Trapping State in a System of Single Three-level Trapped Ion Driven by External Fields *

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We investigate the interaction of a single three-level trapped ion with two laser beams. By applying a unitary transformation and a small rotating transformation, an exact solution to this quantum system is obtained without performing the Lamb–Dicke approximation, and the trapping state is observed.

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In recent years, considerable attention has been devoted to coherent manipulation of trapped ions owing to their potential practical applications such as precision spectroscopy[1] and quantum computation.[2] Most works focused on the generation and detection of nonclassical states of c.m. motion of trapped ions. So far many ways have been proposed to generate these nonclassical states. Fock states of the vibrational motion of the trapped ion can be prepared by using quantum jump effects,[4] by applying two-colour laser fields successively,[5] or by using the adiabatic passage method.[6] In addition, Squeezed states of motion of trapped ions can be generated by applying two standing wave laser fields with different frequencies.[7] These investigations are mostly on basis of operation in the Lamb–Dicke limits (LDL), i.e. a trapped ion is located to a region much smaller than the optical wavelength.[8]

Recently an alternative approach is proposed to avoid constraint on the Lamb–Dicke parameter in the studying system of trapped ion–laser.[9] In this approach, the Hamiltonian of the system can be transformed to the usual Jaynes–Cummings model in context of cavity QED by using a unitary transformation. In Ref. [10], a small rotating transformation furthermore has been carried out to avoid performing the rotating wave approximation (RWA).

In this Letter, we investigate the model of single three-level trapped ion driven by two laser beams with equal frequencies. With applications of unitary transformations and a small rotating transformation, we obtain a complete solution to this problem and find that the trapping state can also be observed in this model.

We consider a single ion trapped in a harmonic potential traps, which interacts with two laser fields with equal frequency. The ion consists of two ground states $|1\rangle$ and $|3\rangle$ with energy E_1 (set $E_1 = 0$) and an excited states $|2\rangle$ with energy $E_2 = \hbar\omega_a$, which is shown in Fig. 1. We restrict our consideration to the quantum mechanical motion of the ion in the x direction. The Hamiltonian is given as

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2m}\nu^2\hat{x}^2 + \hbar\omega_a|2\rangle\langle 2|) + \frac{\hbar\Omega}{2}(e^{i(kx-\omega t)}|2\rangle\langle 1| + e^{i(kx-\omega t)}|2\rangle\langle 3 + \text{H.c.}), \tag{1}$$

where $\hat{x}$ and $\hat{p}$ denote the position and momentum operators of c.m. motion of the ion, respectively; ν is the trapping frequency; k is the wave number of lasers; Ω is the Rabi frequency. Two lasers are assumed to have the equal strength. Applying the optical rotating wave approximation, we reach the Hamiltonian

$$\hat{H} = \hbar\nu\hat{a}^+\hat{a} + \hbar\Delta|2\rangle\langle 2| + \frac{\hbar\Omega}{2}(e^{i\eta(\hat{a}+\hat{a}^+)}|2\rangle\langle 1| + e^{i\eta(\hat{a}+\hat{a}^+)}|2\rangle\langle 3| + \text{H.c.}), \tag{2}$$

where $\hat{a} = (m\nu/2\hbar)^{1/2}(\hat{x} + i\hat{p}/m\nu)$ is the usual annihilation operator of the vibrational mode of the ion, $\Delta = \omega_a - \omega$ is the detuning of the ionic transition from the laser frequencies, $\eta = k\sqrt{\hbar/2m\nu}$ are the Lamb–Dick parameters. In the representation of ionic state vectors $|l\rangle$ $(l = 1, 2, 3)$, the Hamiltonian (2) can be rewritten as $(\hbar = 1)$

$$\hat{H} = \nu\hat{a}^+\hat{a}I + \frac{\Omega}{2}\begin{pmatrix} 0 & \hat{D}^2(\alpha) & 0 \\ \hat{D}^{+2}(\alpha) & \Delta & \hat{D}^{+2}(\alpha) \\ 0 & \hat{D}^2(\alpha) & 0 \end{pmatrix}, \tag{3}$$

where $\hat{D}(\alpha) = e^{(\alpha\hat{a} - \alpha^*\hat{a}^+)}$ are displacement operators, I is the unit matrix, $\alpha = -i\eta/2$.

