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Nonlinear Coupling of Langmuir Wave and Dust-Acoustic Wave in Plasmas with the Variable-charge grains *

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Nonlinear interaction of the Langmuir wave and the dust-acoustic wave in plasmas with the variable-charge grains has been investigated. In the approximation of small amplitude, we find the solution of envelope soliton for the nonlinear Langmuir wave. The results show that the amplitude of envelope soliton increases with the increasing of dust grain density.

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In the past few years, collective phenomena in a dusty plasma have attracted extensive attention due to their importance in fusion study, space environment and material processing. The existence of the strongly charged heavy grains in a plasma makes considerable effects on the properties of a plasma. Some new phenomena in a dusty plasma, such as the dusty acoustic wave[1] and plasma crystals,[2] etc., have been found. Recently, some authors have worked on the nonlinear phenomena in a dusty plasma.[3-6] Pople *et al.*[3] have studied the double-layer in a dusty plasma, and found that there exist shock structures in plasmas involving variable-charge grains due to the dissipation associated with charging of the grains. Liu *et al.*[4] and Xie *et al.*[5] considered the dust-acoustic solitary waves and the modified dust-acoustic solitons in a dusty plasma.

It is well-known that the nonlinear coupling of the Langmuir wave and the ion acoustic waves in unmagnetized plasmas can lead to the formation of Langmuir envelope soliton.[7] We know that the electrostatic perturbations on very slowly variable, such as in the dust acoustic waves or the dust-ion acoustic waves, can be excited in a dusty plasma. The Langmuir wave propagating in a dusty plasma also can be modulated by the extremely low frequency electrostatic waves. In this letter, we study the nonlinear coupling between the Langmuir wave and the dust acoustic waves in plasmas with variable-charge dust grains. Starting from fluid theory, we obtained the nonlinear equation describing the envelope amplitude for the Langmuir waves, and studied the properties of the Sagdeev potential and the solitary waves numerically. We found the analytical solution in the limit of small amplitude. Our results show that for a given Mach number M the amplitude of envelope soliton increases with the increasing of grain number density.

Supposing that a plasma consists of electrons, ions and the strongly charged dust grains, we consider the one-dimensional propagation of the Langmuir wave in unmagnetized dusty plasma. The high-frequency Langmuir wave field can be written as

$$\boldsymbol{E}_{\rm h} = \frac{1}{2}\,\tilde{E}\,(x,t)\boldsymbol{e}_x \exp(-{\rm i}\,\omega_{\rm pe}t) + {\rm c.c.}, \qquad (1)$$

where $\boldsymbol{e}_x$ is the unit vector in the x-direction, $\omega_{\rm pe} = (4\pi n_{\rm eo}e^2/m_{\rm e})^{1/2}$ the electron plasma frequency, $\tilde{E}$ is the complex amplitude of the high-frequency electric field, which is a slowly varying function of space and time due to nonlinear coupling between the Langmuir wave and the low-frequency density perturbations. The equation governing the slowly varying amplitude of the field is[7-8]

$${\rm i}\,\partial_t\,\tilde{E} + \frac{3}{2}\frac{v_{\rm te}^2}{\omega_{\rm pe}}\partial_{xx}^2\,\tilde{E} = \frac{1}{2}\omega_{\rm pe}\frac{n_{\rm el}}{n_{\rm e0}}\,\tilde{E}, \qquad (2)$$

where $n_{\rm el}$ and $n_{\rm e0}$ are the low-frequency electron perturbation density and the background density, respectively; $v_{\rm te} = (T_{\rm e}/m_{\rm e})^{1/2}$ is the electron thermal speed.

The electrons acted by the ambipolar potential and the pondermotive force obey the Boltzmann distribution:[9]

$$n_{\rm e} = n_{\rm e0}\exp\Big(\frac{e\phi}{T_{\rm e}} - \frac{|\,\tilde{E}\,|^2}{4\pi n_{\rm e0}T_{\rm e}}\Big), \qquad (3)$$

where ϕ is the ambipolar electrostatic potential, $T_{\rm e}$ the electron temperature.

