CHIN.PHYS.LETT. Vol. 25, No. 2 (2008) 540

Controllable Kerr Nonlinearity with Perfect Transparency in the Existence of Spontaneously Generated Coherence *

NIU Yue-Ping(钮月萍)**, GONG Shang-Qing(龚尚庆), LI Ru-Xin(李儒新), XU Zhi-Zhan(徐至展)

State Key Laboratory of High Field Laser Physics, Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Shanghai 201800

(Received 7 September 2007)

We investigate the Kerr nonlinearity of a V-type three-level atomic system where the upper two states decay outside to another state and hence spontaneous generated coherence may exist. It is shown that dark state and hence perfect transparency present under certain conditions. Meanwhile, the Kerr nonlinearity can be controlled by manipulation of the decay rates and the splitting of the two excited states. Therefore, enhanced Kerr nonlinearity without absorption can be obtained under proper parameters.

PACS: 42. 65. −k, 42. 50. Ar, 42. 50. Gy

The effect of spontaneously generated coherence (SGC), which is known from the pioneering work of Agarwal,[1] has led to various interesting phenomena. The existence of such coherence relies on the nonorthogonality of the two transitions. In atomic systems, although it is difficult to realize SGC in free space, and several proposals have been put forward to bypass this strict condition.[2,3] For example, Ficek and Swain[3] have developed a project to obtain SGC from a V-type three-level system with perpendicular dipole moments by a dc field. The effect of SGC has been investigated not only in atomic systems[4] and molecular systems,[5] but also in semiconductor quantum dot systems.[6] It has led to suppression of spontaneous emission,[7] self-induced transparency,[8] modified quantum beats,[9] unexpected population inversions,[10] dark state polaritons,[11] giant Kerr nonlinearity,[12] and so on. The giant Kerr nonlinearity induced by SGC recently demonstrated by us[12] dealt with the general three-level atomic systems where strong coupling and weak probe fields were applied. On the other hand, transparency induced by SGC[8] have shown that decay interference from the upper two levels of a four-level system can lead to loss-free propagation without a second coupling field. In this Letter, we study the same four-level atomic system with a single ground state and two excited states which decay outside to another state. A single laser field interacts with this system and the Kerr nonlinear property is explored.

Kerr nonlinearity corresponds to the refractive part of the third-order susceptibility in optical media. As is known, the Kerr-type nonlinear coefficients play a crucial role in nonlinear optics. The large enhancement of nonlinear susceptibilities with small absorption makes the nonlinear optics studied at low light levels.[13−16] Several schemes, such as additional fields,[17] autoionizing resonances,[18] SGC between the pump and probe transitions,[12] interacting double-dark states[19] and coherent tunnelling in quantum wells,[20] etc., have been proposed to realize enhancement of Kerr nonlinearity with reduced absorption. In this study, we focus on the control of Kerr nonlinearity with perfect transparency. When the maximum SGC presents, proper atomic parameters enable the occurrence of dark state and hence perfect transparency. With control over the two upper levels splitting and their decay rates, the Kerr nonlinearity can be built up evidently.

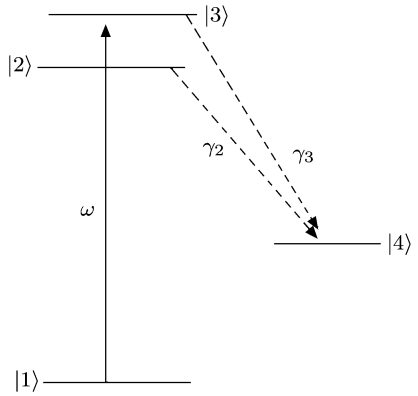

Fig. 1. A schematic diagram of the atomic system considered. The ground state $|1\rangle$ is coupled by a laser field ω to the two close-lying excited states $|2\rangle$ and $|3\rangle$, which decay to a common state $|4\rangle$ and hence SGC between transitions $|2\rangle$-$|4\rangle$ and $|3\rangle$-$|4\rangle$ may present.

