Convection-Dominated Accretion Flows with Radiative Cooling *

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By numerically solving the set of basic equations describing black hole accretion flows with low accretion rates, we show that although the dynamical structure of these flows is essentially unaffected by radiative processes in comparison with the case in which the radiation is not considered, the radiative cooling can be more important than the advective cooling in the flow's convection-dominated zone, and this result may have implications to distinguish observationally convection-dominated accretion flows from advection-dominated accretion flows.

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The advection-dominated accretion flow (ADAF) model is one of the popular models for black hole accretion flows (see Refs. [1,2] for reviews). In this model, most of the gravitational and rotational energies released by the accreted matter is advected inwards in the form of internal energy and is finally absorbed by the black hole. At the same time as this model was proposed, it was realized that ADAFs are possibly to be convectively unstable in the radial direction because of the inward increase of the entropy of accreted gas.[3] Accordingly, in Refs. [4,5] the researchers constructed the so-called convection-dominated accretion flow (CDAF) model, in which accretion is suppressed by strong convection, such that the mass accretion rate $\dot{M}$ is very small and the luminosity is very low. This low-luminosity feature of CDAFs is similar to that of ADAFs, but in the latter case it is because of the inward bulk advection of energies.

The CDAF model of Refs. [4,5] is in the self-similar form. It is only a local, not global, solution, in the sense that it can only be valid for the region of an accretion flow far away from the boundaries. In particular, it cannot reflect the transonic radial motion, i.e. the most fundamental character of black hole accretion. Advection should be important in the vicinity of the black hole because of the large radial velocity of the accretion flow, so advection and convection should be considered in a combined way. With this view, in Ref. [6] the authors solved numerically the set of dynamical equations for black hole accretion flows, and obtained global solutions which can be divided into three types according to the strength of viscosity. Type-I solutions correspond to a large viscosity parameter $\alpha \gtrsim 0.1$ and are purely advection-dominated with no convection. Type-II solutions are for moderate $\alpha \sim 0.01$ and have a three-zone structure: the inner zone is advection-dominated, the middle zone is convection-dominated, and the outer zone is convectively stable and outwardly matches with a Keplerian disc. Type-III solutions are for small $\alpha \lesssim 0.001$ and consist of an inner advection-dominated zone and an outer convection-dominated zone, as was suggested in Ref. [7]. These results are in good agreement with those of analytic works[8] and those of two-dimensional and three-dimensional numerical simulations of black hole accretion flows.[9,10]

However, in Ref. [6] the authors obtained only purely dynamical solutions, the radiative processes in the accretion flow were not considered. In this Letter, we improve the work of Ref. [6] by including the radiative cooling term in the energy equation to see the effect of radiation on the flow behaviour.

We consider a set of stationary, axisymmetric, height-integrated equations describing an accretion flow around a non-rotating black hole.[4,7] The continuity equation, radial momentum equation, and angular momentum equation are the same as those in Ref. [6]. They are

$$\dot{M} = -2\pi R\Sigma v = \text{const}, \tag{1}$$

$$v\frac{dv}{dR} + (\Omega_K^2 - \Omega^2)R + \frac{1}{\rho}\frac{dP}{dR} = 0, \tag{2}$$

$$\frac{(\alpha + \alpha_{\rm con})c_s^2 R^2}{\Omega_K v}\frac{d\Omega}{dR} = \Omega R^2\Big(1 - \frac{2\alpha_{\rm con}c_s^2}{R\Omega_K v}\Big) - j, \tag{3}$$

where R is the radius, $\Sigma = 2H\rho$ is the surface density, ρ is the density, $H = c_s/\Omega_K$ is the scale height, $c_s = (P/\rho)^{1/2}$ is the sound speed, P is the pressure, $\Omega_K = (GM/R)^{1/2}/(R - R_g)$ is the Keplerian angular velocity in the well-known Paczyński–Wiita potential,[11] M is the black hole mass, $R_g = 2GM/c^2$ is the gravitational radius, v is the radial velocity, Ω is the angular velocity, α is the Shakura–Sunyaev viscosity parameter that is assumed to be a constant, $\alpha_{\rm con}$ is a dimensionless parameter that describes the strength of convective diffusion and is not a constant, and j is a integration constant that represents the specific angular momentum accreted by the black hole.

