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Dual Role of Wall Slip on Linear Stability of Plane Poiseuille Flow *

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The linear stability of plane Poiseuille flow is extended to the cases of two wall boundaries maintaining different slip coefficients β_1 and β_2. We determine the slip coefficient pairs, β_{1c} and β_{2c}, which yield the same critical Reynolds number as the classical no-slip case. It is found that the wall slip may stabilize the flow for $\beta_1 > \beta_{1c}$ and $\beta_2 > \beta_{2c}$, whereas slightly destabilize the flow for $\beta_1 < \beta_{1c}$ and $\beta_2 < \beta_{2c}$.

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Finite slip at the solid surface may be considered in many situations such as lubrication and microfluidics. For instance, evidences from both experiments[1] and molecular dynamic simulations[2] suggest the possibility of slip at the wall boundaries. With the research on the evidence and the possible mechanisms of slip, much attention has been given to the relationship between flow stability and wall slip.

We are here interested in the linear stability of plane channel flow with slip boundary conditions (BCs), especially the effect of wall slip on the instability threshold. Gersting,[3] who seems to be the first to examine such problem, concluded that wall slip has a stabilizing effect on the flow stability. Similar conclusions may be found in Spille *et al.*[4] However, Chu[5] indicated that wall slip degrades the flow stability significantly. The disagreement of Chu's results[5] with Gersting's conclusions[3] led Lauga and Cossu[6] to make an analysis of their work. Lauga and Cossu[6] revealed that under the same slip basic flow the incorrect disturbance BCs adopted in Chu,[5] i.e., no-slip ones, lead to incorrect results, and the use of the slip disturbance BCs shows a stabilizing effect of wall slip on the flow stability, in accordance with previous results of Gersting and Spille *et al.*[3,4] Therefore slip disturbance BCs prevail, which encourages us to pursue the work further under slip disturbance BCs.

The surface wettability, surface roughness and surface chemistry etc.[7] at two separate walls are distinct, which may result in the walls maintaining different slip coefficients β_1 and β_2. The linear stability of plane Poiseuille motion with different slip coefficients at the wall boundaries was addressed by Lauga and Cossu.[6] However, they only concerned with the two special cases of symmetric slip ($\beta_1 = \beta_2 = \beta$) and asymmetric slip ($\beta_1 = \beta, \beta_2 = 0$). The present work extends such stability problem to various pairs of β_1 and β_2 under slip disturbance BCs. It may be expected that the wall slip would always stabilize the flow under slip disturbance BCs according to Refs. [3,4,6]. However, surprisingly this is not always found to be the case, and our work shows that some interesting conclusions can be reached.

The plane Poiseuille flow is the term used to describe fluid motion between two infinite parallel plates located at $y^* = \pm H$, where a Newtonian fluid with viscosity μ is driven by a constant pressure gradient dp^*/dx^* in the x^* direction. The asterisked variables are dimensional. For the two-dimensional Poiseuille flow, only the streamwise component of the velocity is non-zero, i.e., $u_x^* = U^*(y^*)$. The continuous equation is satisfied identically and the incompressible Navier–Stokes equations are simplified as $\mu d^2U^*/dy^{*2} = dp^*/dx^*$. Navier's concept of slip is often quoted today, where the relative velocity of the fluid with respect to the wall (slip velocity) is proportional to the wall shear rate.[8] For the plane Poiseuille flow, the Navier slip conditions should be described as $U^* + \beta_1^* dU^*/dy^* = 0$ at $y^* = +H$ and $U^* - \beta_2^* dU^*/dy^* = 0$ at $y^* = -H$.[6]

All variables will be rendered dimensionless using H, $U_{\rm ref}$, $H/U_{\rm ref}$, $\rho U_{\rm ref}^2$ as units for length, velocity, time and pressure. The Reynolds number is defined by $Re = \rho H U_{\rm ref}/\mu$, where the characteristic velocity $U_{\rm ref}$ may be chosen as the maximum velocity of the no-slip case. The dimensionless basic solution satisfying the slip BCs and governing equation can be solved analytically by

$$U(y) = 1 + \frac{2(\beta_1 + \beta_2 + 2\beta_1\beta_2)}{2 + \beta_1 + \beta_2} + \frac{2y(\beta_1 - \beta_2)}{2 + \beta_1 + \beta_2} - y^2. \tag{1}$$

Note that in the special case β_1=β_2=0, the problem reduces to the familiar no-slip case.

