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Negative Refraction of a Symmetrical π-shaped Metamaterial

YAN Chang-Chun(闫长春)[1,2], CUI Yi-Ping(崔一平)[1**], WANG Qiong(王琼)[1], ZHUO Shing-Chuang(卓士创)[1]

[1] *Advanced Photonics Center, School of Electronic Science and Engineering, Southeast University, Nanjing 210096*
[2] *School of Physics and Electronic Engineering, Xuzhou Normal University, Xuzhou 221116*

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A symmetrical π-shaped metamaterial is investigated. Numerical simulations exhibit the negative-refractive property of this structure. The complex refractive index n, wave impedance z, effective permittivity ε and effective permeability μ have been retrieved from the simulated S parameters. The negative-refractive band lies between 11.2 GHz and 13.05 GHz. The frequency band with high transmission with low loss occurs between 11.95 GHz and 12.5 GHz, which is helpful for practical applications. The mechanism of the electromagnetic resonance is also revealed.

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In 1968, a novel matematerial, a left-handed material (LHM) with simultaneously negative permittivity ε and negative permeability μ, was discussed by Veselago.[1] However, increasing interest[2−4] in the LHM was attracted when the novel matematerial was first demonstrated by Smith *et al.*[5] So far, there are a few ways towards the realization of an LHM. For example, split-ring resonators (SRR) and wires,[6−8] pairs of metallic nanorods,[9] fishnet structures,[10−12] and ultrathin waveguides[13] were experimentally confirmed to be LHMs, of which frequency regions have ranged from microwave to visible spectrum, in the past seven years. As is well known, the metamaterials are generally composed of shaped elements with periodic intervals much smaller than the incident electromagnetic wavelengths, and thus they can be referred to as effective homogeneous media with electromagnetic constitutive parameters.

As far as the SRR-wire method is concerned, it has been developed as many forms, such as an Ω-shaped method,[14] an S-shaped method,[15] an H-shaped method,[16] and so on.[17] In this Letter, we propose a new configuration in which every unit cell consists of two symmetrical π-shaped wires. Although Dong *et al.*[18] studied the so-called π-shaped cells, their structure is continuous and unsymmetrical and negative refraction can not take place. Nonetheless, the configuration we discuss in the following is discrete and symmetrical. The retrieved refractive indices show that the negative-refractive band lies between 11.2 GHz and 13.05 GHz, and the frequency band with high transmission occurs between 11.95 GHz and 12.5 GHz.

As shown in Figs. 1(a) and 1(b), the configuration we consider here is a periodic copper array, in which every unit cell consists of two symmetrical π-shaped wires (as outlined by the rectangular dashed lines in Fig. 1(a)). The unit cell is cuboid. A substrate of FR4 (a kind of excellent insulating material with permittivity $\varepsilon = 4.4$, loss tangent of 0.02) is assumed. Two symmetrical π-shaped wires are positioned on the substrate, modelling the structures typically produced by lithographic circuit board techniques. Other parts in the unit cell are occupied by vacuum. All dimensions are displayed in caption under Fig. 1. For geometry, the π-shaped metamaterial can be regarded as a combination of symmetrical metallic C-shaped resonators (note that only a unit cell contains several C-shaped resonators) and continuous wires. The metamaterials with ring resonators generally generate magnetic oscillation with the above gigahertz frequencies.[19,20] Under plasma frequency, the continuous-wire metamaterials also yield negative effective permittivity.[21] Therefore, the symmetrical π-shaped metamaterial can exhibit negative refraction in a certain frequency band, which can be demonstrated in the following simulations.

A commercial electromagnetic mode solver, based on a finite element method, is used in our simulations. To justify the simulations and the retrieved electromagnetic parameters, a copper SRR-wire unit cell, in which the copper SRR and wire are set on opposite sides of a substrate, is firstly investigated. The scales, wave ports and boundary conditions in the unit cells are the same as those in Fig. 2 in Ref. [22]. The simulated S-parameter responses are in agreement with those in Fig. 3 in Ref. [22]. Based on Eqs. (9), (10) and (4) in Ref. [22], namely

$$n = \frac{1}{kd}\cos^{-1}\Big[\frac{1}{2S_{21}}(1 - S_{11}^2 + S_{21}^2)\Big], \tag{1}$$

$$z = \sqrt{\frac{(1+S_{11})^2 - S_{21}^2}{(1-S_{11})^2 - S_{21}^2}}, \tag{2}$$

$$\varepsilon = n/z, \quad \mu = nz, \tag{3}$$

we can retrieve the values for the complex refractive index n, wave impedance z, effective permittivity ε and effective permeability μ. Their frequency responses are completely coincident with those in Ref. [22].

