Simplified Analysis for Leaky Modes in Planar Optical Waveguides *

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In combination with the transfer matrix method and the concept of an equivalent attenuated coefficient, a first-order perturbation theory is proposed to study the leaky modes in planar optical waveguides. Simplified analytical expressions of the perturbed propagation constants are presented in a more explicit and general manner compared to the existing theory.

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There has been great interest in developing numerical and approximate techniques to study the propagation constant of leaky planar structures.[1-6] Recently, Lin *et al.*[5] used a light transmission approach based on the ray theory to find the power losses in leaky planar structures. Im and Kim[6] proposed the concept of a complex incident angle to calculate the perturbed propagation constants in a prism coupling structure. Neither approach, however, can give any analytical solution for the propagation constant of the leaky modes. Ghatak *et al.*[1-4] derived the analytical expressions of the perturbed propagation constants by employing the concept of a Lorentzian response curve. The method involves two deficiencies. First, there is only one exponentially decaying solution in the buffer layer. This is physically unacceptable. Secondly, the application of inequality $k_1^2 \ll k_0^2 n_1^2$ implies that this analysis is only valid for the lower-order modes. In this paper, the transfer matrix method is employed to a four-layered planar waveguide with the help of the new concept, equivalent attenuated coefficient, and a first-order perturbation theory is developed. Consequently, the real and imaginary parts of the perturbed propagation constant β of the leaky waveguide are obtained at the same time. It should be pointed out that except for a weak coupling condition, our method requires no other approximations.

Consider a four-layered planar optical waveguide illustrated in Fig. 1. It is composed of a substrate, a guiding layer, a buffer and a cladding. The refractive index distribution is given by

$$n^2(x) = \begin{cases} n_0^2, & x < 0, \\ n_1^2, & 0 < x < d, \\ n_2^2, & d < x < d+s, \\ n_3^2, & x > d+s. \end{cases} \tag{1}$$

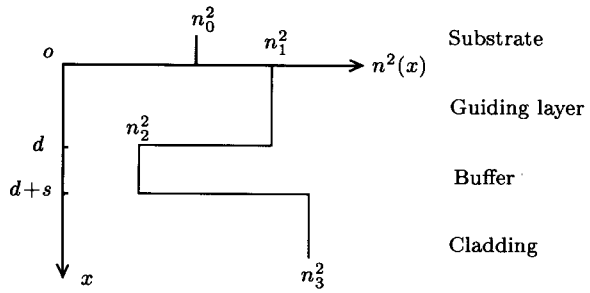

Fig. 1. Index distribution of a four-layered planar optical waveguide.

For a leaky structure, it usually requires

$$n_3^2 > n_1^2 > n_0^2 > n_2^2. \tag{2}$$

We suppose that the wave propagates along the z direction, thus, for the TE mode we may write

$$E_y(x) = \begin{cases} A_0 \mathrm{e}^{p_0 x}, & x < 0, \\ A_1 \mathrm{e}^{\mathrm{i}k_1(x-d)} + B_1 \mathrm{e}^{-\mathrm{i}k_1(x-d)}, & 0 < x < d, \\ A_2 \mathrm{e}^{p_2(x-d)} + B_2 \mathrm{e}^{-p_2(x-d)}, & d < x < d+s, \\ A_3 \mathrm{e}^{\mathrm{i}k_3(x-d-s)}, & x > d+s. \end{cases} \tag{3}$$

where

$$\begin{aligned} p_0^2 &= \beta^2 - k_0^2 n_0^2, & k_1^2 &= k_0^2 n_1^2 - \beta^2, \\ p_2^2 &= \beta^2 - k_0^2 n_2^2, & k_3^2 &= k_0^2 n_3^2 - \beta^2, \end{aligned} \tag{4}$$

By employing the transfer matrix method,[7] we have the following matrix equation:

$$\begin{bmatrix} E_y(0) \\ E_y'(0) \end{bmatrix} = M_1 M_2 \begin{bmatrix} E_y(d+s) \\ E_y'(d+s) \end{bmatrix}, \tag{5}$$

where

$$M_1 = \begin{bmatrix} \cos(k_1 d) & -\dfrac{1}{k_1}\sin(k_1 d) \\ k_1 \sin(k_1 d) & \cos(k_1 d) \end{bmatrix}, \tag{6}$$

$$M_2 = \begin{bmatrix} \cosh(p_2 s) & -\dfrac{1}{p_2}\sinh(p_2 s) \\ -p_2 \sinh(p_2 s) & \cosh(p_2 s) \end{bmatrix}, \tag{7}$$

and the prime "$\prime$" denotes the derivative with respect to x.

Applying the boundary conditions at $x = 0$ and $x = d + s$, we obtain

$$(-p_0 \ 1) M_1 M_2 \begin{pmatrix} 1 \\ \mathrm{i}k_3 \end{pmatrix} = 0. \tag{8}$$

Using Eqs. (6) and (7), Eq. (8) becomes

$$k_1 d = m\pi + \arctan\left(\frac{p_0}{k_1}\right) + \arctan\left(\frac{p_2'}{k_1}\right), \quad m = 0, 1, 2 \cdots, \tag{9}$$

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where p_2' is the so-called equivalent attenuated coefficient and is given by

$$p_2' = p_2 \left[1 + \frac{k_3 - \mathrm{i}p_2}{k_3 + \mathrm{i}p_2} \mathrm{e}^{-2p_2 s}\right] \left[1 - \frac{k_3 - \mathrm{i}p_2}{k_3 + \mathrm{i}p_2} \mathrm{e}^{-2p_2 s}\right]^{-1}. \tag{10}$$

