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Constraints on Deceleration Parameter of a 5D Bounce Cosmological Model from Recent Cosmic Observations *

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We study the constraint on deceleration parameter q from the recent SNeIa Gold dataset and observational Hubble data by using a model-independent deceleration parameter $q(z) = 1/2 - a/(1+z)^b$ under the five-dimensional bounce cosmological model. For the cases of SNeIa Gold dataset, Hubble data, and their combination, the present results show that the constraints on transition redshift z_T are $0.35^{+0.14}_{-0.07}$, $0.68^{+1.47}_{-0.58}$, and $0.55^{+0.18}_{-0.09}$ with 1σ errors, respectively.

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The cosmological observation of High redshift type-Ia supernova (SNeIa) shows that our universe is undergoing an accelerated expansion,[1−3] which is consistent with subsequent observations from WMAP[4−7] and large scale structure survey.[8] The acceleration is either derived by dark energy or the effect of extra dimensions. In the case of dark energy, the cosmological constant is a natural candidate with the constant equation of state (EOS). However, this model exist serious theoretical problems related to the fine tuned value for the cosmological constant. Later, many dark energy models are proposed, such as quintessence,[9] phantom,[10] quintom,[11] and k-essence.[12] On the other hand, if one consider that our universe has more than four dimensions (4D), this accelerated expansion can be explained in high dimensions theories, such as RS,[13] and DGP braneworld model.[14] In these models, we must extend general relativity from 4D to higher dimensions. Another one of such extensions is the five-dimensional (5D) space-time-matter (STM) theory,[15] where a class of cosmological solution of the STM theory is given by Liu, Mashhoon[16] and called the 5D big bounce cosmological model.[17] Furthermore, it was shown that the 5D big bounce model can be constructed according to the deceleration parameter q.[18] In this Letter, we study the constraint on deceleration parameter q from recent SNeIa Gold dataset and observational Hubble data by using a model-independent deceleration parameter q in this 5D bounce cosmology, and present the evolutions of the cosmological quantities. Meanwhile, the transition redshift z_T with 1σ error can also be obtained by using recent observations.

The 5D metric of the bounce solution reads[16]

$$dS^2 = B^2dt^2 - A^2\Big(\frac{dr^2}{1-kr^2} + r^2d\Omega^2\Big) - dy^2, \quad (1)$$

with

$$d\Omega^2 \equiv (d\theta^2 + \sin^2\theta d\phi^2), \quad B = \frac{1}{\eta}\frac{\partial A}{\partial t} \equiv \frac{\dot{A}}{\eta},$$
$$A^2 = (\eta^2 + k)y^2 + 2\nu y + \frac{\nu^2 + K}{\eta^2 + k}, \quad (2)$$

where $\eta = \eta(t)$ and $\nu = \nu(t)$ are two arbitrary functions of t, k is the 3D curvature index ($k = \pm 1, 0$), and K is a constant. This solution satisfies the 5D vacuum equations $R_{AB} = 0$. The Hubble parameter H and deceleration parameter q are given as[19,20]

$$H \equiv \frac{\dot{A}}{AB} = \frac{\eta}{A}, \quad (3)$$
$$q(t,y) \equiv -A\frac{d^2A}{d\tau^2}\Big(\frac{dA}{d\tau}\Big)^{-2} = -\frac{A\dot{\eta}}{\eta\dot{A}}. \quad (4)$$

Furthermore, for simplicity we assume that there are two components: dark matter (DM) and dark energy (DE) in universe. Using the 4D part of the 5D metric Eq. (1) to calculate the 4D Einstein tensor with the relation $A_0/A = 1 + z$ and the definition $\eta_0^2/\eta_z^2 = f(z)$, one can obtain the EOS of dark energy ω_{DE}, dimensionless density parameters of dark matter Ω_{DM} and dark energy Ω_{DE}, Hubble parameter H and deceleration parameter q in terms of $f(z)$ as follows:[21]

$$w_{DE} = \frac{p_{DE}}{\rho_{DE}} = -\frac{1 + (1+z)d\ln f(z)/dz.}{3 - 3\Omega_{DM}}, \quad (5)$$
$$\Omega_{DM} = \Omega_{0DM}(1+z)f(z), \quad (6)$$
$$\Omega_{DE} = 1 - \Omega_{DM}, \quad (7)$$
$$H(z) = H_0(1+z)^2 f(z)^{-1}, \quad (8)$$
$$q = \frac{1 + 3\Omega_{DE}w_{DE}}{2} = -\frac{(1+z)}{2}\frac{d\ln f(z)}{dz}, \quad (9)$$

where H_0 is the current value of Hubble parameter. From Eq. (9) we find that function $f(z)$ can be constructed with respect to the redshift z from a given

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form of $q(z)$, then it is easy to obtain the concrete forms of the parameters ω_{DE}, Ω_{DM}, Ω_{DE} and $H(z)$.

