Influence of Deviation on Optical Transmission through Aperiodic Superlattices *

YIN Hai-Long(殷海龙)[1], YANG Xiang-Bo(杨湘波)[1,2**], LAN Sheng(兰胜)[2], HU Wei(胡巍)[2]

MOE Key Laboratory of Laser Life Science and Institute of Laser Life Science, South China Normal University, Guangzhou 510631

[2]*Laboratory of Photonic Information Technology, South China Normal University, Guangzhou 510631*

(Received 17 May 2007)

We propose a deviation model and study the influences of the relative error and sensitivity of a machine on the transmission coefficients (TCs) of Fibonacci superlattices. It is found that for a system with fewer layers, the influence of deviation can be ignored. When superlattices become more complicated, they may be fabricated by a machine with suitable relative error and possess the designed value of TC. However, when the number of system layers exceeds some critical value, superlattices should be manufactured only by precise machines. The influence of the sensitivity is also discussed.

PACS: 42. 25. Bs, 01. 50. Zv, 78. 67. Pt

Since the quasicrystal was discovered in experiment by Shechtman *et al.*,[1] much attention has been paid to quasilattices and quasiperiodic superlattices. In particular, light propagation through quasiperiodic multilayers has been extensively studied. Kohmoto and co-workers[2] investigated the optical localization in two-tile multilayers following a one-dimensional (1D) Fibonacci quasiperiodic sequence and their theoretical results are confirmed by the dielectric multilayer experiment.[3]

In the past few decades, many quasiperiodic models have been proposed and the corresponding optical transmission properties have also been researched. An incomplete list includes intergrowth quasiperiodic systems,[4,5] Fibonacci and Fibonacci-class ones,[2,3,6–8] generalized Fibonacci ones,[9–12] Thue–Morse and generalized Thue–Morse ones,[13–20] extended Fibonacci ones,[21,22] and SML one,[23] etc. To the best of our knowledge, the aforementioned superlattices were all dealt with ideally without any deviation. In fact, deviations exist everywhere and should be taken into account for the experiments of complex systems. How about the influences of deviation on the optical transmission through aperiodic multilayer systems? It is an interesting and practical problem for optical thin film technology. It is generally known that the thickness of each coating thin film is always fluctuating around the designed value and can be adjusted by monitored control system. Based on this consideration we propose the following model to describe the deviation of practical thin film:

$$d_j = d_{j0}\Big[1 + r \cdot \sin\frac{(j-1)\pi}{N}\Big], \qquad (1)$$

where d_{j0} is the designed value of the jth layer of the thin film, r is the relative error of the system, and N is the sensitivity of a machine (here the larger the N value, the worse the sensitivity). In this study, we take the Fibonacci optical superlattices as an example to study the influence of deviation on the optical transmissions. It is found that the influence of deviation is not simply inversely proportional to the precision of the system. Fortunately, selecting appropriate parameters, it is possible to manufacture optical thin films fitting accurately with the designed values by use of not very precisely machine. It would be useful for the designing and fabricating of optical thin films.

Fibonacci sequence is a well-known quasiperiodic model and can be generated by the substitution rules $B \to A$ and $A \to AB$. Here we set B as the first generation, i.e., $G_1 = B$. To investigate the optical transmission of the multilayer superlattices following a Fibonacci sequence, we consider the simplest experimental settings following the conventional working principle[2] and take into account the cases of vertical incidence and identical optical phase difference for each layer. On the other hand, in order to make the quasiperiodicity of the aperiodic multilayers most effective, wavelengths should satisfy the quasi resonance condition:[2] the optical phase difference should be equal to 1/4 central wavelength, i.e., $n_i d_i = \lambda_c/4$, where n_i is the refraction index of material i and d_i denotes the thickness of the layer. In our calculations, we choose SiO_2 and TiO_2 as media A and B, respectively, and the central wavelength is 700 nm. Their indices of refraction at 700 nm are $n_A = 1.45$ and $n_B = 2.30$, respectively, and the designed physical thicknesses of the corresponding thin films A and B should be $d_{A0} = 120.7$ nm and $d_{B0} = 70.1$ nm, respectively. On the other hand, we sandwich the superlat-

* Supported by the National Natural Science Foundation of China under Grant No 10004003, and the Programme for Innovative Research Team of the Higher Education in Guangdong Province under Grant No C10034.

** To whom correspondence should be addressed. Email: xbyang@scnu.edu.cn or xiangbo.yang@yahoo.com.cn

©2007 Chinese Physical Society and IOP Publishing Ltd

tices between substrates A (i.e., SiO_2) and calculate the optical transmission coefficient (TC) by using the conventional formula[2]

$$T = \frac{4}{2 + m_{11}^2 + m_{12}^2 + m_{21}^2 + m_{22}^2}, \qquad (2)$$

where $m_{i,j}$ $(i, j = 1, 2)$ is an element of the optical propagation matrix. The results are plotted in Figs. 1–4.

