# Nuclear Zpif-Type Plots *

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*Isospin dependent classical molecular dynamics model is used to investigate the nuclear disassembly of $^{129}Xe$. Zpif-type plot in the field of linguistics is tested for the rank-classified cluster arrangement from this nuclear disassembly. It is found that the average cluster charge (or mass) of rank n in the charge (or mass) list is exactly inverse to its rank, i.e. there exists Zpif's law at the point of the liquid gas phase transition. This novel criterion can be used to diagnose the nuclear liquid gas phase transition experimentally and theoretically.*

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In intermediate energy heavy ion collisions (HIC) the hot nuclei with moderate temperature can be formed and finally deexcite by different decay modes, such as multifragmentation. Generally the onset of multifragmentation exhibits that the multiplicity $N_{\rm imf}$ of intermediate mass fragment (IMF) attains to a maximum at a certain beam energy,[1] which is an indication of the coexistence of liquid and gas phases. In relations to the IMF distribution, there exists a minimum of power law parameter, $\tau_{\rm min}$, for the cluster distribution when the liquid gas phase transition takes place.[2]

On other hand, the study on nuclear caloric curve is an attractive subject due to the fact that a temperature plateau exists when the first order liquid gas phase transition occurs. Experimentally the He-Li isotopic temperature from Albergo's thermometer[3] for projectile-like Au spectators reveals a temperature plateau in the excitation energy range of 3 to 10 MeV/u by ALADIN Collaborations,[4] who claimed that the first order nuclear liquid gas phase transition was observed experimentally. Nuclear caloric curves were also experimentally surveyed by other groups, such as INDRA,[5] MSU,[6] TAMU,[7] EOS,[8] and GSI (Ref. 9) etc.

In addition, the possibility to extract critical exponents and critical behavior from finite-size systems has been attempted experimentally in Ref. 10.

However, even though the above trials to search for the liquid gas phase transition are multivariant, these signatures are still in controversial debating.[11-14] Hence it will be meaningful to present any novel signature to characterize the nuclear liquid gas phase transition to guide the experimental search and theoretical predictions.

In this letter we will introduce, for the first time, Zpif's law into the study of nuclear liquid gas phase transition. The Zpif's law[15] has been known as a statistical phenomenon concerning the relation between English words and their frequency in literature in the field of linguistics. In this Zpif-type analysis, one calculates the histogram that gives the total number of occurences of each word in a text. If all the words in the text are arranged in rank $n$ order, from most frequent to least frequent, then such a histogram is found to be linear on double logarithmic paper, with a slope parameter $\lambda \approx 1$ for all languages. Attempts to understand the origin of the Zpif's law are connected to the hierarchical structure of language.[16] Later this relation was found not only in linguistics but also in other fields of sciences. For examples, the law appearing in distributions of populations in cities, distributions of areas of lakes, DNA base pair sequences[17] and cluster-size distribution in percolation process.[18] In this letter, Zpif's law will be tested for the charge (or mass) distribution of nuclear clusters and evidenced to be a novel factor to characterize the phase transition.

An isospin dependent classical molecular dynamics model (I-CMD) was used here to investigate the Zpif's law appearing in nuclear cluster distribution during nuclear disassembly. However, we do not do any *ab initio* molecular dynamics calculation but only use it to simulate nuclear disassembly starting from a thermally equilibrated source. This starting point is that the nucleons have been initialized at their lattice sites with the Metropolis sampling and their momenta are generated by Monte Carlo sampling of Maxwell Boltzmann distribution by the isospin dependent lattice gas model (I-LGM) (the details of the I-LGM can be found in some literature, such as Ref. 19 and our previous papers.[20] Once this is done we will switch to a molecular dynamics calculation in which the system will be evolved under the influence of the chosen force according to classical mechanics. The reason why we switch to the molecular dynamics calculation is that lattice gas model only considers the nearest neighbouring interaction and hence the long range Coulomb field is not amenable to such lattice gas type calculation. The form of the force in the I-CMD is chosen to be isospin dependent as in I-LGM.[19] The potential for unlike-nucleons is