The atomic-level scheme considered here is shown in Fig. 1. Two close-lying excited states $|2\rangle$ and $|3\rangle$ are coupled to the ground state $|1\rangle$ by a coherent driving field ω. The decay rates from the upper pair levels to the outside state $|4\rangle$ are γ_2 and γ_3. When states

*Supported by the National Natural Sciences Foundation of China under Grant No 60708008, the Project of Academic Leaders in Shanghai under Grant No 07XD14030, and the Knowledge Innovation Project of Chinese Academy of Sciences.

** Email: niuyp@mail.siom.ac.cn

© 2008 Chinese Physical Society and IOP Publishing Ltd

$|2\rangle$ and $|3\rangle$ are closely spaced such that the two transitions to state $|4\rangle$ interact with the same vacuum mode, then the two decay pathways are not independent and SGC could be present. Under the rotating wave approximation, the systematic Hamiltonian is given by ($\hbar = 1$ is assumed)

$$H = \sum_{n=1-4} \omega_n |n\rangle\langle n| + \sum_k \omega_k a_k^\dagger a_k + (\Omega_2 |1\rangle\langle 2| + \Omega_3 |1\rangle\langle 3| + \sum_{m=2,3} g_k^{(m)} a_k |m\rangle\langle 4| + \text{H.c.}), \tag{1}$$

where $\Omega_{2(3)} = \mathrm{d}_{12(3)} \cdot \boldsymbol{E}$ represents the Rabi frequency of the respective transition; ω_n is the unperturbed energy of state $|n\rangle$; a_k ($a_k^\dagger$) is the annihilation (creation) operator of the kth vacuum mode with angular frequency ω_k; and $g_k^{(m)}$ is the coupling constant of the atomic transitions $|m\rangle$ to $|4\rangle$ with the kth vacuum mode. In our analysis we neglect any decay to the ground state $|1\rangle$. By making use of the time-dependent Schrödinger equation, formally eliminating the vacuum amplitude and applying the Weisskopf–Wigner theory of spontaneous emission,[21] the corresponding equations of motion for the probability amplitude evolution can be obtained by

$$\frac{\partial}{\partial t} c_1 = -i(\Omega_2 c_2 + \Omega_3 c_3), \tag{2}$$

$$\frac{\partial}{\partial t} c_2 = -i\Big[\Omega_2 c_1 + (\delta_2 - i\frac{\gamma_2}{2}) c_2 - i\frac{p\sqrt{\gamma_2\gamma_3}}{2} c_3\Big], \tag{3}$$

$$\frac{\partial}{\partial t} c_3 = -i\Big[\Omega_3 c_1 - i\frac{p\sqrt{\gamma_2\gamma_3}}{2} c_2 + (\delta_3 - i\frac{\gamma_3}{2}) c_3\Big], \tag{4}$$

where $\delta_{21} = \omega_{21} - \omega$ and $\delta_{31} = \omega_{31} - \omega$ are the laser detunings. The energy level splitting of $|2\rangle$ and $|3\rangle$ is described by ω_{23}. The parameter p denotes the alignment of the two spontaneous emission dipole matrix elements and is defined as $p = \mathrm{d}_{42} \cdot \mathrm{d}_{43}/|\mathrm{d}_{42} \cdot \mathrm{d}_{43}|$. If d_{42} and d_{43} are parallel (antiparallel), then $p = 1$ ($p = -1$), while if d_{42} and d_{43} are orthogonal, then $p = 0$. The terms with $p\sqrt{\gamma_2\gamma_3}/2$ represent the quantum interference results from the cross coupling between spontaneous emission paths $|2\rangle$-$|4\rangle$ and $|3\rangle$-$|4\rangle$. Obviously, the effect of SGC occurs only if $p \neq 0$.

The single atom polarization is given by $\langle d\rangle = (d_{12}c_2c_1^* + d_{13}c_3c_1^*)e^{-i\omega t} + \text{H.c.}$, while the macroscopic polarization with N atoms is described by $P(\omega) = N\langle d\rangle = \varepsilon_0\chi^{(1)}E + \varepsilon_0\chi^{(3)}EEE/2 + \cdots$. In order to derive the linear and third-order nonlinear susceptibilities, we need to obtain the steady-state solution of the amplitude variable equations. In the limit of weak incident field, we tailor the atom dynamics in such a way that the initial condition on the populations remains essentially unaltered, i.e., $|c_1|^2 \approx 1$. After considering the higher-order corrections, we ultimately arrive at an approximate solution for the linear and third-order nonlinear susceptibilities:

