MHD Boundary Layer Flow of a Non-Newtonian Fluid on a Moving Surface with a Power-Law Velocity *

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A theoretical analysis for MHD boundary layer flow on a moving surface with the power-law velocity is presented. An accurate expression of the skin friction coefficient is derived. The analytical approximate solution is obtained by means of Adomian decomposition methods. The reliability and efficiency of the approximate solutions are verified by numerical ones in the literature.

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During the last decades, investigations of boundary layer flow on continuous stretching surface have attracted considerable attention. Sakiadis[1] first studied the boundary layer flow induced by a solid surface with a constant velocity in an otherwise ambient fluid. Erickson *et al.*[2] investigated the temperature distribution of the boundary layer when the surface is maintained at a constant temperature with suction or blowing. Since then, several investigations have concentrated on the problems of a stretched surface and different thermal boundary conditions.[3−7]

The above-mentioned investigations restricted the analyses to flows of a Newtonian fluid. Recently, the non-Newtonian fluids in the presence of a magnetic field have been applied to many areas such as chemical engineering, electromagnetic propulsion, nuclear reactors etc. Motions of non-Newtonian fluids in the presence of a magnetic field were studied earlier by several authors.[8−11] Pavlov[12] studied the MHD boundary layer flow of an electrically conducting fluid due to the stretching of a plane elastic surface. Wu[13] investigated an MHD boundary layer flow with uniform suction or injection. Andersson *et al.*[14] studied the MHD flow of a non-Newtonian power-law fluid over a stretching surface with a linear velocity in an ambient fluid. In this Letter, we investigate the MHD boundary layer flow of a non-Newtonian fluid over a moving surface with power-law velocity.

Consider the flow of an electrically conducting fluid obeying the power-law model in the presence of a magnetic field $B(x) = B_0 x^{(m-1)/2}$. The basic equations for the boundary layer flow are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho}\Big(\frac{\partial \tau_{xy}}{\partial y}\Big) - \frac{\sigma B(x)^2}{\rho}u, \tag{2}$$

where u and v are the velocity components parallel and normal to the plate respectively, ρ is the fluid density, σ is the electrical conductivity, and τ_{xy} is the shear stress tensor defined as

$$\tau_{xy} = -K\Big(-\frac{\partial u}{\partial y}\Big)^n. \tag{3}$$

The boundary conditions are

$$u\mid_{y=0} = Cx^m, \quad v\mid_{y=0} = 0, \quad u\mid_{y=\infty} = 0, \tag{4}$$

where C is a positive constant.

We now define the following similarity transformations:

$$\psi(x,y) = \Big[\frac{(n+1)C^{2n-1}K/\rho}{2mn-m+1}\Big]^{\frac{1}{n+1}} \cdot x^{(2mn-m+1)/(n+1)} f(\eta),$$
$$\eta = y\Big[\frac{(2mn-m+1)C^{2-n}}{(n+1)K/\rho}\Big]^{\frac{1}{n+1}} \cdot x^{(2m-mn+1)/(n+1)}. \tag{5}$$

Substituting Eq. (5) into Eqs. (1) and (2), we obtain the following nonlinear boundary value problem:

$$n(-f'')^{n-1}f''' + ff'' - \beta(f')^2 - Mf' = 0, \tag{6}$$

with the boundary conditions

$$f(0) = 0, \quad f'(0) = 1, \quad f'(+\infty) = 0, \tag{7}$$

where

$$\beta = \frac{m(n+1)}{2mn-m+1}, \quad M = \frac{\sigma B_0(n+1)}{\rho C(2mn-m+1)}. \tag{8}$$

The skin friction coefficient C_f at the wall is given by

$$C_f = 2[-f''(0)]^n\Big(\frac{2mn-m+1}{n+1}\Big)^{1/(n+1)}$$

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$$\cdot \left[\frac{(Cx^m)^{2-n}\cdot x^n}{K/\rho}\right]^{-1/(n+1)}. \tag{9}$$

Here $\frac{(Cx^m)^{2-n}\cdot x^n}{K/\rho}$ is the local Reynolds number based on the surface velocity Cx^m, and C_f is directly related to the dimensionless velocity gradient $f''(0)$ at the wall.

