Electrostatic Instabilities at High Frequency in a Plasma Shock Front *

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(Received 11 December 2006)

New electrostatic instabilities in the plasma shock front are reported. These instabilities are driven by the electrostatic field which is caused by charge separation and the parameter gradients in a plasma shock front. The linear analysis to the high frequency branch of electrostatic instabilities has been carried out and the dispersion relations are obtained numerically. There are unstable disturbing waves in both the parallel and perpendicular directions of shock propagation. The real frequencies of both unstable waves are similar to the electron electrostatic wave, and the unstable growth rate in the parallel direction is much greater than the one in the perpendicular direction. The dependence of growth rates on the electric field and parameter gradients is also presented.

PACS: 52. 35. Tc, 52. 35. Py

Plasma shock and shock front have been studied widely since the 1950s. In most cases, except for a few early studies,[1−3] the plasma shocks are treated as quasi-neutral shocks and described by the single plasma fluid or magneto-hydrodynamic equations. In the early 1960s, Greenberg *et al.*[1,2] and Jaffrin and Probstein[3] found that the plasma shock front is usually non-neutral, and there are charge separation and electrical field in the shock front. Greenberg *et al.*[1] indicated that the electrical field can reach 4.17×10^6 V/m at low Mach numbers. These electric characteristics of a plasma shock front have been further confirmed by the numerical simulation of implosion shock in single bubble sonoluminescence.[4] In the simulation, the authors found that there is an extreme electrostatic field in the shock front (the typical value can reach about 10^{10} V/m). This is caused by the charge separation due to the difference of the electron fluid velocity and ion fluid velocity. The difference of velocities is caused by the huge inertial difference and different diffusion rates of electron and ion fluid. In the shock front, extremely high density and temperature gradients appear. Thus, the particle and heat diffusive fluxes caused by the density and temperature gradients are very large and the corresponding difference between the electron and ion diffusive fluxes will produce large charge separation in the front. The steady state profiles of plasma parameters in a non-neutral pure plasma shock front have been shown in Refs. [5–7] (one-dimensional case) and Ref. [8] (two-dimensional case). The one of the effects of this electrostatic field on the non-neutral shock is that itself or it together with the parameter gradients can drive several instabilities in the equilibrium (steady state) shock front. These instabilities are a kind of new electrostatic mode, which are qualitatively different from those well-known neutral modes (e.g. Rayleigh–Taylor or Richtmyer–Meshkov instability) driven only by the parameters gradient in the neutral shock front or discontinuity. In this Letter, as a primary example, we discuss the mentioned electrostatic instability at high frequency ($\omega \geqslant \omega_{pe}$) in a plane (one-dimensional) plasma shock front.

For the purpose to show the electrostatic instability driven by the electric field or/and parameter gradients, we take shock wave propagating in the $\boldsymbol{e}_x$ direction. Therefore, the basic equations are composed of the two-fluid equations and the Poisson equation in the one-dimensional case,[9]

$$\partial_t n_\alpha + \nabla \cdot (n_\alpha \boldsymbol{u}_\alpha) = 0, \tag{1}$$

$$\partial_t (m_\alpha n_\alpha \boldsymbol{u}_\alpha) + \nabla \cdot (m_\alpha n_\alpha \boldsymbol{u}_\alpha \boldsymbol{u}_\alpha) + \nabla P_\alpha - e_\alpha n_\alpha \boldsymbol{E} = 0, \tag{2}$$

$$\partial_t \Big(\frac{1}{2} m_\alpha n_\alpha u_\alpha^2 + \frac{3}{2} P_\alpha\Big) + \nabla \cdot \Big[\Big(\frac{1}{2} m_\alpha n_\alpha u_\alpha^2 + \frac{5}{2} P_\alpha\Big) \boldsymbol{u}_\alpha\Big] - e_\alpha n_\alpha \boldsymbol{u}_\alpha \cdot \boldsymbol{E} = 0, \tag{3}$$

$$\nabla \cdot \boldsymbol{E} = 4\pi e (Z n_i - n_e), \tag{4}$$

where n_α, $\boldsymbol{u}_\alpha$, P_α, are the number density, velocity, and pressure. $\boldsymbol{E}$ is the electrostatic field in the shock front, e_α is the charge of the ion and electron ($e_i = Ze, e_e = -e$). In this study, we take the equation of state as the perfect plasmas $P_\alpha = n_\alpha T_\alpha$.