$$v_{\rm np}(r)(\frac{r}{r_0}<a) = A\Big[B(\frac{r_0}{r})^p - (\frac{r_0}{r})^q\Big]\exp(\frac{1}{(r/r_0)-a}),$$
$$v_{\rm np}(r)(\frac{r}{r_0}>a) = 0, \qquad (1)$$

where $r_0 = 1.842$ fm is the distance between the centers of two adjacent cubes. The parameters of the potentials are $p = 2$, $q = 1$, $a = 1.3$, $B = 0.924$, and $A = 1966$ MeV. With these parameters the potential is minimum at $r_0$ with the value $-5.33$ MeV, is zero when the nucleons are more than $1.3r_0$ apart and becomes stronger repulsive when $r$ is significantly less than $r_0$. We have taken the nuclear force between two like-nucleons to be the same expressions as above plus 5.33 MeV up to $r = 1.842$ fm and zero afterwards:

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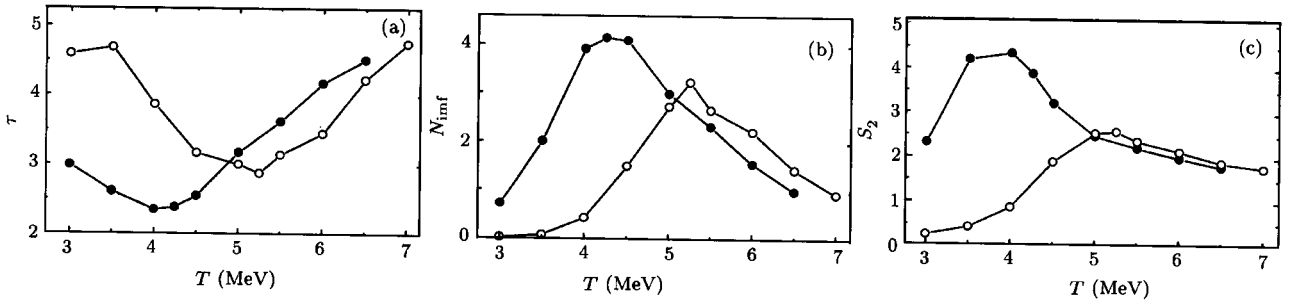



**Fig. 1.** Effective power-law parameter $\tau$ (a), the multiplicity of intermediate mass fragments, $N_{\rm imf}$ (b), and the second moment of the cluster distribution, $S_2$ (c) as a function of temperature for the disassembly of $^{129}$Xe in the framework of I-CMD with (solid circles) /without (open circles) Coulomb interaction.