The low-frequency response of ions in a plasma is discribed by the continuous equation and the momentum balance equation:

$$\partial_t n_{\rm i} + \partial_x(n_{\rm i}\,v_{\rm i}) = 0, \qquad (4)$$

$$\partial_t v_{\rm i} + v_{\rm i}\,\partial_x v_{\rm i} = -\frac{e}{m_{\rm i}}\partial_x\phi, \qquad (5)$$

where $n_{\rm i}$ $m_{\rm i}$, and v_i are the ion density, mass and speed, respectively. Note that here we assume the ions are cold, and ignore the pondermotive force acting on ions, which is an inverse measure of the ion mass.

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The charge on a grain is governed by the relaxation equation $\partial_t q_\alpha = I_e + I_i$, where I_e and I_i are the electron and ion currents flowing into the grain surface. We suppose that the time scale of the relaxation of process is much smaller than the characteristic time for the motion of ion, on which the dust charge reaches local equilibrium. Thus, we have[4]

$$I_e + I_i = 0, \tag{6}$$

$$I_e = -\pi r^2 e n_e \overline{v}_e \frac{e q_d}{r T_e}, \tag{7}$$

$$I_i = \pi r^2 e n_i v_i \left(1 - \frac{2 e q_d}{r T_e}\right), \tag{8}$$

where r ($r \ll \lambda_{De}$) and q_d are the radius and the charge of a grain, $\lambda_{De} = [T_e/(4\pi n_{eo} e^2)]^{1/2}$ is the Debye radius; $\overline{v}_e = [8T_e/(\pi m_e)]^{1/2}$ is the average thermal speed of electrons. Note that the charge on a grain is a function of local electrostatic potential. To close the system of Eqs. (2)–(6), we make use of the quasi-neutral condition

$$e(n_i - n_e) + q_d n_d = 0, \tag{9}$$

where n_d is the grain density. For the situation under our consideration, the dust grains are treated as an immobile background, so n_d is a constant in the absence of the creation and annihilation of grains.

The system of Eqs. (2)–(9) can be written in the normalized form:

$$i\epsilon \partial_t E + \frac{3}{2}\partial_{xx}^2 E = \frac{1}{2}(N_e - 1)E, \tag{10}$$

$$N_e = \exp(\Phi - E^2), \tag{11}$$

$$-N_e \exp Q_d + \delta_i \mu N_i v_i \left(1 - \frac{2Q_d}{v_i^2}\right) = 0, \tag{12}$$

$$\partial_t N_i + \partial_x (N_i v_i) = 0, \tag{13}$$

$$\partial_t v_i + v_i \partial_x v_i = -\partial_x \Phi, \tag{14}$$

$$\delta_i N_i - N_e + Z Q_d \delta_d = 0, \tag{15}$$

where $\epsilon = (m_e/m_i)^{1/2}$, $\mu = \epsilon(\pi/8)^{1/2}$, $Z = rT_e/e^2$, $\delta_i = n_{i0}/n_{e0}$, $\delta_d = n_{d0}/n_{e0}$. In obtaining Eqs. (10)–(15), we have normalized the variables as follows: $x/\lambda_{De} \to x$, $e\phi/T_e \to \Phi$, $eq_d/(rT_e) \to Q_d$, $v_i/c_s \to v_i$, $|\widetilde{E}|/(4\pi n_{e0} T_e)^{1/2} \to E$, $n_\alpha/n_{\alpha 0} \to N_\alpha$ (α = e, i).

From Eqs. (12) and (15), we can get the expressions in the unperturbed case:

$$v_0 \exp Q_{d0} - \delta_i \mu (v_0^2 - 2Q_{d0}) = 0, \tag{16}$$

$$\delta_i + Z\delta_d Q_{d0} = 1, \tag{17}$$

where Q_{d0} is the charge on a grain in equilibrium, v_0 the background speed of ions.