The dynamics of the model corresponding to the Hamiltonian (3) can be solved exactly. By applying the unitary transformation

$$\hat{T}_1 = \frac{1}{2}\begin{pmatrix} \hat{D}^+(\alpha) & \sqrt{2}\hat{D}(\alpha) & \hat{D}^+(\alpha) \\ \sqrt{2} & 0 & -\sqrt{2} \\ \hat{D}^+(\alpha) & -\sqrt{2}\hat{D}(\alpha) & \hat{D}^+(\alpha) \end{pmatrix} \tag{4}$$

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to the Hamiltonian (3), we can obtain the following linearized Hamiltonian $H' = THT^+$,

$$\hat{H}' = \nu\hat{a}^+\hat{a}I + \begin{pmatrix} \lambda_1 & 0 & K(\hat{a},\hat{a}^+) \\ 0 & 0 & 0 \\ K(\hat{a},\hat{a}^+) & 0 & \lambda_2 \end{pmatrix}, \quad (5)$$

with

$$\lambda_1 = \sqrt{2}\Omega/2 + \eta^2\nu/4 + \Delta/2, \quad (6a)$$
$$\lambda_2 = -\sqrt{2}\Omega/2 + \eta^2\nu/4 + \Delta/2, \quad (6b)$$
$$K(\hat{a},\hat{a}^+) = i\eta\nu(\hat{a}-\hat{a}^+)/2 - \Delta/2. \quad (6c)$$

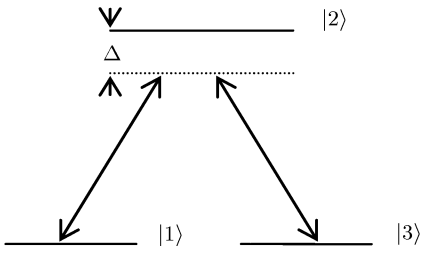


Fig. 1. Level scheme of the trapped ion interacting with two laser beams.

From Eq. (5), we can see that the energy level $|2\rangle$ of the ion is decoupled from the energy levels $|1\rangle$ and $|3\rangle$. In order to avoid application of the RWA, based on the idea given in Ref. [9], we further apply a small unitary transformation to the Hamiltonian (5), which has the form of

$$\hat{T}_2 = \begin{pmatrix} \cos\hat{w} & 0 & i\sin\hat{w} \\ 0 & 1 & 0 \\ i\sin\hat{w} & 0 & \cos\hat{w} \end{pmatrix}, \quad (7)$$

where $\hat{w} = -\varepsilon(\hat{a}+\hat{a}^+)$, $\varepsilon\langle\langle 1$. The transformed Hamiltonian $\hat{H}'' = \hat{T}_2\hat{H}'\hat{T}_2^+$ can be represented as

$$\hat{H}''_{11} = \nu\hat{a}^+\hat{a} + \frac{\Omega}{2}\lambda_1 - \eta\nu\varepsilon + o(\varepsilon^2), \quad (8a)$$
$$\hat{H}''_{12} = \hat{H}''_{21} = \hat{H}''_{23} = \hat{H}''_{32} = 0, \quad (8b)$$
$$\hat{H}''_{13} = i[\eta\nu/2 - \nu\varepsilon + \sqrt{2}\Omega\varepsilon]\hat{a} + i[\sqrt{2}\Omega\varepsilon + \nu\varepsilon - \eta\nu/2]\hat{a}^+ - \frac{\Delta}{2} + o(\varepsilon^2), \quad (8c)$$
$$\hat{H}''_{31} = \hat{H}''_{13}, \quad (8d)$$
$$\hat{H}''_{33} = \nu\hat{a}^+\hat{a} + \frac{\Omega}{2}\lambda_2 + \eta\nu\varepsilon + o(\varepsilon^2). \quad (8e)$$

By setting

$$\varepsilon = \frac{\eta\nu}{2(\sqrt{2}\Omega+\nu)} \quad (9)$$

and omitting the higher terms on ε, we finally obtain

$$\hat{H}'' = \nu\hat{a}^+\hat{a}I + \begin{pmatrix} w_1 & 0 & ig\hat{a} \\ 0 & 0 & 0 \\ -ig\hat{a}^+ & 0 & w_2 \end{pmatrix}, \quad (10)$$

where

$$w_j = \lambda_j + (-1)^j\frac{\eta^2\nu^2}{2(\sqrt{2}\Omega+\nu)}, \quad (11a)$$
$$g = \frac{\sqrt{2}\eta\nu\Omega}{(\sqrt{2}\Omega+\nu)}. \quad (11b)$$