* To whom correspondence should be addressed. Email: cyp@seu.edu.cn
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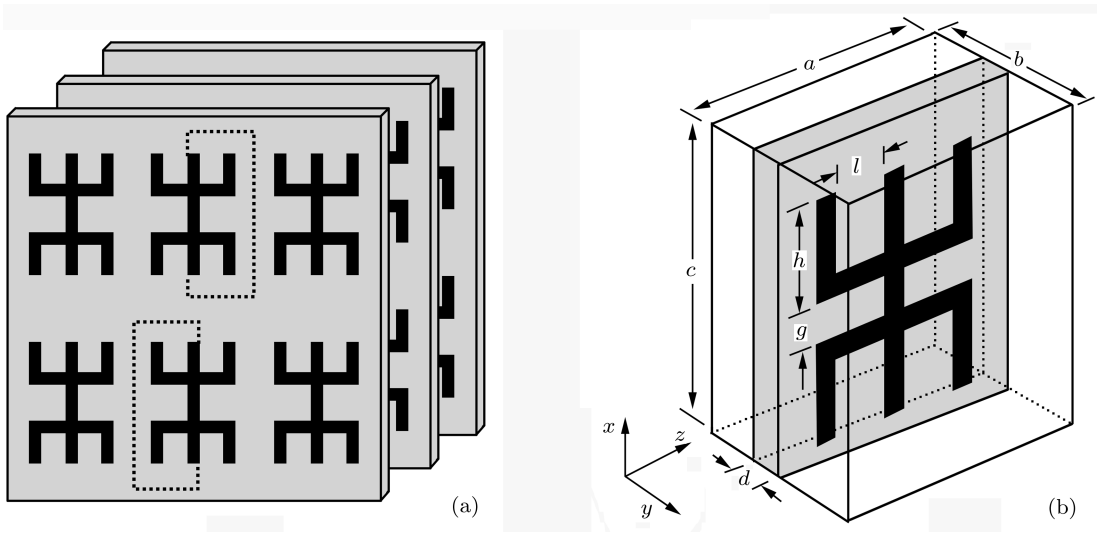

Fig. 1. (a) Schematic for the array of a π-shaped metamaterial. The rectangular dashed lines outline the so-called π shape. (b) A unit cell with geometric dimensions $a = 6\,\text{mm}$, $b = 4\,\text{mm}$, $c = 10\,\text{mm}$, $d = 0.1\,\text{mm}$, $e = 0.6\,\text{mm}$, $l = 1.6\,\text{mm}$ $h = 2.5\,\text{mm}$, and $g = 2.2\,\text{mm}$. The metal (copper) is 0.6 mm in width and 0.02 mm in thickness. The propagation of the polarized electromagnetic wave is along the z axis, and the electric field and the magnetic field are in the x and y directions, respectively.

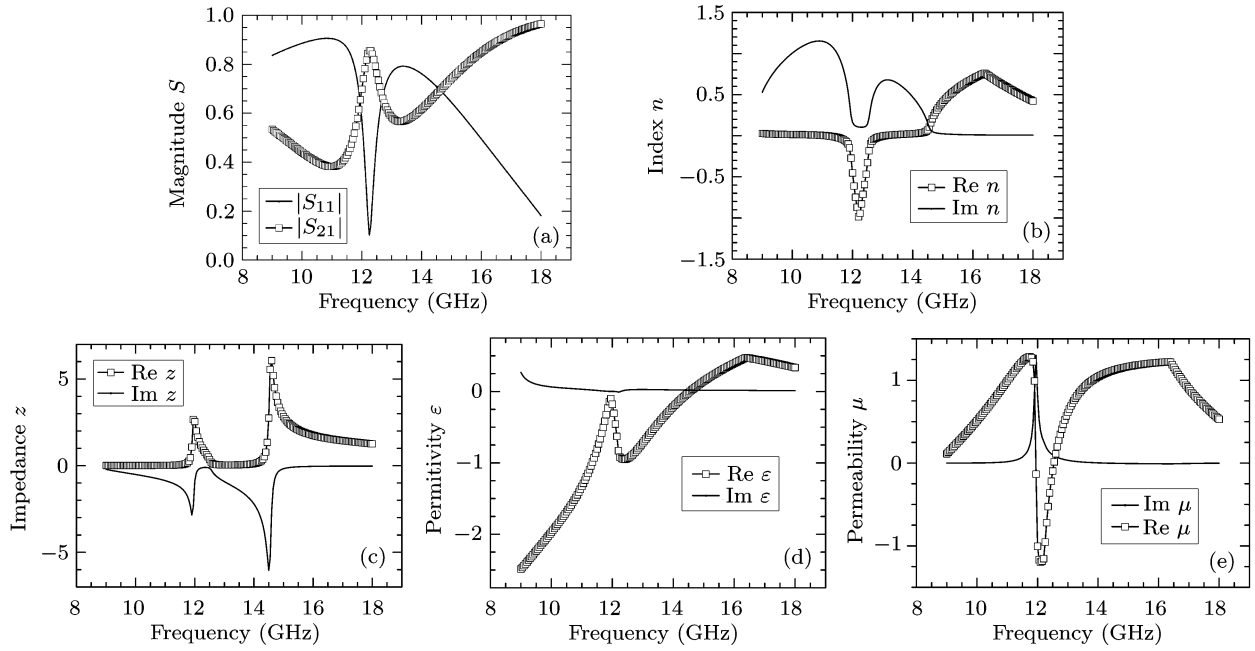

Fig. 2. (a) The magnitude of the simulated S parameters for the unit cell in Fig. 1(b); (b) the retrieved refractive index; (c) the retrieved impedance; (d) the retrieved permittivity; (e) the retrieved permeability.