$$\chi^{(1)} = \frac{2NA}{\varepsilon_0[2\gamma_3\delta_2 + \gamma_2(-i\gamma_3 + 2\delta_3) + 4i(\eta^2 + \delta_2\delta_3)]}, \tag{5}$$

$$\chi^{(3)} = \frac{32NiA[C + D - 4\eta(\gamma_2 + \gamma_3)d_{12}d_{13}]}{\varepsilon_0[2i\gamma_3\delta_2 + \gamma_2(\gamma_3 + 2i\delta_3) - 4(\eta^2 + \delta_2\delta_3)^2 B]}, \tag{6}$$

with $A = \gamma_3 d_{12}^2 + 2i\delta_3 d_{12}^2 + d_{13}[-4\eta d_{12} + (\gamma_2 + 2i\delta_2)d_{13}]$, $B = 2i\gamma_3\delta_2 - \gamma_2(\gamma_3 - 2i\delta_3) + 4(\eta^2 + \delta_2\delta_3)$, $C = [\gamma_3^2 + 4(\eta^2 + \delta_3^2)]d_{12}^2$, $D = [\gamma_2^2 + 4(\eta^2 + \delta_2^2)]d_{13}^2$ and $\eta = p\sqrt{\gamma_2\gamma_3}/2$.

We now study the dynamics of the linear and Kerr nonlinear susceptibilities of the V-type four-level system in the presence of SGC. Following Paspalakis *et al.*,[8] the above expressions can be simplified under the condition of dark state. In the current case, if

$$p = 1, \quad \text{and} \quad d_{13} = d_{12} \tag{7}$$

are satisfied, then electromagnetically induced transparency (EIT) presents at the detuning of

$$\delta_2 = -\frac{\sqrt{\gamma_2}}{\sqrt{\gamma_3}}\delta_3 = -\frac{\sqrt{\gamma_2}\omega_{23}}{\sqrt{\gamma_2} + \sqrt{\gamma_3}}. \tag{8}$$

Substituting Eq. (8) into Eq. (6), $\mathrm{Re}[\chi^{(3)}]$ can be rewritten as

$$\mathrm{Re}[\chi^{(3)}] = \frac{2Nd_{12}^4(\sqrt{\gamma_2} - \sqrt{\gamma_3})(\sqrt{\gamma_2} + \sqrt{\gamma_3})^3(\gamma_2 + \gamma_3)}{\varepsilon_0\gamma_2^{3/2}\gamma_3^{3/2}\omega_{23}^3}. \tag{9}$$

Together with Eq. (8), it is clearly found that the value of Kerr nonlinearity and the position of perfect transparency are absolutely determined by the decay rates and the energy level splitting. When the two decay rates are equal, the Kerr nonlinearity is zero at the transparency point. However, except for this case, the Kerr nonlinearity can be controlled by the optimization of atomic parameters. On the one hand, the smaller the energy level splitting is, the larger the Kerr nonlinearity will be. On the other hand, the dependence of Kerr nonlinearity on γ_3/γ_2 exhibits different evolutions in the region of $\gamma_3/\gamma_2 > 1$ and $\gamma_3/\gamma_2 < 1$. As shown in Fig. 2, in the region of $\gamma_3/\gamma_2 > 1$, the higher the ratio is, the larger the Kerr nonlinearity will be; while in the region of $\gamma_3/\gamma_2 < 1$, the lower the ratio is, the larger the Kerr nonlinearity will be. Furthermore, the Kerr nonlinearity builds up more sharply in the region of $\gamma_3/\gamma_2 < 1$ than that of $\gamma_3/\gamma_2 > 1$.