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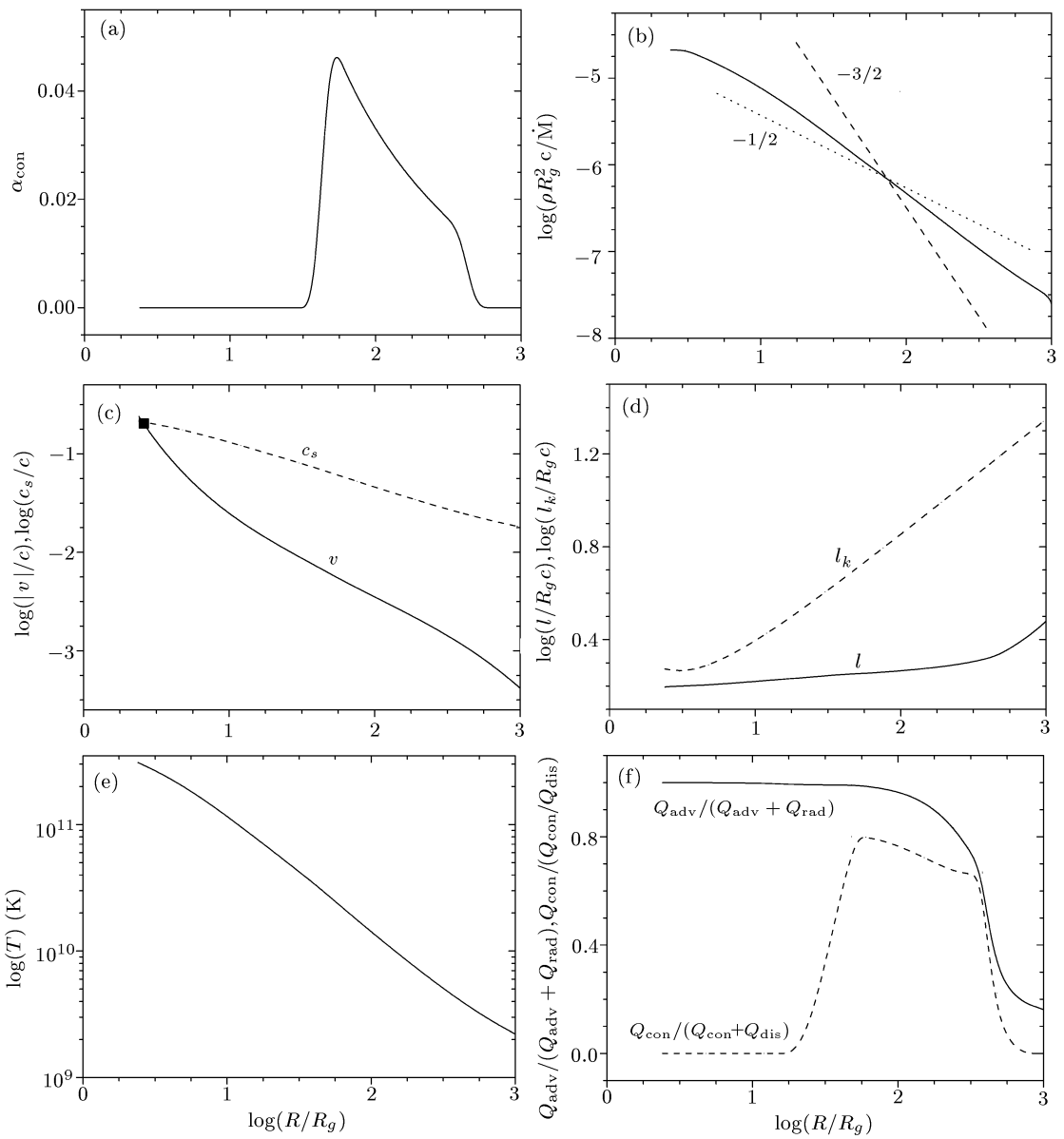


Fig. 1. Example of the type-Ⅱ three-zone solution for the viscosity parameter $\alpha = 0.01$ for (a) the convective diffusion parameter $\alpha_{\rm con}$, (b) the density ρ, (c) the radial velocity v and the sound speed c_s, (d) the specific angular momentum l and the Keplerian specific angular momentum l_K, (e) the temperature T, and (f) the ratio of convective heating $Q_{\rm con}$ to total heating of convection and dissipation ($Q_{\rm con}+Q_{\rm dis}$) and the ratio of advective cooling $Q_{\rm adv}$ to total cooling of advection and radiation ($Q_{\rm adv}+Q_{\rm rad}$).