Considering the temporal evolution of the infinitesimal disturbances, two-dimensional disturbance is expected to be applicable according to Squire's theorem since we are only interested in the instability threshold. In the linear normal mode analysis, the disturbance stream function may be assumed as the form

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of Tollmien–Schlichting wave. The dimensionless governing equation for the disturbance amplitude $\varphi(y)$ is the classical Orr–Sommerfeld (O-S) equation, in the form of

$$(D^2-\alpha^2)^2\varphi(y) = i\alpha Re[(U-\omega/\alpha) \cdot (D^2-\alpha^2) - D^2U]\varphi(y), \quad (2)$$

where $D = d/dy$, the wave number α is taken to be real, the frequency $\omega = \omega_r + i\omega_i$ treated as complex, and U the slip basic solution. The slip BCs satisfied by the disturbance amplitudes are $\varphi = D\varphi + \beta_1 D^2\varphi = 0$ at $y = +1$ and $\varphi = D\varphi - \beta_2 D^2\varphi = 0$ at $y = -1$.

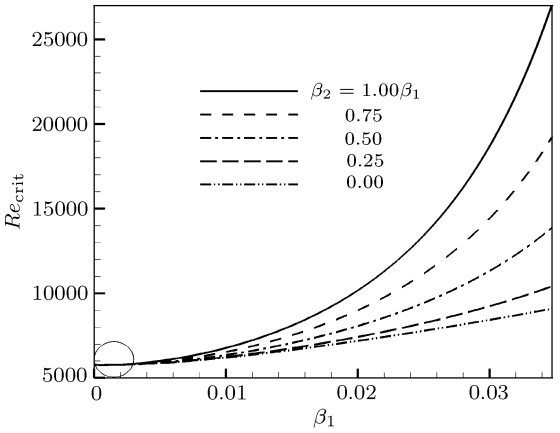


Fig. 1. Critical Reynolds number $Re_{\rm crit}$ as a function of slip coefficients β_1 for different relationships of β_2 with β_1.

Consideration of the O-S equation subject to the BCs at the walls forms an eigenvalue problem, whose solution can be expressed in the form $\omega = \omega(\alpha, Re, \beta_1, \beta_2)$ and can be solved by using a spectral collocation method based on N-th Chebyshev polynomials, of which a comprehensive description can be found in Schmid and Henningson.[9] The basic flow is unstable under disturbances for positive values of $(\omega_i)_{\max}$, the imaginary part of the first eigenvalue which crosses the real axis. For given β_1 and β_2, the curves of $(\omega_i)_{\max} = 0$ in the $(Re,\ \alpha)$-plane are termed neutral stability curves, where the minimum Reynolds number is called the critical Reynolds number, $Re_{\rm crit}$, and the corresponding value of α is the critical wavenumber $\alpha_{\rm crit}$.

Table 1. Critical values as a function of the spectral approximation order for the classical no-slip case.

N	$Re_{\rm crit}$	$\alpha_{\rm crit}$
50	5772.1752 8782	1.0204 6793
66	5772.2230 8608	1.0206 0063
80	5772.2229 9247	1.0206 0164
100	5772.2229 9247	1.0206 0164

The verification of the numerical algorithm is the computation of the specific critical values of no-slip case, the accepted values of $Re_{\rm crit}$ and $\alpha_{\rm crit}$ to be 5772.22 and 1.02056.[10] Table 1 presents $Re_{\rm crit}$ and $\alpha_{\rm crit}$ as a function of the number of spectral modes N. Throughout this work, $N = 66$ is used for the computation of the linear stability, that provides six exact figures both in the critical Reynolds number and in the critical wavenumber, and is in good agreement with the classical values.