Now we introduce the variable $\xi = x - Mt$, where M is the Mach number. From Eqs. (11)–(15), eliminating Φ, Q_d and v_i, we get

$$(1 - MF)N_e \exp[(Z\delta_d)^{-1}(N_e - u_0\delta_i F)] + \delta_i \mu u_0 \cdot [(1 - MF)^2 - 2F^2(Z\delta_d)^{-1}(N_e - u_0\delta_i F)] = 0, \tag{18}$$

where $F = [u_0^2 - 2(\ln N_e + E^2)]^{-1/2}$, $u_0 = M - v_0$. In obtaining Eq. (18), we have had use of the boundary condition: Φ, $\partial_\xi \Phi$, $E \to 0$, N_i, $N_e \to 1$, $v_i \to v_0$, at $|\xi| \to \infty$.

The slowly varying amplitude is written as

$$E(x,t) = E(\xi)\exp[i(\Omega t + \kappa t)], \tag{19}$$

where $E(\xi)$ is a real function of ξ. Substituting Eq. (19) into Eq. (10), from its imaginary part we can get:

$$\kappa = \frac{\epsilon M}{3},$$

and from its real part the equation for E:

$$3\partial_{\xi\xi}^2 E = (N_e - 1 + A)E, \tag{20}$$

where $A = 2\epsilon\Omega + \epsilon^2 M^2/3$ is the nonlinear frequency shift. Equations (18) and (20) discribe the nonlinear coupling of the slowly varying amplitude and the slow frequency density perturbation for the dust acoustic wave in plasmas with the variable-charge grains. Obviously, it is impossible to solve it analytically.

Now we consider the solution of envelope soliton for Eqs. (18) and (20) in the limit of small amplitude. Let $N_e = 1 + N$, $|N| \ll 1$. Expanding Eq. (18), and using Eq. (16), we can get

$$N = -\frac{Q}{P+Q}E^2, \tag{21}$$

where

$$P = \left(1 + \frac{1}{Z\delta_d}\right)\exp Q_{d0} + \frac{2\mu\delta_i}{v_0 Z\delta_d}, \tag{22}$$

$$Q = \frac{1}{(M - v_0)^2}\left\{\frac{\delta_i}{Z\delta_d}\exp Q_{d0} + \mu\delta_i\left[M + \frac{Z\delta_i}{v_0 Z\delta_d} + \frac{2Q_{d0}}{v_0^2}(M - 2v_0)\right]\right\}. \tag{23}$$

Substituting Eq. (21) into Eq. (20), we obtain the equation for the amplitude E in the form of the energy integral:

$$(d_\xi E)^2 + V(M, \delta_d, E^2) = 0, \tag{24}$$

where

$$V(M, \delta_d, E^2) = -\frac{1}{3}AE^2 + \frac{1}{6}\frac{Q}{P+Q}E^4, \tag{25}$$

$V(M, \delta_d, E^2)$ is the Sagdeev potential. We know that the solution of Eq. (24) depends strongly on the behaviour of the Sagdeev potential $V(M, \delta_d, E^2)$, which

is related to the dust grain density δ_d. So the existance of the dust grains in a plasma gives observable effects on the properties of solutions for Eq. (24). In the following, we study Eq. (24) numerically. For concreteness, we consider an argon plasma with the dust grains, and take $T_e = 4ev$, the dust grain radius $r = 1\,\mu m$, $v_0 = 0.8$, $A = 0.002$.

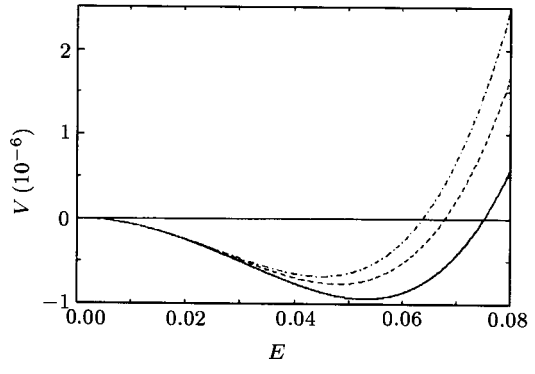

Fig. 1. Sagdeev potential for $\delta_d = 4\times10^{-5}$ with different Mach number M. The solid, dashed and dot-dashed lines correspond to $M = 0.3$, 0.5, and 0.7, respectively.