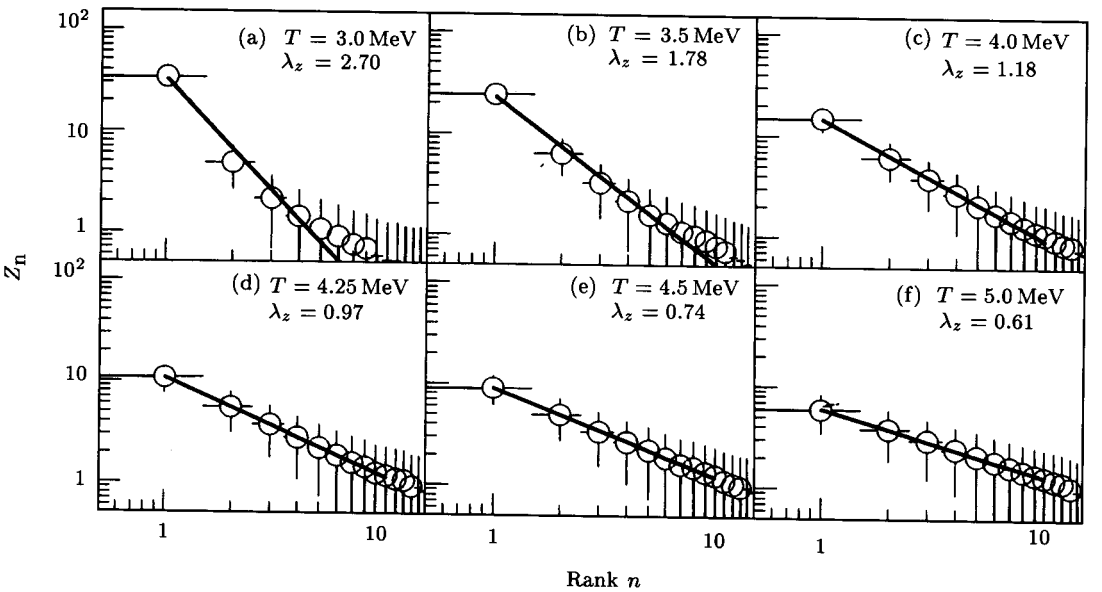



**Fig. 2.** Average charge $Z_n$ with rank $n$ as a function of $n$ in the I-CMD with Coulomb force at different temperature. The open circles are the original calculation results with statistical errors and the straight lines are their fits with $Z_n \propto n^{-\lambda z}$.

$$v_{\rm pp}(r)(r < r_0) = v_{\rm np}(r) - v_{\rm np}(r_0),$$
$$v_{\rm pp}(r)(r > r_0) = 0. \qquad (2)$$

The system evolves with the above potential. The particles are propagated in phase space by integrating Newton's equations of motion through a "leap-frog" algorithm. At asymptotic times the clusters are easily recognized. In the case of proton–proton interactions, the Coulomb interaction can be added separately. By switching on or off the Coulomb interaction, we can investigate the influence of Coulomb force on the features of the cluster distribution and nuclear liquid gas phase transition.

As an example, we chose the medium size nuclei $^{129}$Xe to analyze the phase transition behavior in nuclear disassembly and present its Zpif's law. The "freeze-out" density $\rho_{\rm f}$ is chosen to be $0.38\rho_0$ due to the data that can be best fitted by a $\rho_{\rm f}$ between $0.3\rho_0$ and $0.4\rho_0$ in the previous LGM calculations[21] which correspond to $7 \times 7 \times 7$ cubic lattice.

In order to check the phase transition behavior in the I-CMD, we will first show some observable, namely the effective power-law parameter $\tau$, the multiplicity of intermediate mass fragments, $N_{\rm imf}$, and the second moment of the cluster distribution, $S_2$ (Ref. 22) for the disassembly of $^{129}$Xe in Fig. 1. The observable has been evidenced useful in previous works for judging the liquid gas phase transition, as shown in Refs. 20 and 23. The valley of $\tau$, the peaks of $N_{\rm imf}$ and $S_2$ happen around $T \sim 4 - 4.25$ MeV in the case with Coulomb interaction, and $5 - 5.25$ MeV in the case without Coulomb interaction. For each case, the turning temperatures extracted from these extreme values of $\tau$, $N_{\rm imf}$, and $S_2$ happen in a fixed point and indicate the onset of nuclear liquid gas phase transition. The transition temperature in the case with Coulomb interaction is smaller than the one without it, i.e. the long range repulsion will make the phase transition temperature lower. This transition temperature will be used as a reference of the following novel signature of Zpif's law.