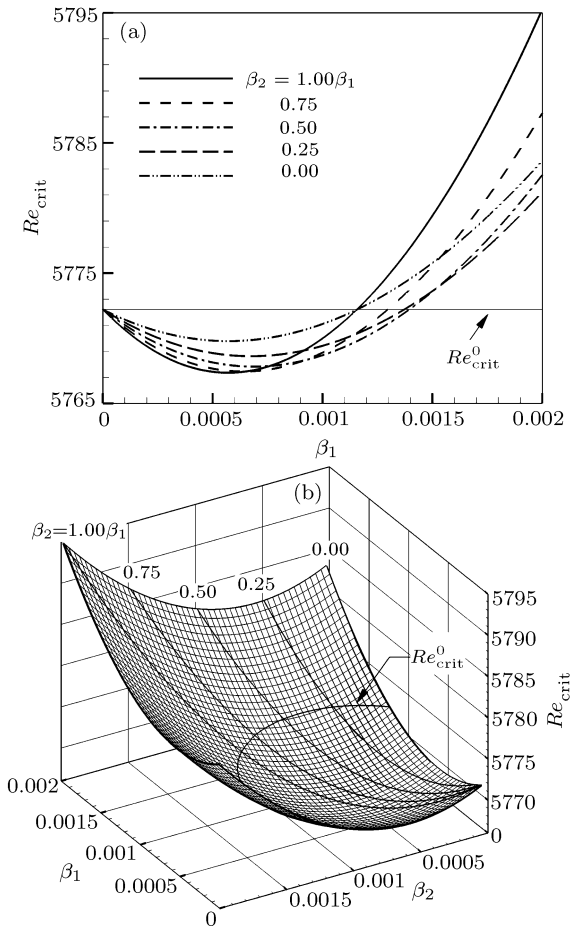


Fig. 2. (a) A closer view of the circled area in Fig. 1(b) The corresponding critical surface $Re_{\rm crit}(\beta_1, \beta_2)$ for extreme low slip coefficients.

Without loss of generality, we restrict the discussions to the cases $0 \le \beta_2 \le \beta_1$. Figure 1 displays $Re_{\rm crit}$ as a function of slip coefficient β_1 for different relationships between β_1 and β_2. It can be seen that $Re_{\rm crit}$ increases monotonically with increasing slip coefficients, where the increase of slip coefficients means that both β_1 and β_2 increase unless one of them equals to zero. This shows that the values of $Re_{\rm crit}$ are larger than the classical no-slip one $Re^0_{\rm crit}$ when slip occurs at the walls, and we may thus conclude that the effect of wall slip is always stabilizing. These tendencies may also be found in recent studies of Lauga and Cossu,[6] where only the curves of $Re_{\rm crit}$ against β_1 for $\beta_2 = \beta_1$ and $\beta_2 = 0$ are given.

However, a closer view of the circled area in Fig. 1 shows that the stability behaviour for extreme low slip coefficients is somewhat more complicated than that aforementioned. This is further illustrated in

Fig. 2(a), where the horizontal line represents the classical no-slip value. Evidently for fixed relationship of β_2 with β_1, as the slip coefficients increase from zero, $Re_{\rm crit}$ initially decreases, before reaching a minimum and subsequently increasing monotonically. For each relationship of β_2 with β_1, we can identify a local minimum of the critical Reynolds number, whereas a global minimum $Re^c_{\rm crit}$ occurs at β_m where $\beta_{1m} = \beta_{2m} = 0.00057$, for which any flow with $Re < Re^c_{\rm crit}$ is stable to any infinitesimal disturbance for any pair of β_1 and β_2 and for which any flow with $Re > Re^c_{\rm crit}$ will be unstable to some disturbances for some values of β_1 and β_2. This unusual behaviour has not been considered to be possible by other authors. An expanded view of the critical surface $Re_{\rm crit}(\beta_1, \beta_2)$ for extreme low slip coefficients is depicted in Fig. 2(b) where the concave shape is obvious and the curves in Fig. 2(a) and the contour line of $Re^0_{\rm crit}$ are given. The critical surface reaches its global minimum 5767.36 at β_m. The critical wavenumbers are smaller than those in the no-slip case, $\alpha^0_{\rm crit}$. For instance, $\alpha_{\rm crit} \approx 1.01900 < \alpha^0_{\rm crit} \approx 1.02060$ when $\beta_1 = \beta_2 = 0.00116$ and $Re_{\rm crit} \approx Re^0_{\rm crit}$.