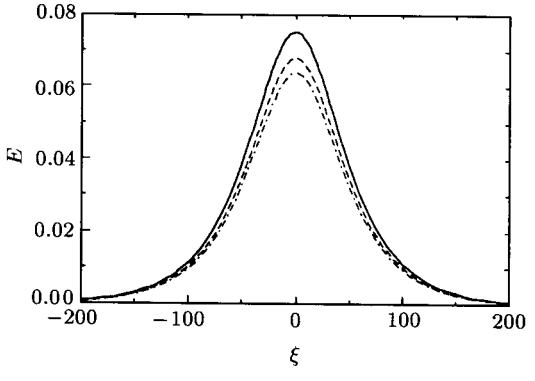

Fig. 2. Profiles of the amplitude E for $\delta_d = 4\times10^{-5}$ with different Mach number M. The solid, dashed, and dot-dashed lines correspond to $M = 0.3$, 0.5, and 0.7, respectively.

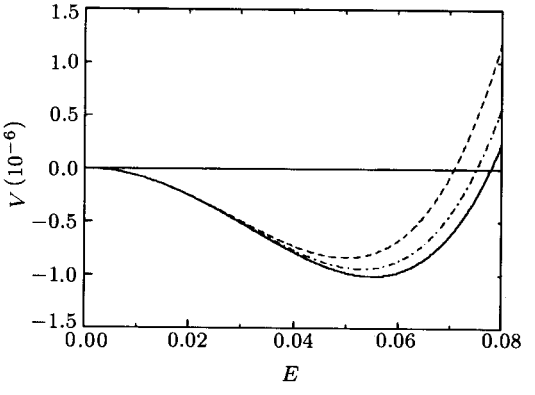

Fig. 3. Sagdeev potential for Mach number $M = 0.3$ with different δ_d. The dashed, dot-dashed, and solid lines correspond to $\delta_d = 1\times10^{-6}$, 4×10^{-5}, and 5×10^{-5}, respectively.

Figures 1 and 2 show the Sagdeev potential curves and the corresponding curves of the field amplitude E for given δ_d with different Mach number M. We can see that with the increasing Mach number, the amplitude of the localized waves decreases. Figures 3 and 4 depict the Sagdeev potential curves and the corresponding curves of the field amplitude E for the given Mach numbers with the different grain density δ_d. From these figures, we can see that the amplitude of the envelope soliton becomes larger with the increasing dust grain content in a plasma.

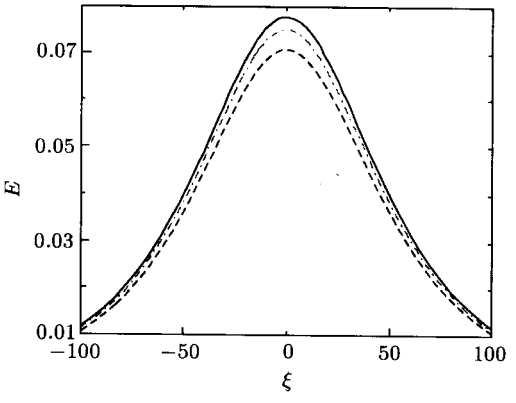

Fig. 4. Profiles of the amplitude E for $M = 0.3$ with different δ_d. The dashed, dot-dashed, and solid lines correspond to $\delta_d = 1\times10^{-6}$, 4×10^{-5}, and 5×10^{-5}, respectively.

Using the standard method, we can find the analytical solution of Eq. (24):

$$E = E_0 \mathrm{sech}^2\left(\sqrt{\frac{A}{3}}\xi\right), \tag{26}$$

where $E_0 = [2A(P+Q)/P]^{1/2}$ is the amplitude of envelope soliton.

In conclussion, modulation of Langmuir wave propagating in plasmas with the variable-charge grains by the dust acoustic waves can also lead to the formation of the Langmuir envelope soliton. The amplitude of envelope soliton increases with the increasing dust grain content in a plasma.

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