In this Letter, we consider the deceleration parameter[21]

$$q(z) = 1/2 - a/(1+z)^b, \tag{10}$$

where a and b are constants determined by the current observational constraints. From Eq. (10), it can be seen that at the limit of $z \gg 1$, the deceleration parameter $qarrow1/2$, which is the value of deceleration parameter at matter dominated epoch. The current value of deceleration parameter is determined by $q_0 = 1/2 - a$. By using Eqs. (9) and (10), function $f(z)$ can be expressed concretely as

$$f(z) = C \exp\Big(- \frac{2a(1+z)^{-b}}{b}\Big)/(1+z), \tag{11}$$

where $C = \exp(2a/b)$ is an integral constant determined by $f(0) \equiv 1$.[18] Then according to Eq. (11), the Hubble parameter, i.e., Eq. (8) can be written as

$$H(z) = H_0(1+z)^{3/2} \exp[a((1+z)^{-b} - 1)/b]. \tag{12}$$

Next, we constrain the model parameter a and b by using Eq. (12) from the SNeIa Gold dataset and observational Hubble data, then show out the concrete evolution of 5D big bounce universe. For the case of SNeIa Gold dataset, constraints can be obtained by fitting the distance modulus $\mu(z)$,

$$\mu_{th} = 5\log_{10}(D_L(z)) + \mathcal{M}, \tag{13}$$

where $D_L(z) = H_0 d_L(z)$ is the Hubble free luminosity distance,

$$d_L(z) = (1+z)\int_0^z \frac{dz'}{H(z')}, \tag{14}$$

$$\mathcal{M} = 5\log_{10}\Big(\frac{H_0^{-1}}{Mpc}\Big) + 25 = -5\log_{10} h + 42.38. \tag{15}$$

Thus the best fit values of parameters in dark energy models can be determined by minimizing

$$\chi^2_{\rm SNeIa}(a,b) = \sum_{i=1}^{N} \frac{(\mu_{\rm obs}(z_i) - \mu_{th}(a,b,z_i))^2}{\sigma_i^2}, \tag{16}$$

where $N = 182$ for SNeIa Gold dataset, $\mu_{\rm obs}(z_i)$ are the moduli obtained from observations, σ_i is the total uncertainty of the SNeIa data.

Simon *et al.*[22] obtained several values of $H(z)$ in the range of $0 \lesssim z \lesssim 1.8$ by using the differential ages of passively-evolving galaxies from the Gemini Deep Deep Survey (GDDS)[23] and archival data.[24] The observational Hubble data from Ref. [22] are listed in Table 1.

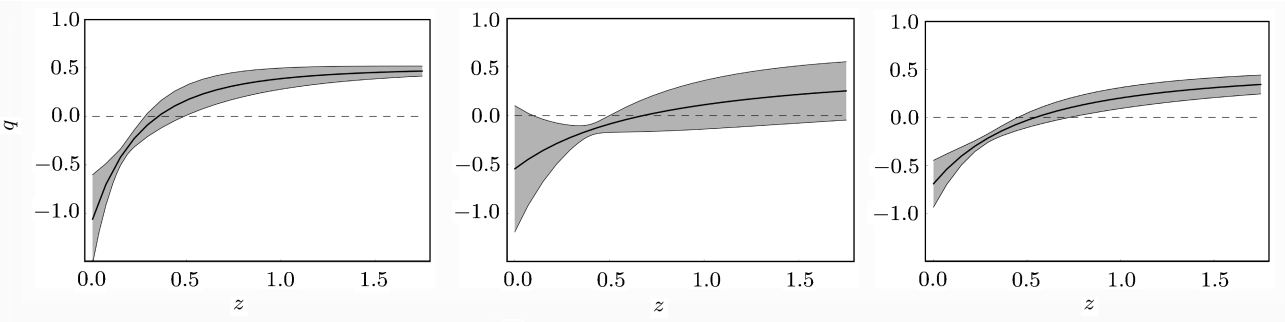

Fig. 1. Evolutions of the decelerated parameter q with respect to the redshift z, which are constrained by 182 SNeIa data, the observational Hubble data and their combination, respectively.

Table 1. The observational $H(z)$ data.

z	0.09	0.17	0.27	0.40	0.88	1.30	0.43	1.53	1.75
$H(z)$ ($\mathrm{km\,s^{-1}\,Mpc^{-1}}$)	69	83	70	87	117	168	177	140	202
1σ uncertainty	±12	±8.3	±14	±17.4	±23.4	±13.4	±14.2	±14	±40.4

We can obtain the best fit values of the model parameters a and b from observational Hubble data by minimizing

$$\chi^2_{\rm Hub}(a,b,z_i) = \sum_{i=1}^{N} \frac{[H_{th}(a,b,z_i) - H_{\rm obs}(z_i)]^2}{\sigma^2(z_i)}, \tag{17}$$

where H_{th} is the predicted value for the Hubble parameter and can be given by Eq. (12), $H_{\rm obs}$ is the observed value, $\sigma(z_i)$ is the standard deviation measurement uncertainty.