The energy equation is

$$Q_{\rm dis}+Q_{\rm con}=Q_{\rm adv}+Q_{\rm rad}. \tag{4}$$

In this equation, $Q_{\rm dis}$ is the dissipative heating rate due to both the viscous and the convective shear stress, $Q_{\rm con}$ is the heating rate due to the convective transfer, $Q_{\rm adv}$ is the advective cooling rate, and $Q_{\rm rad}$ is the radiative cooling rate. They are expressed as

$$Q_{\rm dis}=\frac{c_s^2\Sigma R}{\Omega_K}\Big[(\alpha+\alpha_{\rm con})R\Big(\frac{d\Omega}{dR}\Big)^2+2\alpha_{\rm con}\Omega\frac{d\Omega}{dR}\Big], \tag{5}$$

$$Q_{\rm con}=\frac{1}{R}\frac{d}{dR}\Big(\frac{\alpha_{\rm con}c_s^2R\Sigma T}{\Omega_K}\frac{ds}{dR}\Big), \tag{6}$$

$$Q_{\rm adv}=\Sigma vT\frac{ds}{dR}=\Sigma v\Big(\frac{1}{\gamma-1}\frac{dc_s^2}{dR}-\frac{c_s^2}{\rho}\frac{d\rho}{dR}\Big), \tag{7}$$

$$Q_{\rm rad}=8\sigma T^4\Big(\frac{3\tau_R}{2}+\sqrt{3}+\frac{8\sigma T^4}{Q_{br}}\Big)^{-1}, \tag{8}$$

where T is the temperature, s is the specific entropy, and γ is the specific heat ratio. Equation (5)–(7) are the same as those in Ref. [6]. Equation (8) is a bridging formula valid for both the optically thick and thin regimes,[12] where $\tau_R = 0.34\Sigma(1+6\times10^{24}\rho T^{-7/2})$ is the Rosseland optical depth, and $Q_{br} = 1.24\times10^{21}H\rho^2T^{1/2}$ ergs s^{-1} cm ^{2} is the bremsstrahlung cooling. Note that in Eqs. (6)–(8) electrons and protons are assumed to have the same temperature (the

one-temperature flow), and only the bremsstrahlung radiation is considered. Strictly speaking, electrons and protons are more likely to have different temperatures (the two-temperature flow), and other radiative processes (the synchrotron radiation and the Comptonization) should also be considered. However, previous works on ADAFs[13,14] have shown that the dynamical and thermal behaviour of two-temperature flows with other radiative processes is not qualitatively different from those of one-temperature flows with only bremsstrahlung radiation, the difference is only quantitative. Therefore, we follow Ref. [12] to use Eq. (8) for simplicity. Of course, if one tries to calculate the flow's radiative spectrum in order to compare with the observational data, then the distinction between the electron temperature and the proton temperature and the inclusion of other radiative processes will be necessary.

The equation of state is

$$P = P_g + P_r, \tag{9}$$

where $P_g = k\rho T/\mu m_p$ is the gas pressure, and $P_r = Q_{\rm rad}(\tau_R + 2/\sqrt{3})/4c$ is the radiation pressure. Note that Eqs. (4) and (9) include the contribution of radiation, whereas in Ref. [6] the radiative cooling term was omitted in the energy equation and only the gas pressure was considered in the state equation.