In the linear evolving phase, the disturbed part $A_1(\boldsymbol{r}, t)$ of each parameter $A(\boldsymbol{r}, t)$ is always much less than the corresponding steady (equilibrium) state part $A_0(x)$. Thus, we have

$$A(\boldsymbol{r}, t) = A_0(x) + A_1(\boldsymbol{r}, t), \quad A_1(\boldsymbol{r}, t) \ll A_0(x). \tag{5}$$

The steady quantity A_0 is x (one-dimensional) dependent, while the disturbed quantity A_1 is $\boldsymbol{r}$ (three-dimensional) dependent. Then, let the disturbed

* Supported by the National Natural Science Foundation of China under Grant No 10175025.

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quantities behave as a plane wave

$$A_1(\boldsymbol{r},t) = A_1 \exp(i\boldsymbol{k}\cdot\boldsymbol{r} - i\omega t), \tag{6}$$

where ω and $\boldsymbol{k}$ are the frequency and the wave vector, respectively.

Firstly, we consider the case that the wave vector is parallel to the shock propagating direction ($\boldsymbol{k} = k_x\boldsymbol{e}_x$). In this case, let

$$\begin{aligned} &A_1(x,t) = A_1 \exp(ikx - i\omega t),\\ &A_1 = (n_{e1}, u_{e1x} = u_{e1}, T_{e1}, E_1). \end{aligned} \tag{7}$$

The profiles of the steady parameters $(n_{e0}, u_{e0}, T_{e0}, E_0)$ in the shock front are taken from Fig. 3 in Ref. [6], as shown in Fig. 1. The scale length of the parameters gradient in the shock front is $L = 2d = 8000\lambda_{De}$ (d is the thickness of shock). In our calculation, the wave vector of the perturbations is $k = (0.01 - 1)/\lambda_{De}$, and the wave length $\lambda = (200\pi - 2\pi)\lambda_{De}$, which is much less than L ($\lambda \ll (d\ln A_0/dx)^{-1}$). Due to the fact that the steady parameters are almost invariant in a disturbed wavelength λ, we ignore the parameter gradients in this case. Furthermore, considering that the ions do not respond to the high frequency ($\omega > \omega_{pe}$) disturbance, the ion distributions to the unstable disturbance will be omitted, i.e. let $n_{i1} = 0, u_{i1} = 0, T_{i1} = 0$).

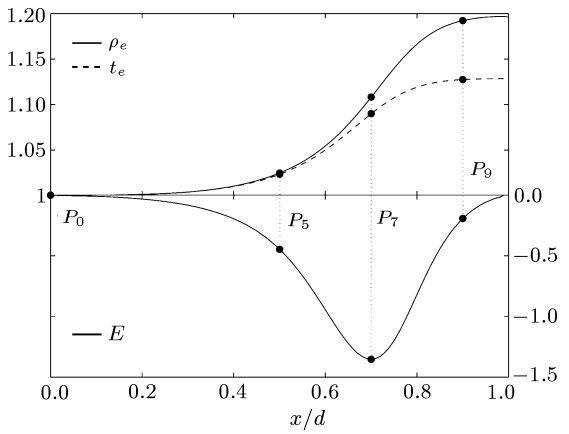


Fig. 1. Steady profiles of a one-dimensional plasma shock front, $P_0 - P_9$ indicate the point at different positions in the front, respectively. The abscissa is x/d (d is the calculation width of the shock front), and the ordinates are $\rho_e(x) = n_{e0}(x)/n_{\text{eupstream}}$, $t_e(x) = T_{e0}(x)/T_{\text{eupstream}}$, and $E = E_0(x)ed/k_BT_{\text{eupstream}}$ ($u_{e0}(x) = \Gamma_e/n_{e0}(x)$, Γ_e = constant).