Now we present the results for testing Zpif-type plots in the charge distribution of clusters. In the simulations, we averaged the rank-classified charges for each rank in charge lists for all events. The so-called rank-classified charge is that we classify the clusters from the largest charge ($n = 1$) to the second largest charge ($n = 2$), and to the third largest charge ($n = 3$), and so on. From the charges averaged, we examined the relation between the charges $Z_n$ and their ranks $n$. Figure 2 shows such relationships of $Z_n$ and $n$ (i.e. Zpif-type plots) for Xe in different

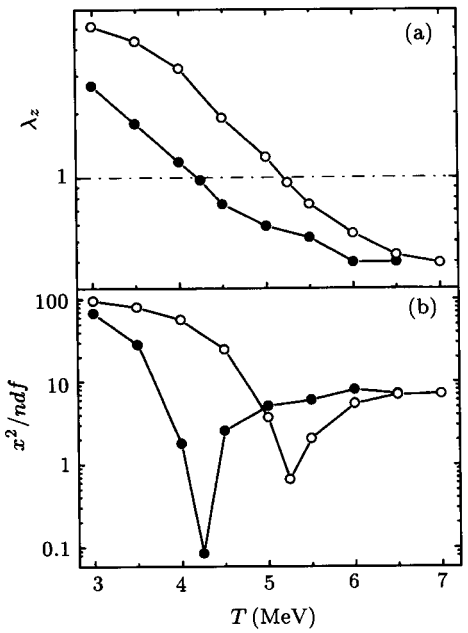



**Fig. 3.** Slope parameter $\lambda_Z$ (a) and the $\chi^2$ test for the Zpif's law (b) as a function of temperature in the framework of I-CMD with (solid circles) /without (open circles) Coulomb interaction.

temperature for I-CMD with the Coulomb interaction. The histogram is the simulated results and the straight lines represent the fit with $Z_n \propto n^{-\lambda_Z}$ in the range of $1 \leq n \leq 10$, where $\lambda_Z$ is the slope parameter. Clearly $\lambda_Z$ decrease with the increasing temperature and it reaches to 1 at $T = 4.25\,\mathrm{MeV}$, i.e. the relation is satisfied to the Zpif's law. The corresponding temperature having the Zpif's law is consistent with the phase transition temperature obtained in Fig. 1, illustrating that the Zpif's law is a good judgement to phase transition. Figure 3(a) summarizes the parameter $\lambda_Z$ as a function of temperature for disassembly of $^{129}$Xe. Clearly the Zpif's law ($\lambda_Z = 1$) only happens at the phase transition temperature.

In order to further illustrate that the Zpif's law exists most probably in phase transition point, we directly fit the histograms with Zpif's law: $Z_n = c/n$. In this case, $c$ is a sole parameter, but what we are interested in is its truth of the hypothesis of Zpif's law: the $\chi^2$ test. Figure 3(b) demonstrates $\chi^2$ for the $Z_n - n$ relations at different $T$. As expected, there exists the minimum $\chi^2$ around the respective phase transition temperature, which further supports that the Zpif's law of the fragment distribution holds at the phase transition point.

In addition, we also analyze the Zpif-type plots for the larger systems and with I-LGM model in different freeze-out densities to see if the system behaves as expected. The results show that Zpif's law still holds at the same phase transition temperature as the extreme values of $\tau$, $N_{\mathrm{imf}}$ and $S_2$. It illustrates that this novel criterion is suitable as the diagnosis of liquid gas phase transition independently of system size and the model. However, we would like to point out that Zpif's law is still a phenomenological device with no foundation in thermodynamics. The hidden physics under phenomenon is waiting for being explored. Even though, the observed fact that Zpif's law agrees with the calculated data at the phase transition and nowhere else, is a new and interesting discovery.

In conclusion, the Zpif-type analysis in the field of linguistics is introduced, for the first time, into the cluster charge (mass) distribution in the nuclear disassembly and is taken to be a novel probe to the liquid gas phase transition of nuclei. The charge/mass distribution at the transition point is exactly inverse to its rank, i.e. there exists the Zpif's law. This criterion ought to be useful in searching the nuclear liquid gas phase transition experimentally and theoretically in combination with the other observable, such as $\tau$, $S_2$, $N_{\mathrm{imf}}$ and information entropy[24] etc. In the light of this work, we propose to do some data analysis on such Zpif-type plots in $4\pi$ nuclear reaction experiments.

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