Imposing the weak-coupling condition $\mathrm{e}^{-2p_2 s} \ll 1$, we have

$$p_2' = p_2 + \Delta p_2, \tag{11}$$

where

$$\Delta p_2 = 2p_2 \frac{(k_3^2 - p_3^2) - \mathrm{i}\,2p_2 k_3}{k_3^2 + p_2^2} \mathrm{e}^{-2p_2 s}, \tag{12}$$

so that

$$\begin{aligned} \arctan\left(\frac{p_2'}{k_1}\right) &= \arctan\left(\frac{p_2 + \Delta p_2}{k_1}\right) \\ &\approx \arctan\left(\frac{p_2}{k_1}\right) + \frac{k_1}{k_1^2 + p_2^2}\Delta p_2. \end{aligned} \tag{13}$$

The perturbation equation for the leaky modes is now given by

$$k_1 d = m\pi + \arctan\left(\frac{p_0}{k_1}\right) + \arctan\left(\frac{p_2}{k_1}\right) + \frac{k_1}{k_1^2 + p_2^2}\Delta p_2. \tag{14}$$

It is obvious that we have $\Delta p_2 = 0$ in the limit of $s \to \infty$, i.e., the four-layered leaky structure reduces to a simple three-layered waveguide. Furthermore, Eq. (14) reduces to the eigenvalue equation for bound modes, which is

$$k_1^0 d = m\pi + \arctan\left(\frac{p_0^0}{k_1^0}\right) + \arctan\left(\frac{p_2^0}{k_1^0}\right), \tag{15}$$

where superscript "0" denotes the eigenvalue of the bound modes.

Let us define the shift of the propagation constant:

$$\Delta\beta = \beta - \beta^0. \tag{16}$$

Under the weak-coupling condition, $\Delta\beta \ll \beta$ is a physically expected result. Now we may expand the perturbed parameters around β^0 and retain only the first-order term, which gives

$$k_1 \approx k_1^0 - \frac{\beta^0}{k_1^0}\Delta\beta, \tag{17}$$

$$\arctan\left(\frac{p_0}{k_1}\right) \approx \arctan\left(\frac{p_0^0}{k_1^0}\right) + \frac{\beta^0}{p_0^0 k_1^0}\Delta\beta, \tag{18}$$

$$\arctan\left(\frac{p_2}{k_1}\right) \approx \arctan\left(\frac{p_2^0}{k_1^0}\right) + \frac{\beta^0}{p_2^0 k_1^0}\Delta\beta. \tag{19}$$

Substituting the above approximations into Eq. (14) and using Eq. (15) yields

$$\Delta\beta = -\frac{k_1^2}{\beta(k_1^2 + p_2^2)(d + 1/p_0 + 1/p_2)}\Delta p_2. \tag{20}$$

For the sake of simplicity, we have neglected the superscript "0" in Eq. (20) and also in the following equations. Since Δp_2 is a complex number, the real and imaginary parts of the perturbed propagation constant are given by

$$\mathrm{Re}(\Delta\beta) = -\frac{2p_2 k_1^2 (k_3^2 - p_2^2)\mathrm{e}^{-2p_2 s}}{\beta(k_1^2 + p_2^2)(k_3^2 + p_2^2)(d + 1/p_0 + 1/p_2)}, \tag{21}$$

$$\mathrm{Im}(\Delta\beta) = \frac{4p_2^2 k_1^2 k_3 \mathrm{e}^{-2p_2 s}}{\beta(k_1^2 + p_2^2)(k_3^2 + p_2^2)(d + 1/p_0 + 1/p_2)}, \tag{22}$$

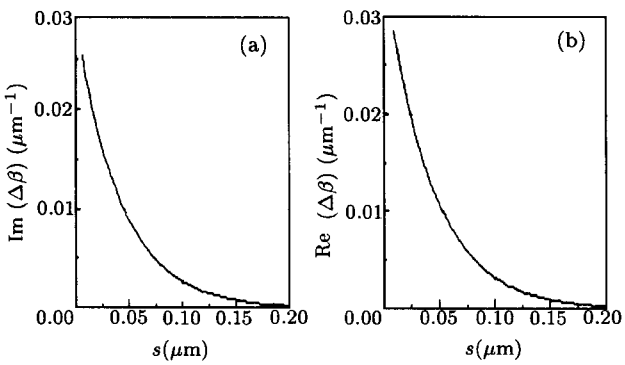

Fig. 2. Shift of propagation constant versus buffer thickness, (a) loss coefficient $\mathrm{Im}(\Delta\beta)$, (b) perturbed propagation constant $\mathrm{Re}(\Delta\beta)$.

Figure 2 shows how the loss coefficient $\mathrm{Im}(\Delta\beta)$ and the perturbed propagation constant $\mathrm{Re}(\Delta\beta)$ drop with increasing thickness of the buffer layer for TE_0 mode. When the buffer thickness is large enough ($s > 0.25\,\mu\mathrm{m}$) with $d = 1.0\,\mu\mathrm{m}$, $n_0 = 1.49$, $n_1 = 1.56$, $n_2 = 1.0$, and $n_3 = 1.60$ at $\lambda = 0.6328\,\mu\mathrm{m}$, light leakage can be shielded effectively.

It is shown that we arrive at above results without any assumptions of complex constant as in usual way. The derivations of the loss coefficient and the perturbed propagation constant are quite natural. We should mention here that the results given in Ref. 3 could be reduced to our simplified form with the eigenvalue equation for the bound modes.

In conclusion, we propose the concept of the equivalent attenuated coefficient based on the transfer matrix method. Analytical expressions for the loss coefficient and the perturbed propagation constant for a leaky planar waveguide have been derived in a more explicit and general manner. It is important to point out that the present method can be easily extended to a structure which has complex refractive indices.

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