By using Eqs. (5) and (7), the linearized mode equations of Eqs. (1)–(4) are

$$-i\omega n_{e1} + iku_{e0}n_{e1} + ikn_{e0}u_{e1} = 0, \tag{8}$$

$$\begin{aligned} &-i\omega m_e(u_{e0}n_{e1} + n_{e0}u_{e1})\\ &\quad + ikm_eu_{e0}(2n_{e0}u_{e1} + u_{e0}n_{e1})\\ &\quad + ik(T_{e0}n_{e1} + n_{e0}T_{e1})\\ &\quad + e(n_{e0}E_1 + E_0n_{e1}) = 0, \end{aligned} \tag{9}$$

$$\begin{aligned} &-\frac{1}{2}i\omega m_eu_{e0}(2n_{e0}u_{e1} - u_{e0}n_{e1})\\ &\quad - \frac{3}{2}i\omega(n_{e0}T_{e1} + T_{e0}n_{e1})\\ &\quad + \frac{1}{2}ikm_eu_{e0}^2(3n_{e0}u_{e1} + u_{e0}n_{e1})\\ &+ \frac{5}{2}ik(n_{e0}T_{e0}u_{e1} + n_{e0}u_{e0}T_{e1} + u_{e0}T_{e0}n_{e1})\\ &\quad + e(n_{e0}u_{e0}E_1 + u_{e0}E_0n_{e1} + n_{e0}E_0u_{e1})\\ &= 0, \end{aligned} \tag{10}$$

$$ikE_1 = -4\pi en_{e1}. \tag{11}$$

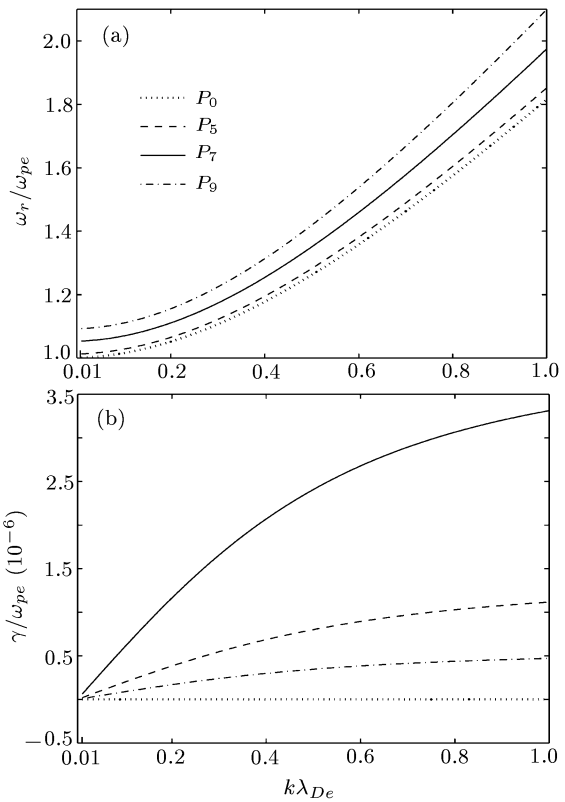


Fig. 2. Normalized real frequency and normalized growth rate as function of $k\lambda_{De}$ when instable wave propagates in shock propagating direction ($\boldsymbol{k} = k_x\boldsymbol{e}_x$). The instability is driven by the steady electric field in shock front.

The dispersion relation of unstable wave A_1 can be obtained from the numerical solutions of Eqs. (8)–(11)

$$\omega(k) = \omega_r(k) + \gamma(k).$$

The normalized real frequencies ω_r/ω_{pe} vs $k\lambda_{De}$ and growth rates γ/ω_{pe} vs $k\lambda_{De}$ at different positions in the shock front are shown in Fig. 2. P_0, P_5, P_7 and P_9 in Fig. 2 indicate that the values of steady parameters $(n_{e0}, u_{e0}, T_{e0}, E_0)$ are taken from the profiles in Fig. 1.[6] Figure 2(a) shows the real part of frequencies of unstable wave ω_r/ω_{pe} vs $k\lambda_{De}$, which are very similar to those of the electron plasma wave. Figure