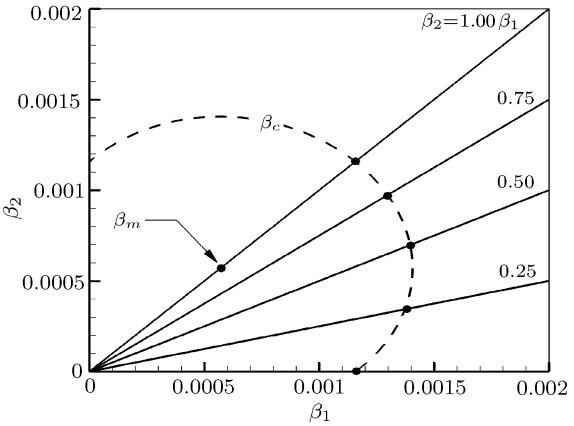


Fig. 3. Projection of the contour line of $Re^0_{\rm crit}$ in Fig. 2(b) on the (β_1, β_2) plane.

Furthermore, we can identify (β_1, β_2)-pairs which yield the same critical Reynolds number as the no-slip case, defined as $\beta_c(\beta_{1c}, \beta_{2c})$. The curves in Fig. 2(a) cross with $Re^0_{\rm crit}$ (the horizontal line therein) at the points β_c, where $\beta_1 = \beta_{1c}$ and $\beta_2 = \beta_{2c}$. Similarly, the critical surface in Fig. 2(b) intersects the horizontal surface of $Re^0_{\rm crit}$ at the contour line. The projection of the contour line in Fig. 2(b) on the (β_1, β_2)-plane is presented in Fig. 3. Furthermore both the point β_m of the critical surface reaching to its global minimum and the locations depicted by dots of the critical Reynolds number same as the no-slip one for the relationships of β_2 with β_1 in Fig. 2(a) are also shown in Fig. 3. Define $\beta_1 \le \beta_{1c}$ and $\beta_2 \le \beta_{2c}$ as $\beta < \beta_c$, similar definition of $\beta_1 \ge \beta_{1c}$ and $\beta_2 \ge \beta_{2c}$ for $\beta > \beta_c$ (the equal sign only holds for $\beta_{1c} = 0$ or $\beta_{2c} = 0$). It is shown that the values of $Re_{\rm crit}$ are less than $Re^0_{\rm crit}$ for $\beta < \beta_c$, whereas larger than $Re^0_{\rm crit}$ for $\beta > \beta_c$. That is to say, the slip case is more unstable than the no-slip case for $\beta < \beta_c$, and stable for $\beta > \beta_c$. Therefore, the influence of wall slip on the linear stability is slightly destabilizing for $\beta < \beta_c$, in contrast to the stabilizing feature for $\beta > \beta_c$. This provides a supplement to the conclusions reported in Spille and Lauga *et al.*,[4,5] who show that wall slip only plays a stabilizing role in the flow stability.

The physical mechanism of the dual role of wall slip on the linear stability of plane Poiseuille flow can be explained by an examination of the energy balance. The interaction of the disturbance with the basic flow via the Reynolds stresses and viscous dissipation of kinetic energy are responsible for the change of the disturbance energy. As far as the destabilizing role of wall slip is concerned, the energy is being transferred from the basic flow to the disturbance. According to our numerical results the transferred energy and viscous dissipation both increases first and then decreases with increasing slip coefficients compared with the no-slip critically stable state. However, the transferred energy is more dominant rather than the viscous dissipation, resulting in the growth of disturbance energy so that wall slip destabilizes the flow. Furthermore, the temporal evolution of disturbance energy increases and then decreases with the increase of the slip coefficients. As a result, the concave area presented in Fig. 2(b) comes into being. As far as the stabilizing role of wall slip is concerned, when slip coefficients increase, the disturbance energy transferred from the basic flow decreases even the energy is transferred from the disturbance to the basic flow, whereas viscous dissipation decreases, resulting in the decay of the disturbance energy so that wall slip stabilizes the flow. Because of the length limitation of this Letter, a more detailed analysis of the instability mechanisms of slip flows will be reported elsewhere.

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