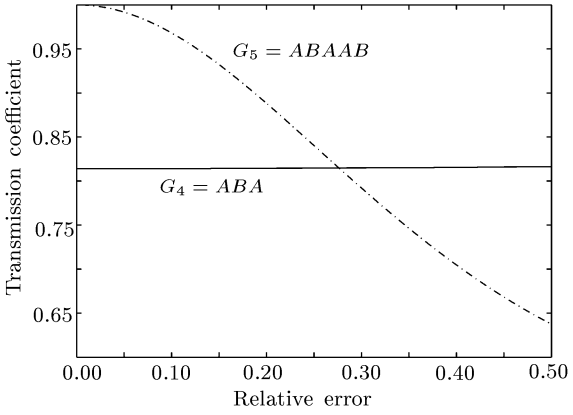

Fig. 1. Relationship between transmission coefficient and relative error for the 4th and 5th generations of Fibonacci superlattices with $N = 20$.

Figure 1 shows the numerical simulations for the 4th and 5th generations, where $N = 20$. It shows that when the parameter r changes from 0 to 0.5, the TCs of the 4th generation system ($G_4 = ABA$) change from 0.8141 to 0.8164, the absolute error is 0.0023 and the corresponding relative error is only equal to 0.29%. This is mainly because of the substrates A and the double outer layers A would be fused into the two underlayers on the left and right. Under this condition, the system $G_4 = ABA$ would be equivalent to the first generation $G_1 = B$ and consequently, the influence of deviation can be decreased quite a few and can be

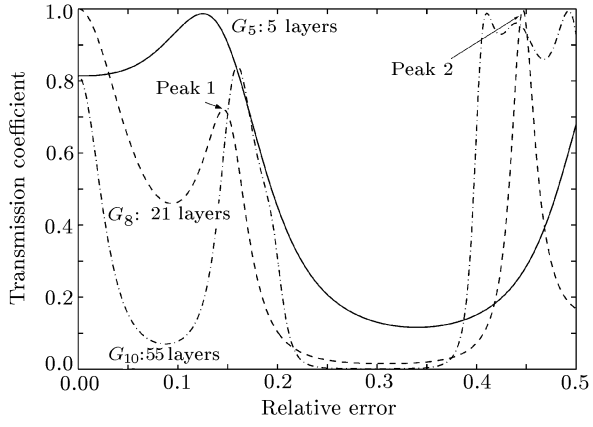

Fig. 2. Relationship between transmission coefficient and relative error for the 7th, 8th, and 10th generations of Fibonacci superlattices with $N = 20$.