2(b) shows that the electric field can drive electrostatic instabilities in shock front, and the corresponding growth rates γ/ω_{pe} vs $k\lambda_{De}$ is in order of magnitude 10^{-6}. Compared Fig. 2(b) with Fig. 1, it is found that the growth rates increase (decrease) with the increasing (decreasing) electric field $E_0(x)$. Near P_7, the electric field $E_0(x)$, hence the growth rates γ/ω_{pe}, reaches its absolute maximum, respectively.

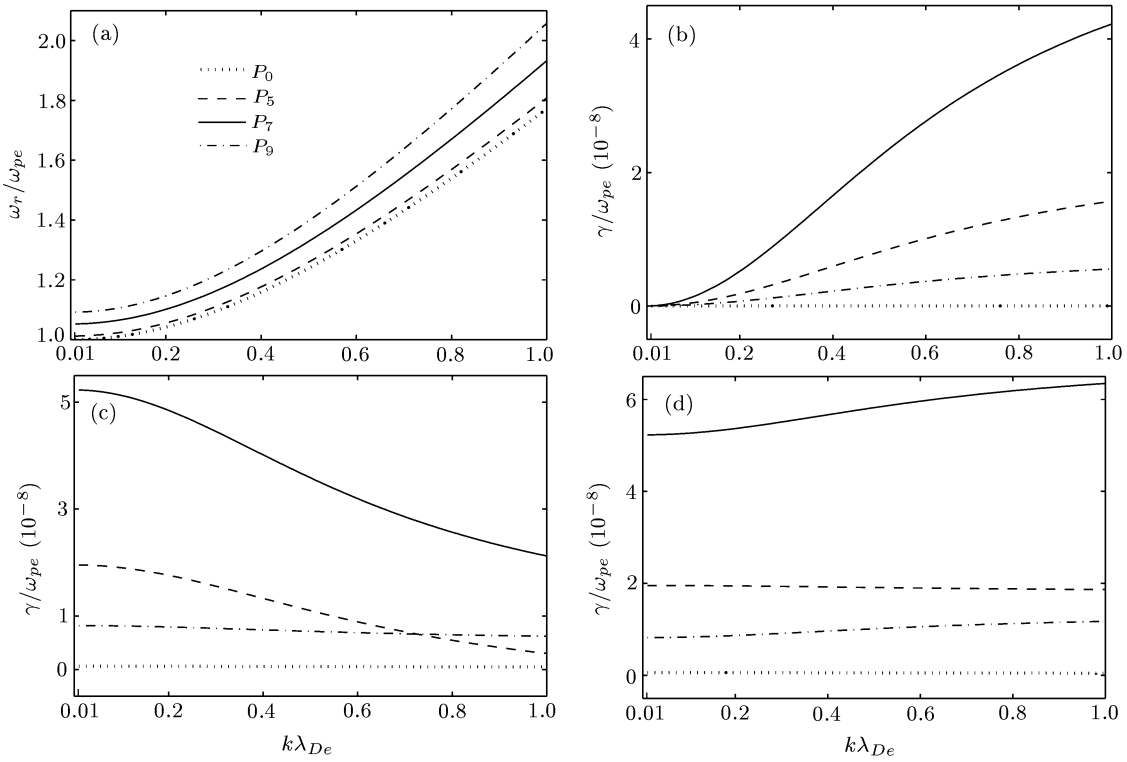


Fig. 3. Normalized real frequency and normalized growth rate as the function of $k\lambda_{De}$ when instable wave propagates perpendicular to the shock direction ($\boldsymbol{k} = k_y \boldsymbol{e}_y$). (a) The real part of frequency vs wave-vector. (b), (c) and (d) The growth rates vs wave-vector in the case that the instabilities driven by electric fields, parameter gradients, and electric field plus parameter gradients, respectively. The steady profiles of the shock front are taken from Fig. 1.