Define the Crocco variable transformation as[15,16]

$$t = f'(\eta), \quad w(t) = (-f''(\eta))^n, \quad 0 < t < 1, \tag{10}$$

where t is the dimensionless tangential independent variable and $w(t)$ is the shear stress of the dependent variable. Then we have

$$w(1) = (-f''(0))^n. \tag{11}$$

Substituting Eq. (10) into Eqs. (6) and (7) and applying the chain rule, we obtain

$$w''(t) = \left(\frac{\beta t^2 + Mt}{(w(t))^{1/n}}\right)' - \frac{t}{(w(t))^{1/n}}, \quad 0 < t < 1. \tag{12}$$

The boundary conditions (7), in view of Eq. (10), are reduced to

$$w(0) = 0, \quad w'(1) = \frac{\beta + M}{(w(1))^{1/n}}. \tag{13}$$

In the following, we investigate Eqs. (12) and (13) with a view to obtain the analytical approximate solutions by utilizing the Adomian decomposition method.[17−19]

Let $w(1) = \alpha$, the inverse operator L_{tt}^{-1} can be formally defined as

$$L_{tt}^{-1} = \int_1^t dt' \int_1^{t'} dt''. \tag{14}$$

Applying L_{tt}^{-1} to Eq. (12), we obtain

$$w(t) = w(1) + \int_1^t \frac{\beta t'^2 + Mt'}{(w(t'))^{1/n}} dt' + \int_1^t \int_1^{t'} \frac{-t''}{(w(t''))^{1/n}} dt'' dt'. \tag{15}$$

The decomposition technique expands the solution of $w(t)$ in a series form

$$w(t) = \sum_{m=0}^{\infty} w_m(t), \tag{16}$$

and the nonlinear term can be decomposed into

$$\sum_{m=0}^{\infty} A_m = \frac{\beta t'^2 + Mt'}{(w(t'))^{1/n}}, \quad \sum_{m=0}^{\infty} B_m = \frac{-t''}{(w(t''))^{1/n}}, \tag{17}$$

where A_m and B_m are the Adomian polynomials which represent the nonlinear operator $\frac{\beta t'^2 + Mt'}{(w(t'))^{1/n}}$ and $\frac{-t''}{(w(t''))^{1/n}}$, respectively. The Adomian decomposition method employs the recursive relation

$$\begin{aligned} w_0(t) &= \alpha, \\ w_1(t) &= \int_1^t A_0 dt' + L_{tt}^{-1} B_0, \\ w_{k+1}(t) &= \int_1^t A_k dt' + L_{tt}^{-1} B_k, \text{ for } k \geq 1. \end{aligned} \tag{18}$$

The practical solution will be the l-terms approximation $\theta_l(t)$ to $w(t)$, which is usually written as

$$\theta_l = \sum_{i=0}^{l-1} w_i(t) = w_0(t) + w_1(t) + w_2(t) + \cdots + w_{l-1}(t), \quad l \geq 1. \tag{19}$$

Now our concern is to determine the constant α for special parameter values β and M using the condition (13). Once the value of α is determined, Eq. (19) will give an approximation analytical solution of $w(t)$.

Table 1. Comparison of values of $f''(0)$ for $n = 1$.