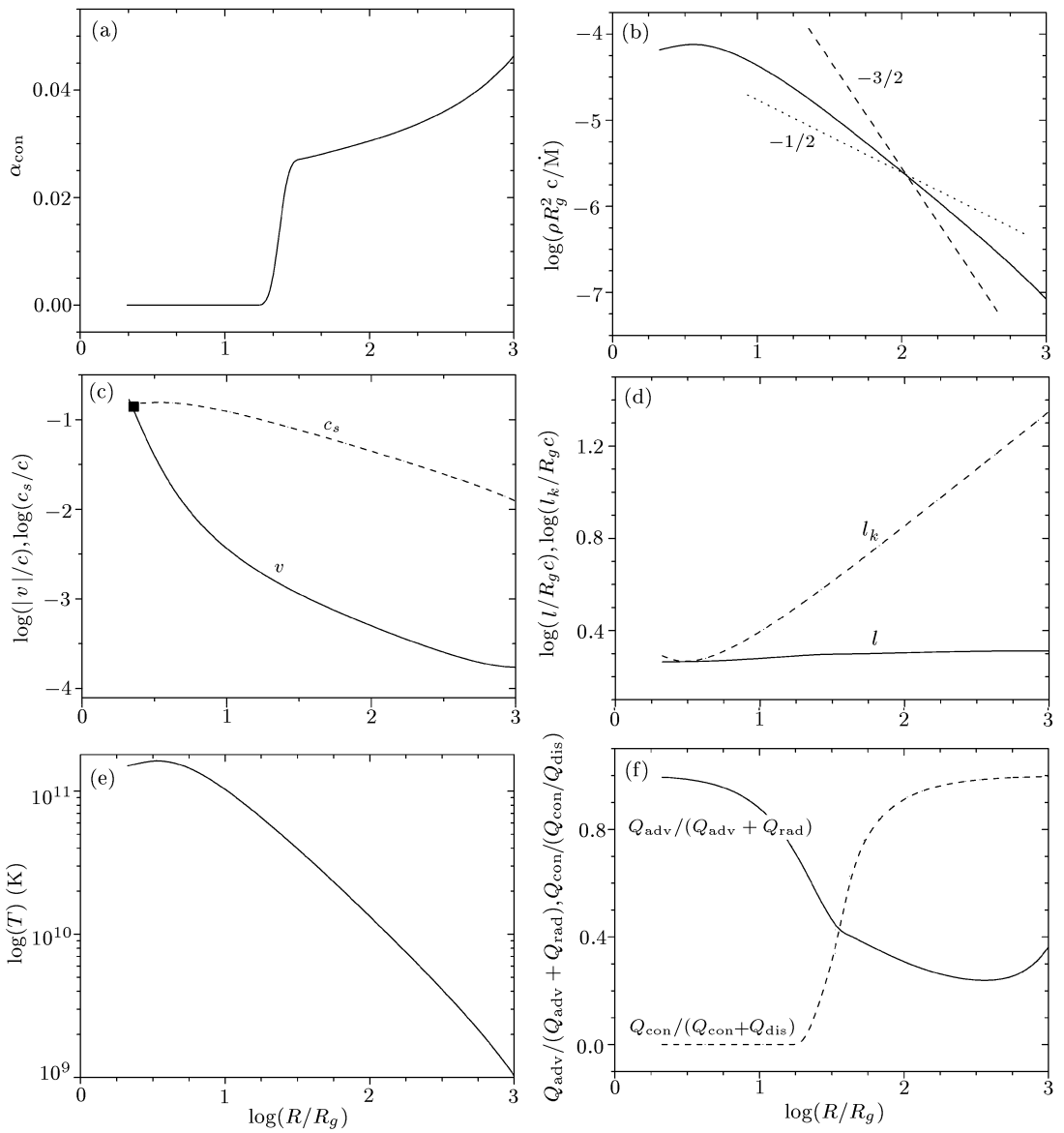


Fig. 2. Example of the type-Ⅲ two-zone solution for $\alpha = 0.001$. The arrangements are the same as those in Fig. 1.

The five equations, namely Eqs. (1)–(4) and (9), can be numerically solved for five unknown quantities v, c_s, Ω, ρ, and T as functions of R, provided that the constant parameters M, $\dot{M}$, α, γ, and j are given. The numerical method we use to solve these equations is the same as that in Ref. [6]. Note that $\alpha_{\rm con}$ is not

another unknown quantity, it can be obtained self-consistently at each radius, as described in Ref. [6]. In our calculations we fix $\dot{M} = 0.0001\dot{M}_{\rm crit}$ ($\dot{M}_{\rm crit}$ is the critical accretion rate corresponding to the Eddington luminosity) and $\gamma = 5/3$. As in Ref. [6], we also obtain three types of global solutions depending on the value of α.

Type I: Pure ADAF solution for $\alpha \gtrsim 0.1$. In this case viscous action is so strong that the flow is totally advection-dominated, and no convection develops at all.

