two metastable states of $^{40}Ca^+$ with lifetimes in the order of 1 s. S_1 and S_2 are the two sublevels of $D_{5/2}$ with $m = -5/2$ and $m = -3/2$, which can be coupled to e by σ^+ and σ^- light at 854 nm. Then decays from e to S_1, S_2, $D_{3/2}$ and $S_{1/2}$ are all possible. However, the branching ratio for $P_{3/2} \to D_{5/2}$ (854 nm) versus $P_{3/2} \to S_{1/2}$ (393 nm) can be estimated to be 1 : 30, the probability is $0.5 \times 10^{-7}\mathrm{s}^{-1}$.[31] Thus, in most cases, e will decay into $S_{1/2}$.

In order to enhance the emission efficiency of the photons from the ions, we can also introduce a optical resonant cavity corresponding to every MZI.[26,27] The probability P_{cav} for a photon to be emitted into the cavity mode after excitation to e can be expressed as $P_{\mathrm{cav}} = 4\gamma\Omega^2/(\gamma+\Gamma)(\gamma\Gamma + 4\Omega^2)$,[33] where $\Omega = (D/\hbar)\sqrt{\hbar c/2\epsilon_0\lambda V}$ and $\gamma = 4\pi c/F_{\mathrm{cav}}L$, and γ is the decay rate of the cavity, $Fcav$ is the finesse, L is the length, Ω is the coupling constant between the transition and the cavity mode, D is the dipole element, λ is the wavelength of the transition, V is the mode volume (which can be taken as small as $L^2\lambda/4$ for a confocal cavity with waist $\sqrt{L\lambda/\pi}$), and Γ is the noncavity related loss rate.[31] From the discussion of Ref. [31], the photon package is about 100 ns, which makes it easy to achieve good overlap for the wave function of the photon on the beam splitter, and we can choose $F_{\mathrm{cav}} = 1900$ and $L = 3\,\mathrm{mm}$, thus $\gamma = 9.9 \times 10^{-6}\mathrm{s}^{-1}$ and $P_{\mathrm{cav}} = 0.01$. For the detection of photons, the effect can be expressed as $\eta^2/2$, where η is the efficiency of a photon detection. Here we can choose $\eta = 0.7$, which can be reached with the current technology. Coupling the photon out of the cavities will introduce another error ξ, which can be modulated to be close to unit. Furthermore, the different transition times for the ions and the consequent pulse broadening will affect the efficiency of the

scheme slightly, because the ions have been placed on the MZIs symmetrically. In the present scheme, the process of generating a multipartite entangled ionic state is fairly complex. However, we can divide the whole process into many small steps. Every step includes a detection of photon, i.e. a circular polarized photon entering an MZI. Thus the success probability of every small step is $P = 1/6P_{\rm cav}\eta = 0.0012$. Although the probability is not very large, we can add enclosure cavity, the step will success in a shorter time as compared with the case of scheme without optical cavity, which will enhance the successful probability of experimental generation. The main drawback of the scheme is how to avoid the dark count during the process of detections. In a word, our generation scheme can be implemented more easily than schemes based on atomic interference in experiment, whereas it is still difficult within current experimental technology.

In conclusion, we have proposed a scheme for generating multipartite ionic GHZ states only by single-qubit measurements via linear optics. The successful probability of generation is $P = (0.125)^{N-1}$. If N is very large, P will be very small. However, we can use the failed states again with the help of entering polarized photons, which can improve the successful probability to $P = (0.24)^{N-1}$. Our scheme avoids the difficulty of synchronizing the arrival time of the two scattered photons, which is faced by the previous schemes. We offer an alternative scheme to generate ionic entangled states via single photon detections.

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