Next, because the structural symmetry of the π-shaped unit cell in this work is similar to that of the SRR-wire unit cell in Fig. 2 in Ref. [22], the same method is employed to investigate the above symmetrical π-shaped metamaterial. To excite the negative magnetic response of the C resonators as well as the negative electric response of the continuous wires, the incident electromagnetic waves are polarized with the magnetic field perpendicular to the C-resonator plane ($\boldsymbol{H}//y$ axis) and the electric field parallel to the x axis ($\boldsymbol{E}//x$ axis). Consequently, the propagation direction (wave vector $\boldsymbol{k}$) is along the z axis. The two wave ports are set in the planes perpendicular to the z axis. The perfect magnetic and electric boundaries (corresponding to the boundaries perpendicular to the y and x axes, respectively) are applied because of the symmetry of this metamaterial, which means that $\boldsymbol{E}$ and $\boldsymbol{H}$ always parallel to the x and y axes, respectively. Both the S parameters and retrieved material parameters are described in Fig. 2. Figure

2(a) shows the magnitude of the computed S parameters. The retrieved index in Fig. 2(b) confirms the negative-refractive band lying between 11.2 GHz and 13.05 GHz. Attentively, $-n_1/n_2 > 1$ is met between 11.95 GHz and 12.5 GHz, which indicates high transmission with low loss in the frequency band. Here n_1 and n_2 are the real and imaginary parts of the refractive index. Therefore, the negative-refractive metamaterial in the band is helpful to practical applications. In addition, the retrieved impedance, the retrieved permittivity and retrieved permeability are shown in Figs. 2(c), 2(d) and 2(e), respectively. From Figs. 2(d) and 2(e), we obtain negative real parts of permittivity and permeability (i.e. Re $\varepsilon < 0$ and Re $\mu < 0$ simultaneously) between 11.95 GHz to 12.5 GHz, differing from the band of the negative index. The reason is that a wider negative Re n range is still achieved as the condition $\varepsilon_1 \cdot \mu_2 + \mu_1 \cdot \varepsilon_2 < 0$ is satisfied,[10,23] where $\varepsilon = \varepsilon_1 + i\varepsilon_2$ and $\mu = \mu_1 + i\mu_2$. Moreover, Fig. 2(e) shows the strong magnetic resonance (Re $\mu = -1.2$ at 12.1 GHz), occurring around frequency of 12.1 GHz. However, the magnetic absorption is weaker (Im $\mu = 0.39$ at the same frequency). Around the magnetic-resonance frequency, there is also an electric resonance (see Fig. 2(d)), which indicates the existence of LC resonance circuit.

Finally, to give the explanation of the electromagnetic resonance, the distributions of induced current in the π-shaped copper wires are simulated around the resonant frequency. Figure 3 displays the distributions of different phases in a resonant period. The curves with arrows represent the flowing directions of induced current. We can see clearly from the figure the process of resonance.

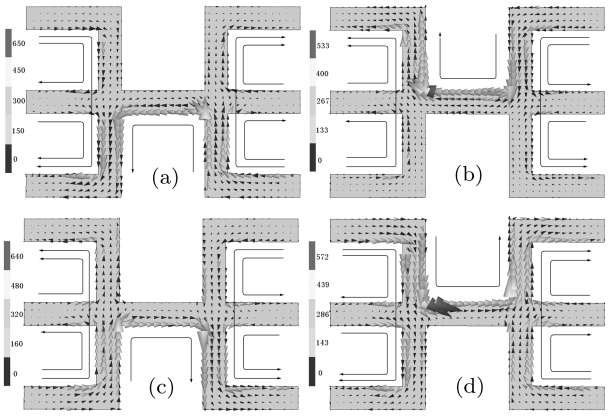

Fig. 3. Distributions of induced current during a resonate period in the -shaped copper wires at the frequencies of 12.25 GHz. (a) Phase equals 10 deg; (b) phase equals 80 deg; (c) phase equals 180 deg; (d) phase equals 280 deg.

In conclusion, a novel symmetrical π-shaped metamaterial has been investigated. Numerical simulations exhibit the negative-refractive property of this structure. To justify the numerical simulations, we have firstly simulated the SRR and wires structure in Ref. [22]. The S parameters and the retrieved values are completely coincident with those in Ref. [22]. After that, the same method has been applied to study the metamaterial designed. The complex refractive index n, wave impedance z, effective permittivity ε and effective permeability μ have been retrieved from the simulated S parameters. The strong magnetic resonance is found around frequency of 12.1 GHz, and the negative-refractive band lies between 11.2 GHz and 13.05 GHz. Furthermore, the negative-refractive band with high transmission with low loss occurs between 11.95 GHz and 12.5 GHz, which is helpful to practical application. In addition, we also give the explanation of the mechanism of the electromagnetic resonance.

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