Furthermore, the combined constraint can be given by minimizing the total χ^2

$$\chi^2_{\rm total}(a,b) = \chi^2_{\rm SNeIa}(a,b) + \chi^2_{\rm Hub}(a,b). \tag{18}$$

In this study, $H_0 = 72\,\mathrm{km{\cdot}s^{-1}{\cdot}Mpc^{-1}}$ is taken as a prior (in this treatment, the corresponding errors of the parameters will be underestimated), which is the centre value given by recent observation from the Hubble Space Telescope (HST) key project.[25] Thus, by fitting the 182 SNeIa Gold data, the observational Hubble data, and their combination respectively, the values of minimum χ^2, best fit parameters a and b, transition redshifts z_T, and current deceleration parameter q_0 can be obtained and listed in Table 2.

Next we plot the evolutions of the decelerated parameter $q(z) = 1/2 - a/(1+z)^b$ with 1σ errors in Fig. 1, and the 1σ and 2σ contours of parameters a and b are plotted in Fig. 2.

Table 2. The best fit parameters with 1σ errors.

data	$\chi^2_{\min}$	a	b	z_T	q_0
SNeIa data	156.66	$1.56^{+0.99}_{-0.55}$	$3.82^{+3.70}_{-2.27}$	$0.35^{+0.14}_{-0.07}$	$-1.06^{+0.46}_{-0.46}$
Hubble data	8.87	$1.05^{+1.69}_{-0.66}$	$1.43^{+3.31}_{-2.21}$	$0.68^{+1.47}_{-0.58}$	$-0.55^{+0.65}_{-0.65}$
Combination	167.64	$1.29^{+0.44}_{-0.31}$	$1.99^{+1.35}_{-1.08}$	$0.55^{+0.18}_{-0.09}$	$-0.69^{+0.24}_{-0.24}$

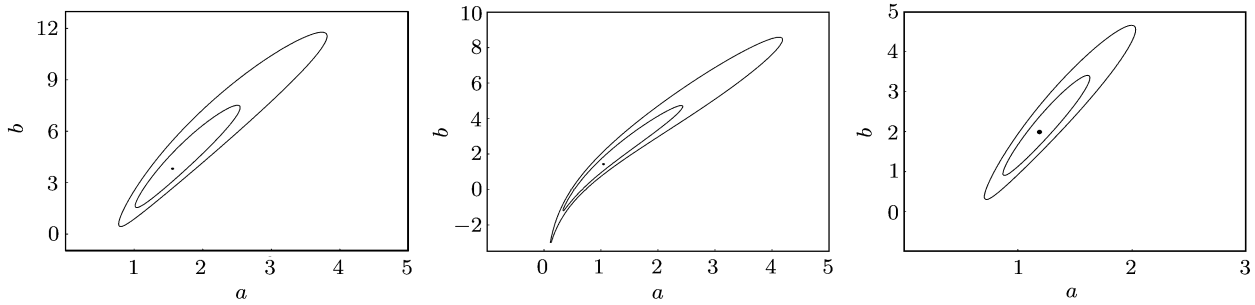

Fig. 2. Contour plots of the parameter a and b with 1σ and 2σ confidence level according with 182 SNeIa data, the observational Hubble data and their combination, respectively.

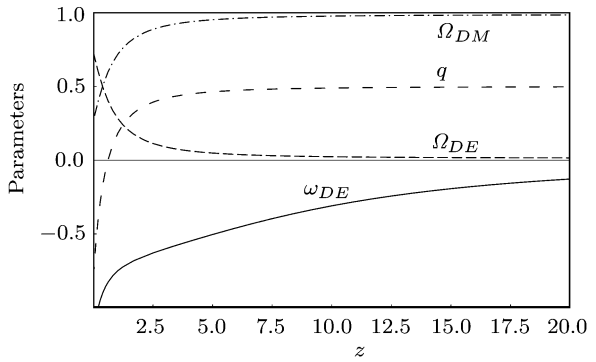

Fig. 3. Evolutions of the cosmological parameters with respect to the redshift z, where $a = 1.29$, $b = 1.99$ from the combined data in Table 2.

From Fig. 1 we can see that the combined constraint is stricter than independent constraints given by SNeIa dataset or the observational Hubble data, respectively. The best fit transition redshift $z_T = 0.55^{+0.18}_{-0.09}$ with 1σ error for the case of combined constraint. In addition, it is shown that the constraint to the deceleration parameter q is looser in 1σ error by using Hubble data than SNeIa data, which may be attributed to a fewer data points from Hubble data. From Fig. 2, it can be seen that the two independent constraints are weaker than the combined constraint. Furthermore, we find that the results of this model is consistent with other models[26] with 1σ error. Then we hope that precise datasets can discriminate them, especially high redshift data in the future, and more attention and study to diminish the effect of contingency error and system error should be considered by using more combined datasets.

Lastly, the evolutions of the cosmological parameters are plotted in Fig. 3 according to Eqs. (5)–(9), where we use the best fit values of a and b in Table 2 and take $\Omega_{0DM} = 0.27$[27] a prior.

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