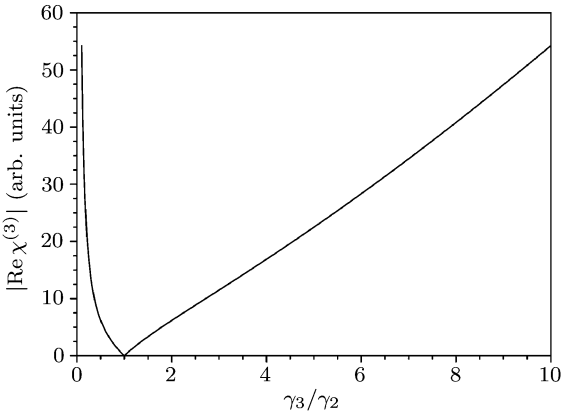

Fig. 2. Dependence of $|\ \mathrm{Re}[\chi^{(3)}]\ |$ on γ_3/γ_2 when $p = 1$ and $\omega_{23} = \gamma_2$.

Figure 3 displays the numerical simulation of $\mathrm{Im}[\chi^{(1)}]$ and $\mathrm{Re}[\chi^{(3)}]$ with and without SGC. As can be seen, both the linear absorption and Kerr nonlinearity behave differently under different values of γ_3/γ_2. Meanwhile, the energy level splitting ω_{23} has a substantial effect on the Kerr nonlinearity when SGC is considered. Just as shown in Figs. 3(a)–3(c), at the detuning of perfect transparency, higher ratio of γ_3/γ_2 and smaller level splitting ω_{23} lead to the distinct enhancement of Kerr nonlinearity. For comparison, we plot the three sets of parameters without SGC in Figs. 3(d)–3(f). It is obvious that in the existence of SGC, not only the Kerr nonlinearity is enhanced to about one order of magnitude but also perfect transparency presents simultaneously.

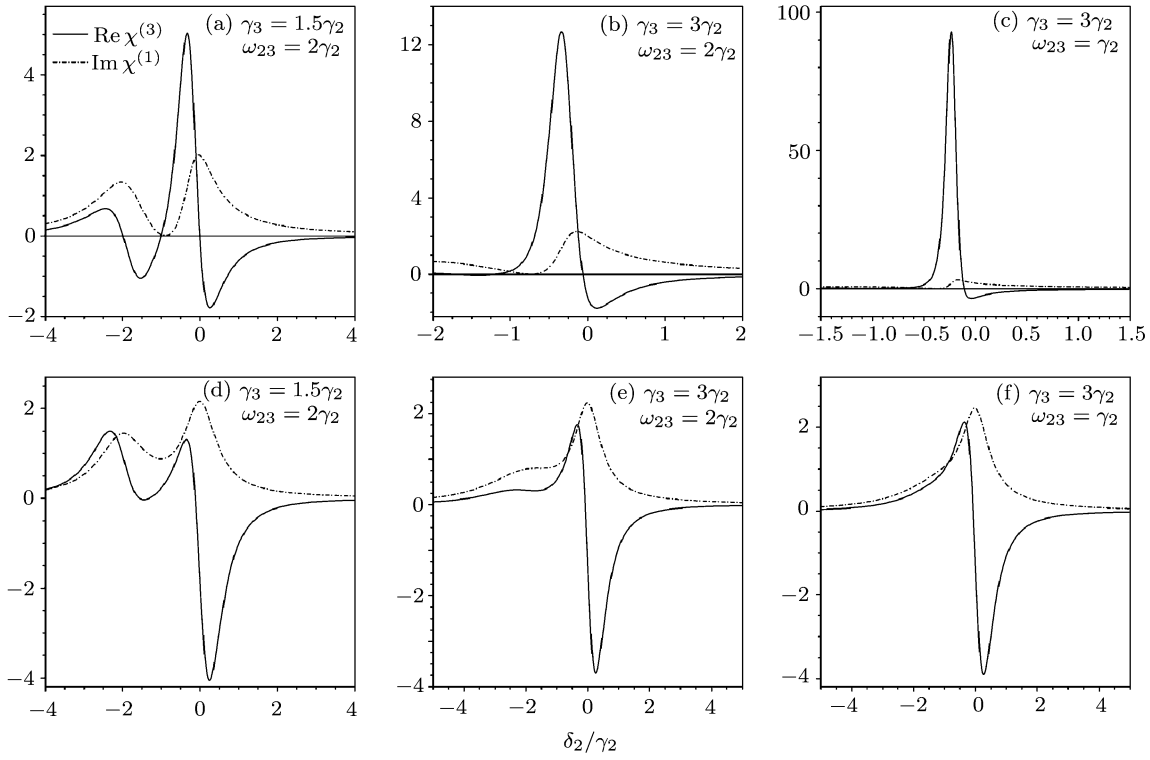

Fig. 3. Linear absorption $\mathrm{Im}\chi^{(1)}$ and Kerr nonlinearity $\mathrm{Re}\chi^{(3)}$ as a function of δ_2 for $p = 1$ [(a)–(c)] and $p = 0$ [(d)–(f)].