β	M	$f''(0)_{\text{app}}$	$f''(0)_{\text{com}}$	Error$\left(\frac{\lvert f''(0)_{\text{app}} - f''(0)_{\text{com}}\rvert}{f''(0)_{\text{app}}}\right)$
	0.5	−0.51787	−0.52395	1.17×10^{-2}
−1.0	1.0	−0.83424	−0.85111	2.02×10^{-2}
	5.0	−2.08665	−2.16287	3.65×10^{-2}
	1.0	−0.64101	−0.65298	1.87×10^{-2}
−1.5	5.0	−2.01214	−2.08524	3.63×10^{-2}
	10.0	−2.94042	−3.05623	3.94×10^{-2}

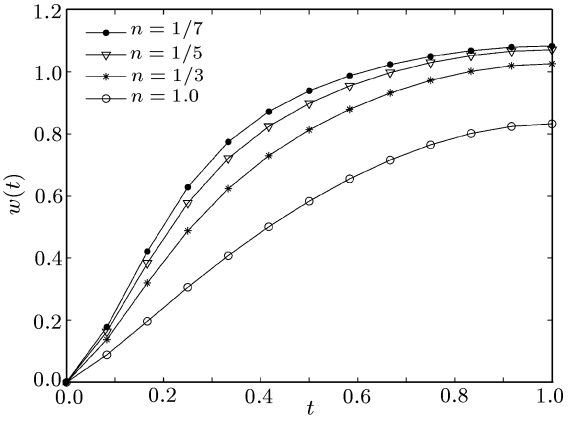


Fig. 1. Shear force distribution for $\beta = -1.0$ and $M = 1$.

Table 2. Comparison of values of α for $M = 0$ and $n = 1$.

l	α ($\beta = 0.5$)	α ($\beta = 1.0$)	α ($\beta = 1.5$)	α ($\beta = 2.0$)	α ($\beta = 5.0$)
6	0.81919	0.97899	1.11694	1.23991	1.81107
7	0.82152	0.98311	1.12271	1.24717	1.82492
8	0.82314	0.98607	1.12692	1.25250	1.83531
9	0.82432	0.98827	1.13010	1.25657	1.84339
10	0.82522	0.98997	1.13258	1.25977	1.84986

Table 3. Comparison of values of α for $\beta = 1.0$ and $n = 1/3$.

M	α ($l = 6$)	α ($l = 7$)	α ($l = 8$)	α ($l = 10$)
0.0	1.10811	1.11536	1.12083	1.12854
0.5	1.22611	1.23561	1.24284	1.25319
1.0	1.31717	1.32812	1.33648	1.34847
1.5	1.39243	1.40446	1.41367	1.42687
2.0	1.45713	1.47003	1.47991	1.49409
2.5	1.51418	1.52781	1.53827	1.55327
3.0	1.56542	1.57970	1.59064	1.60635

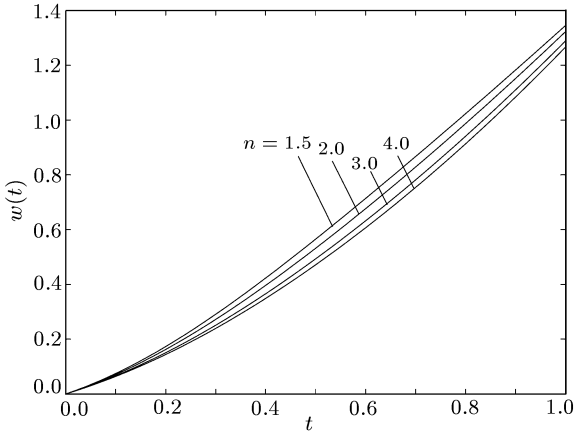


Fig. 2. Shear force distribution for $\beta = 1.0$ and $M = 1$.

The approximate results are presented in Tables 1–3 and Figs. 1 and 2. In Table 1, the results of the present work ($f''(0)_{\rm app}$) are compared with the numerical solutions of Chiam ($f''(0)_{\rm com}$).[20] It is obvious that the approximate solutions are in agreement with the numerical ones of Chiam. Tables 2 and 3 present the effect of parameter β and the magnetic field parameter M on the skin friction. The approximate results indicate that the skin friction $\alpha = w(1)$ increases with the increasing β and M. Figures 1 and 2 show the dimensionless shear force profiles $w(t)$ for pseudoplasticity fluids ($n < 1$), Newtonian fluid ($n = 1$) and dilatant fluids ($n > 1$). It can be seen that the skin friction α decreases with an increase of t in the power-law index n.

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