Then, we consider the case that the wave vector is perpendicular to shock propagating direction ($\boldsymbol{k} = k_y \boldsymbol{e}_y$). Assuming that there is isotropy in the perpendicular direction, we can only take the following disturbed electrostatic mode ($k_y = k$)

$$A_1(\boldsymbol{r}, t) = A_1(y, t) = A_1 \exp(iky - i\omega t),$$
$$A_1 = (n_{e1}, u_{e1y} = u_{e1}, T_{e1}, E_1). \tag{12}$$

In this case, both the electric field and parameter (velocity u_{e0} and temperature T_{e0}) gradients can play important roles in driving the instability. With Eq. (5) and (12), we can obtain the linearized mode equations from Eqs. (1)–(4) with $d_x = d/dx$,

$$-i\omega n_{e1} + ikn_{e0}u_{e1} + (d_x u_{e0})n_{e1} = 0, \tag{13}$$
$$-i\omega m_e n_{e0} u_{e1} + ikT_{e0}n_{e1} + ikn_{e0}T_{e1} + en_{e0}E_1 = 0, \tag{14}$$
$$-\frac{1}{2}i\omega m_e u_{e0}^2 n_{e1} - \frac{3}{2}i\omega(n_{e0}T_{e1} + T_{e0}n_{e1}) + \frac{1}{2}ikm_e n_{e0}u_{e0}^2 u_{e1} + \frac{3}{2}m_e n_{e0}u_{e0}^2(d_x u_{e0})n_{e1} + \frac{5}{2}ikn_{e0}T_{e0}u_{e1} + \frac{5}{2}T_{e0}(d_x u_{e0})n_{e1} + \frac{5}{2}u_{e0}(d_x T_{e0})n_{e1} + eu_{e0}E_0 n_{e1} = 0, \tag{15}$$
$$ikE_1 = -4\pi e n_{e1}. \tag{16}$$

The dispersion relation can be obtained from the numerical solution of Eqs. (13)–(16). Figure 3(a) shows the normalized real frequency ω_r/ω_{pe} vs $k\lambda_{De}$, which is also very similar to the one of electron electrostatic (plasma) wave. Figures 3(b)–3(d) show the growth rates γ/ω_{pe} vs $k\lambda_{De}$ for the steady electric field and parameter gradients taken from different positions of shock front. In Fig. 3(b), the growth rates is calculated when the unstable wave driven only by electric field $E_0(x)$, which is favourable to the shorter wavelength (larger k). Figure 3(c) shows the instability is driven only by the parameter gradients [i.e. $d_x A_0(x), A_0 = (u_{e0}, T_{e0})$], which is favourable to the longer wavelength (smaller k). Figure 3(d) gives the total growth rate driven by both $E_0(x)$ and $d_x A_0(x)$, which is insensitive to the wavelength (or k). We can

also find that the growth rates increase (decrease) with the increasing (decreasing) electric field or/and the parameters gradient. However, the growth rate in perpendicular direction is only in the order of magnitude $\gamma/\omega_{pe} \approx 10^{-8}$, which is much less than the one in the parallel direction (where $\gamma/\omega_{pe} \approx 10^{-6}$).

In conclusion, we have given examples that the electrostatic field or/and parameter gradients in a non-neutral plasma shock front can drive new electrostatic instabilities at high frequency ($\omega \geqslant \omega_{pe}$). In both the parallel and perpendicular directions, there are unstable waves. Their real parts of dispersion relations are similar to the dispersion relation of the well-known electron electrostatic wave, and their growth rates increase with the electric field or/and parameter gradients. However, the value of growth rate ($\gamma/\omega_{pe} \approx 10^{-6}$) in the direction of shock propagation is much greater than the one ($\gamma/\omega_{pe} \approx 10^{-8}$) in the perpendicular direction of shock propagation. The dependencies of real frequencies and growth rates on the steady profiles of $n_{e0}(x), u_{e0}(x), T_{e0}(x)$ and $E_0(x)$ are shown in Figs. 2 and 3 by a numerical solution in a one-dimensional shock front given in Ref. [6] and shown in Fig. 1.

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