We now proceed to give a solution to the Hamiltonian (3), which reads

$$|\psi(t)\rangle = \hat{T}\hat{U}\hat{T}^+|\psi(0)\rangle, \quad (12)$$

where $|\psi(0)\rangle$ is the initial wave vector and $\hat{T} = \hat{T}_2\hat{T}_1$ has the form of

$$\hat{T} = \frac{1}{2}\begin{pmatrix} \hat{D}^+(\alpha') & \sqrt{2}\hat{D}(\alpha') & \hat{D}^+(\alpha') \\ \sqrt{2} & 0 & -\sqrt{2} \\ \hat{D}^+(\alpha') & -\sqrt{2}\hat{D}(\alpha') & \hat{D}^+(\alpha') \end{pmatrix} \quad (13)$$

with $\alpha' = \alpha - i\varepsilon$ and $\hat{U}$ being the time evolution operator corresponding to the Hamiltonian (10). In order to derive $\hat{U}$, the Hamiltonian (10) can be rewritten in the form of direct sum of two terms as

$$\hat{H}'' = \hat{H}''_1 \oplus \hat{H}''_2, \quad (14)$$

with

$$\hat{H}''_1 = (\nu\hat{a}^+\hat{a} + \delta_0)(|1\rangle\langle 1| + |3\rangle\langle 3|) + \frac{\delta}{2}(|1\rangle\langle 1| - |3\rangle\langle 3|) + ig\hat{a}|1\rangle\langle 3| - ig\hat{a}^+|3\rangle\langle 1|), \quad (15a)$$
$$\hat{H}''_2 = \nu\hat{a}^+\hat{a}|2\rangle\langle 2|, \quad (15b)$$

where $\delta_0 = (w_1+w_2)/2$, which can be disregarded because it just represents an overall phase, $\delta = (w_1 - w_2)/2$. Noting that the Hamiltonian in Eq. (15a) describes the off-resonant Jaynes–Cummings model, we can easily deduce the time evolution operator

$$\begin{aligned}\hat{U} =& e^{-it\hat{H}''_1} \oplus e^{-it\hat{H}''_2} \\ =& e^{-i\nu t\hat{a}^+\hat{a}}e^{-i\nu t(1/2)_1(1\rangle\langle 1|-|3\rangle\langle 3|)} \\ & \cdot e^{-it[\bar{\delta}(|1\rangle\langle 1|-|3\rangle\langle 3|)+ig(\hat{a}|1\rangle\langle 3|-\hat{a}^+|3\rangle\langle 1|)]} \\ & \oplus e^{-i\nu t\hat{a}^+a|2\rangle\langle 2|} \\ =& e^{-i\nu t\hat{a}^+\hat{a}}\Big\{\sum_{j=1}^{3} e^{-i\bar{\nu}_j t/2}\hat{U}_{jj}|j\rangle\langle j| \\ & + e^{-i\nu t/2}\hat{U}_{13}|1\rangle\langle 3| + e^{i\nu t/2}\hat{U}_{31}|3\rangle\langle 1|\Big\},\end{aligned} \quad (16)$$

where $\bar{\delta} = \delta - \nu$, $\bar{\nu}_j = (2-j)\nu$, and

$$\hat{U}_{11} = \cos(\hat{\chi}_{\hat{n}+1}t) - i\bar{\delta}\frac{\sin(\hat{\chi}_{\hat{n}+1}t)}{\hat{\chi}_{\hat{n}+1}}, \quad (17a)$$
$$\hat{U}_{22} = 1, \quad (17b)$$

$$\hat{U}_{33} = \cos(\hat{\chi}_{\hat{n}}t) + i\bar{\delta}\frac{\sin(\hat{\chi}_{\hat{n}}t)}{\hat{\chi}_{\hat{n}}}, \tag{17c}$$

$$\hat{U}_{13} = g\hat{a}\frac{\sin(\hat{\chi}_{\hat{n}}t)}{\hat{\chi}_{\hat{n}}}, \tag{17d}$$

$$\hat{U}_{31} = -g\hat{a}^{+}\frac{\sin(\hat{\chi}_{\hat{n}+1}t)}{\hat{\chi}_{\hat{n}+1}}, \tag{17e}$$

where $\hat{\chi}_{\hat{n}} = \sqrt{\bar{\delta}^2 + g^2\hat{n}} = \sqrt{\bar{\delta}^2 + g^2\hat{a}^{+}\hat{a}}$.

Now let us suppose that the internal state of ion is in the superposition states of between $|1\rangle$ and $|3\rangle$, and its vibrational mode is prepared in the coherent state $|\beta\rangle$ at the initial time. Then initial state vector $|\psi(0)\rangle$ of the system can be expressed as

$$|\psi(0)\rangle = \frac{\sqrt{2}}{2}(|1\rangle - |3\rangle) \otimes |\beta\rangle. \tag{18}$$

We can compute the state of the system with any time according to Eq. (12), which is given by

$$|\psi(t)\rangle = \frac{\sqrt{2}}{2}e^{-i\nu t/2(|1\rangle\langle 1| - |3\rangle\langle 3|)}(|1\rangle - |3\rangle) \otimes e^{-i\nu t\hat{a}^{+}\hat{a}}|\beta\rangle. \tag{19}$$

Equation (19) displays that the trapping state will occur in this system. However, unlike cavity QED, the trapping state of the ion is independent of the coupling parameters between the ion and the external field [see Eq. (19)].

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