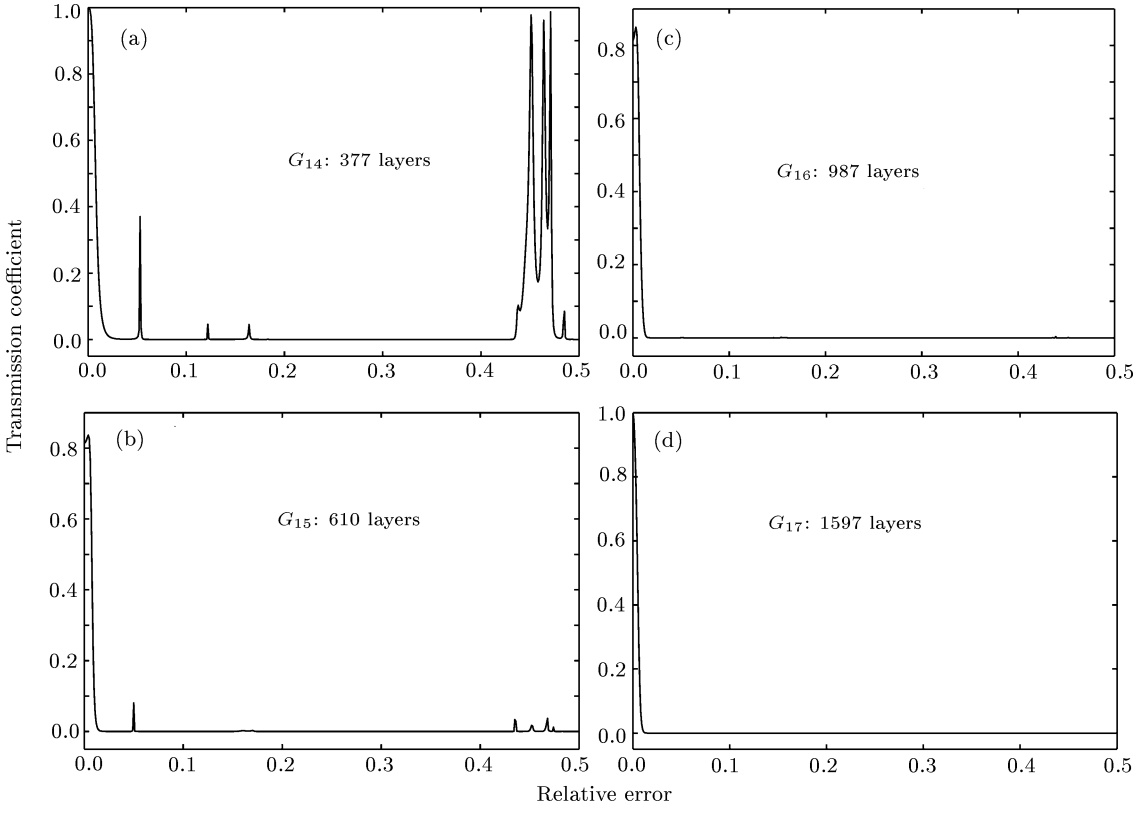

Fig. 3. Relationship between transmission coefficient and relative error for the 14th to 17th generations of Fibonacci superlattices with $N = 20$.

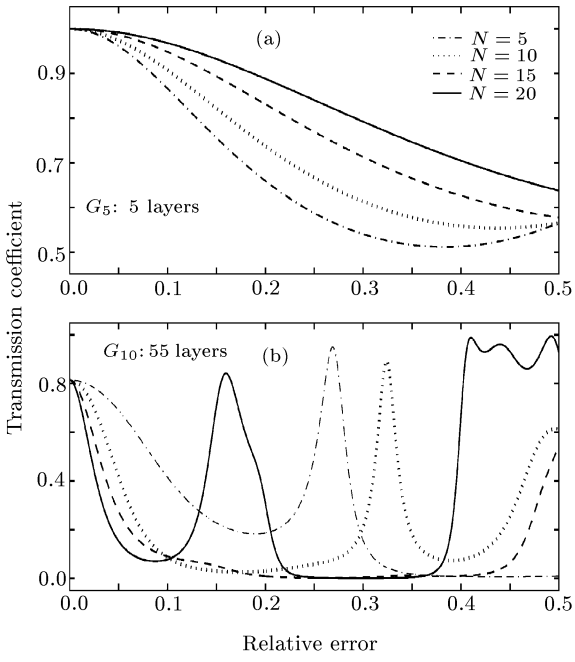

Fig. 4. Relationship between transmission coefficient and relative error for the 5th and 10th generations of Fibonacci superlattices with $N = 5, 10, 15, 20$.

ignored, i.e., for this simple system the precision of machine is not very important. However, for the 5th generation system ($G_5 = ABAAB$), the TCs decrease monotonically from 1 to 0.6376 and the corresponding relative error equals 36.24%. Obviously, one can see that when systems become more complex, the influence of deviation increases. The larger the relative error of the system is, the more seriously influence it would cause on the optical TC.

When the superlattices become more complicated, the properties are quite different. Figure 2 shows the numerical simulations of the 7th, 8th and 10th generations of Fibonacci multilayer systems, where there exist 13, 21 and 55 layers, respectively. It shows that when the parameter r changes from 0 to 0.5, the TC functions are no longer monotonic ones and on each curve there exist more than one point with the value as the same as that of $r = 0$. The reason is as follows: With the increment of the relative error of a machine, the accumulative error of the total optical phase of the complicated thin films would be increased. When the value of the error reaches some point, the total optical phase would increase (decrease) integer times of 2π and the TC for the practical thin films would be equal to that of the designed system. Because the optical phase is related to $\sum n_i d_i$, the peak value would be related to the structure parameters of the system, including the indices of refraction of media, the number of multilayers, and the order of the sequence, etc. For example, for the 8th generation system, the TCs at $r = 0.4770$ and $r = 0$ are all equal to 1.0. This means that if one controls the relative error of a machine to be equal to 44.7%, then a multilayer system possessing the designed value of TC can be manufactured. This would provide more choices for experimenters.