According to the above discussion, atomic system with maximum SGC should be satisfied. Usually, the external driving field applied to the upper two excited states cannot make the dipole moments parallel or anti-parallel. In order to obtain the maximum SGC (parallel or anti-parallel dipole moments), an alternative method is to apply a strong laser field to one of the two transitions in a lambda-type atom.[2] That is to say, the upper two states in our analysis should be the two dressed levels of one excited state. Thus, the sub-system with the upper dressed levels $|2\rangle$, $|3\rangle$ and state $|4\rangle$ behaves as a V-type system with parallel dipole moments. This system has the advantage that the magnitudes of the transition dipole moments and the upper level splitting can be controlled by the Rabi frequency and detuning of the applied laser field. Therefore, the amplitude of the Kerr nonlinearity can be manipulated by the atomic parameters under the condition of perfect transparency.

In conclusion, we have suggested an atomic scheme for enhancing Kerr nonlinearity with perfect transparency. In the dressed state picture, the maximum SGC between two transitions from the upper close-lying states to an outside state can be generated. In the case of $p = 1$ and $d_{12} = d_{13}$, proper detuning of $\delta_2 = -\omega_{23}\sqrt{\gamma_2}/(\sqrt{\gamma_2} + \sqrt{\gamma_3})$ makes dark state exist.

By manipulation of the decay rates and the upper level splitting, Kerr nonlinearity with perfect transparency can be enhanced evidently.

References

[1] Agarwal G S 1974 *Quantum Statistical Theories of Spontaneous Emission and Their Relation to Other Approaches* ed Hohler G *Springer Tracts in Modern Physics* (Berlin: Springer) vol 70

[2] Ficek Z and Swain S 2002 *J. Mod. Opt.* **49** 3

[3] Ficek Z and Swain S 2004 *Phys. Rev.* A **69** 023401

[4] Patnaik A K and Agarwal G S 1999 *Phys. Rev.* A **59** 3015

[5] Xia H R, Ye C Y and Zhu S Y 1996 *Phys. Rev. Lett.* **77** 1032

[6] Gurudev Dutt M V, Cheng J, Li B et al 2005 *Phys. Rev. Lett.* **94** 227403

[7] Zhu S Y and Scully M O 1996 *Phys. Rev. Lett.* **76** 388

[8] Paspalakis E, Kylstra N J and Knight P L 1999 *Phys. Rev. Lett.* **82** 2079

[9] Zhou P and Swain S 1998 *J. Opt. Soc. Am.* B **15** 2593

[10] Gong S Q, Paspalakis E and Knight P L 1998 *J. Mod. Opt.* **45** 2433

[11] Joshi A and Xiao M 2005 *Eur. Phys. J.* D **35** 547

[12] Niu Y P and Gong S Q 2006 *Phys. Rev.* A **3** 053811

[13] Hemmer P R, Katz D P, Donoghue J et al 1995 *Opt. Lett.* **20** 982

[14] Imamoglu A, Schmidt H, Woods G and Deutsch M 1997 *Phys. Rev. Lett.* **79** 1467

[15] Wu Y and Deng L 2004 *Phys. Rev. Lett.* **93** 143904

[16] Harris S E and Yamamoto Y 1998 *Phys. Rev. Lett.* **81** 3611

[17] Schmidt H and Imamoglu A 1996 *Opt. Lett.* **21** 1936

[18] Nakajima T 2000 *Opt. Lett.* **25** 847

[19] Niu Y P, Gong S Q, Li R X, Xu Z Z and Liang X Y 2005 *Opt. Lett.* **30** 3371

[20] Sun H, Gong S Q, Niu Y P et al 2006 *Phys. Rev.* B **74** 155314

[21] Scully M O and Zubairy M S 1997 *Quantum Optics* (Cambridge: Cambridge University Press)