Type II: Three-zone solution for moderate $\alpha \sim 0.01$.

Figure 1 provides an example of this type of solution, with $\alpha = 0.01$, $j = 1.57cR_g$, and the sonic radius $R_s = 2.4R_g$. Figure 1(a) shows how $\alpha_{\rm con}$ varies with R, from which a three-zone structure is clearly seen. In the middle zone ranging from $R = 41R_g$ to $R = 465R_g$, $\alpha_{\rm crit}$ is larger than α, indicating that convection is dominant. For the inner zone ($R < 41R_g$) and the outer zone ($R > 465R_g$) convection ceases to exist ($\alpha_{\rm con} = 0$). Figure 1(b) shows the radial profile of ρ (solid line), it is between the profiles $\rho \propto^{3/2}$ of the self-similar ADAF solution[3] (dashed line) and $\rho \propto R^{-1/2}$ of the self similar CDAF solution[4,5] (dotted line). Figure 1(c) is for v (solid line) and c_s (dashed line), the crossing point of the two lines is the sonic point. Figure 1(d) is for the specific angular momentum $l = \Omega R^2$ (solid line) and the Keplerian specific angular momentum $l_K = \Omega_K R^2$ (dashed line). In the inner zone, l behaves as in the pure ADAF solution of type I; in the middle zone the l profile is greatly flattened compared with that of the pure ADAF solution, because of the strong inward transport of angular momentum by the convective flux; while in the outer zone, l increases with increasing R. All these results are qualitatively similar to those of Ref. [6], the differences are only quantitative. This means that the effect of radiation on the flow's dynamical structure is unimportant. It is noticed that Figs. 1(e) and 1(f) are new, which were not given in Ref. [6]. As shown in Fig. 1(e), T increases from about 1.8×10^9 K at $R = 1000R_g$ to about 3×10^{11} K at the sonic point. Figure 1(f) may be more meaningful. It is seen that in the inner zone, the heating is only due to dissipation $Q_{\rm dis}$, and the advective cooling $Q_{\rm adv}$ is dominant; in the middle convection-dominated zone the convective heating $Q_{\rm con}$ indeed becomes more important than $Q_{\rm dis}$, and the radiative cooling $Q_{\rm rad}$ becomes comparable, or even larger than $Q_{\rm adv}$; and in the outer zone, $Q_{\rm con}$ is zero as in the inner zone, however the cooling is only due to radiation $Q_{\rm rad}$, indicating that the outer zone has features similar to those of the Shakura–Sunyaev discs.[15]

Type III: Two-zone solution for small $\alpha \lesssim 0.001$. An example of this type of solution is given by Fig. 2, with $\alpha = 0.001$, $j = 1.84cR_g$, and $R_s = 2.1R_g$. It is seen from Fig. 2(a) that the solution has only two zones with the transition radius at $R = 25R_g$. In the inner zone there is no convection ($\alpha_{\rm con} = 0$), and the outer zone is convection-dominated ($\alpha_{\rm con} > \alpha$). Figures 2(b) and 2(c) are essentially similar to Figs. 1(b) and 1(c), respectively. Figure 2(d) is qualitatively different from Fig. 1(d), in the sense that l keeps to be almost constant in the outer zone, instead of increasing outwards for large radii as in the type-II solution. Figure 2(e) shows that T is lower than that in the type-II solution, because of smaller α. As seen from Fig. 2(f), in the outer zone, convection and radiation play even more important roles than those in the middle zone of the type-II solution, in the sense that $Q_{\rm con}$ is dominant over $Q_{\rm dis}$ and that $Q_{\rm rad}$ greatly exceeds $Q_{\rm adv}$, again because the advective action becomes weaker for smaller α.

We have shown that radiative processes do not affect much the dynamical structure of CDAFs, proving that the dynamical solutions obtained in Ref. [6] are essentially correct. As the main new result of this study, it has been demonstrated that the radiative cooling can exceed the advective cooling in CDAFs. It is expected that this result may be important from the observational point of view. It is worth further calculating the luminosity and the output spectrum of the CDAF model, and compare with those obtained in the ADAF model by applying the both models to astrophysical systems which are believed to be powered by black hole accretion and have low luminosities.

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