With the increase of layers the superlattices become more and more complex. When the number of layers exceeds some critical value and the system tends to be very sophisticated, the deviation of a machine will influence the transmission properties of thin films seriously. Figure 3 illustrates the numerical simulations of the 14th to 17th generation systems. From Fig. 3(a) one can see that even for the 14th generation superlattices (with 377 layers), there still exist some chances to choose a machine with suitable relative error fabricating them with designed optical transmission. Figs. 3(b)–3(d) show that from the 15th generation (with 610 layers) on, superlattices should be manufactured only by precise machines. Otherwise, the TCs of systems may be equal to zero. We can explain this as follows: when the number of layers exceeds some critical value, tiny relative error of a machine would cause huge change of the optical phase of the coated films. On the other hand, for all systems with $T \neq 1$ the transmissivity decays exponentially with the increment of the number of layers. Consequently, even if a designed transparent multilayer film, the practical TC would tend to be zero rapidly when the optical phase of the very complex system has been mismatched because of tiny relative error of a machine. Thus there exist some critical value for the number of layers. To estimate the order of magnitude of the critical layer number n, we suppose that the transmissivity would decay from 0.99 to 0.01 after passing through n layers of the thin film (i.e. $0.99^n \leq 0.01$). Then one can obtain that $n \approx 458$. This means that the order of magnitude of the critical layer number is 10^2.

Additionally, we discuss the influence of the sensitivity of a machine on the TCs and plot the results in Fig. 4. From Fig. 4(a) one can see that for the 5th generation system (only 5 layers), the curve with smaller N is under the curves with larger N. This means that for simple systems, the lower the sensitivity of a machine, the smaller the influence on the TC. However, when systems become more complicated, the influence of N will be indeterminate (see Fig. 4(b)).

In conclusion, we have proposed a deviation model to study the influences of the relative error r and sensitivity N of a machine on the TCs of Fibonacci superlattices. It is found that when a system is very simple, the precision of a machine is not very important. When superlattices become more and more complicated, one may choose a machine with suitable relative error to manufacture the system possessing the designed value of TC. However, when the number of

system layers exceeds some critical value, superlattices should be manufactured only by precise machines. On the other hand, we have also discussed the influence of the sensitivity of a machine on the TCs.

References

[1] Shechtman D, Blech I, Gratias D and Cahn J 1984 *Phys. Rev. Lett.* **53** 1951

[2] Kohmoto M, Sutherland B and Iguchi K 1987 *Phys. Rev. Lett.* **58** 2436

[3] Gellermann W, Kohmoto M, Sutherland B and Taylor P 1994 *Phys. Rev. Lett.* **72** 633

[4] Huang X, Liu Y and Mo D 1993 *Solid State Commun.* **87** 601

[5] Wang Y, Huang X and Gong C 2000 *Chin. Phys. Lett.* **17** 498

[6] Peng R, Huang X, Qiu F, Wang M, Hu A and Jiang S 2002 *Appl. Phys. Lett.* **80** 3063

[7] Qin Y, Su H and Tang S 2003 *Appl. Phys. Lett.* **83** 1071

[8] Yang X, Liu Y and Fu X 1999 *Phys. Rev.* B **59** 4545

[9] Riklund R and Severin M 1988 *J. Phys.* C **21** 3217

[10] Klauzer-Kruszyna, Salejda W and Tyc M 2004 *Optik* (Stuttgart) **115** 257
Klauzer-Kruszyna, Salejda W and Tyc M 2004 *Optik* (Stuttgart) **115** 267

[11] Qin Y, Zhu Y, Zhu S, Luo G, Ma J and Ming N 1999 *Appl. Phys. Lett.* **75** 448

[12] Li F and Yang X 2005 *Physica* B **368** 64

[13] Tamura S and Nori F 1989 *Phys. Rev.* B **40** 9790

[14] Liu N 1997 *Phys. Rev.* B **55** 3543

[15] Wang X, Grimm U and Schreiber M 2000 *Phys. Rev.* B **62** 14020

[16] Chattopadhyay S and Chakrabarti A 2000 *J. Phys.: Condens. Matter* **12** 5681

[17] Negro L, Stolfi M, Yi Y, Michel J, Duan X, Kimerling L, LeBlanc J and Haavisto J 2004 *Appl. Phys. Lett.* **84** 5186

[18] Jiang X, Zhang Y, Feng S, Huang K, Yi Y and Joannopoulos J 2005 *Appl. Phys. Lett.* **86** 20110

[19] Chen F and Yang X 2005 *Mod. Phys. Lett.* B **19** 655

[20] Chen F and Yang X 2005 *Phys. Status Solidi* B **242** 2509

[21] Peng R, Wang M, Hu A, Jiang S, Jin G and Feng D 1998 *Phys. Rev.* B **57** 1544

[22] Huang X, Wang Y and Gong C 1999 *J. Phys.: Condens. Matter* **11** 7645

[23] Chen F, Yang X and Xing D 2004 *Physica* B **351** 19