Vacuum-Induced Quantum Interference in a Trapped Λ-Configuration Three-Level System *

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In consideration of quantization of centre-of-mass motion, we derive the second-order solution of the dynamic equation of a Λ-configuration three-level atom confined in an approximately harmonic trap by using the time-dependent perturbation theory. It is found that there are a series of dark lines in the second-order probability spectrum with multi-peak structures, which is the result of the quantum interference from the same vacuum mode in the spontaneous decay process of the trapped atom from the upper level to the two nearby lower levels. Our study shows that the second-order spectrum may be modified by the oscillation frequency Ω of the trap and the frequency difference Δ between two lower levels of the three-level atom, and the depth of the dark lines from the vacuum-induced quantum interference effect is strongly dependent on the above two parameters (Ω and Δ).

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Recently, many schemes implementing the spontaneous emission have been studied theoretically and experimentally.[1−13] In particular, three-level atoms have induced much attention in the spontaneous emission modification schemes due to quantum interference effects, where such atoms exhibit many interesting phenomena such as lasing without population inversion (LWI),[6−8] electromagnetically induced transparency (EIT),[9,10] enhanced index of refraction without absorption,[11] and so on. In 1995, Zhu *et al.*[12] investigated the vacuum-induced quantum interference effect of the V-configuration three-level atom system. Meanwhile, Javanainen analysed theoretically the vacuum-induced coherences in a Λ-configuration three-level atom where the two lower levels are nearly degenerate.[13] However, the effect of the c.m. oscillation was not considered in Refs. [12,13].

In the experiments of laser cooling and trapping, the quantization of centre-of-mass oscillation state has been observed and the quantum effect of centre-of-mass motion has been paid much attention in atom and quantum optics.[14−18] Wilkens[19] first investigated the role of the c.m. motion in the process of spontaneous emission. Then Stoop *et al.*[20] studied the spontaneous emission from a two-level atom trapped in a harmonic binding potential with several c.m. initial states. In this Letter, we study a new quantum interference effect induced by vacuum fluctuation and first consider the effect of the quantization of the c.m. motion on the spontaneous emission of a Λ-configuration three-level atom, which is the basis for analysis of the effect of the quantization of the c.m. motion on the quantum interference effects such as LWI, EIT, and so on. We also qualitatively explain the cause of the above quantum interference effect, and quantitatively study the dependence of the quantum interference effect on the parameters of atom–trap systems.

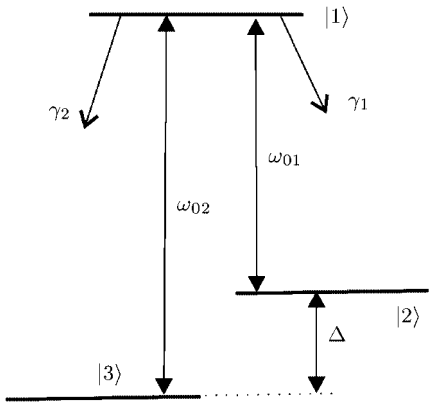

Fig. 1. Three-level atom diagram. Here ω_{01} and ω_{02} are the two transition frequencies, γ_1 and γ_2 are the two spontaneous emission rates from $|1\rangle$ to $|2\rangle$ and $|3\rangle$, respectively.

We consider a Λ-configuration three-level atom with one upper level $|1\rangle$ and two near-by lower levels $|2\rangle$ and $|3\rangle$, as shown in Fig. 1, which is confined in a trap that has approximately harmonic binding potential with an oscillation frequency Ω. The transition between $|1\rangle$ and $|2\rangle$ ($|3\rangle$) with resonant transition frequency ω_{01} (ω_{02}) is assumed to be optically allowed, while the transition between $|2\rangle$ and $|3\rangle$ is forbidden on the basis of parity considerations. The radiation electromagnetic field, we assumed, has quasi-continuum modes, and the jth mode with a frequency ω_j has $\hat{a}_j^+$

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and $\hat{a}_j$ as the creation and annihilation operators. In our three-level atom model, the frequency difference Δ between two lower levels is far smaller than the resonant transition frequency ω_{01} (ω_{02}) and the frequency ω_j of radiation electromagnetic field.

The interaction Hamiltonian of the atom with the radiation electromagnetic field, taking the electronic dipole and rotating wave approximation and in the interaction picture, may be given by

$$\begin{aligned} H_I = &\sum_j \hbar g_{j1}\{\hat{\pi}^+\hat{a}_j \exp[i(\omega_{01}-\omega_j)t] \\ &\cdot \exp[i\boldsymbol{\xi}_j\cdot(\boldsymbol{A}^+e^{i\Omega t}+\boldsymbol{A}e^{-i\Omega t})] + \text{H.c.}\} \\ &+\sum_j \hbar g_{j2}\{\hat{\sigma}^+\hat{a}_j \exp[i(\omega_{02}-\omega_j)t] \\ &\cdot \exp[i\boldsymbol{\xi}_j\cdot(\boldsymbol{A}^+e^{i\Omega t}+\boldsymbol{A}e^{-i\Omega t})] + \text{H.c.}\}, \end{aligned} \tag{1}$$

where $g_{j1} = (\omega_j/2\varepsilon_0\hbar V)^{1/2}\boldsymbol{\varepsilon}_j\boldsymbol{D}_{12}$ and $g_{j2} = (\omega_j/2\varepsilon_0\hbar V)^{1/2}\boldsymbol{\varepsilon}_j\cdot\boldsymbol{D}_{13}$ are taken as two real coupling coefficients; $\boldsymbol{\varepsilon}_j$ is the polarization vector of the jth mode of electromagnetic field, V is the quantization volume, $\boldsymbol{D}_{12}$ and $\boldsymbol{D}_{13}$ are the dipole moments of the radiation transition, and the transition operators $\hat{\pi}^+$ and $\hat{\sigma}^+$ represent the transitions from $|2\rangle$ to $|1\rangle$ and $|3\rangle$ to $|1\rangle$ respectively, while the transition operators $\hat{\pi}$ and $\hat{\sigma}$ represent transitions from $|1\rangle$ to $|2\rangle$ and $|1\rangle$ to $|3\rangle$ respectively. Here $\boldsymbol{A}^+$ and $\boldsymbol{A}$ are the raising and lowering operators describing the excitation of the c.m. oscillation states. The position and momentum operators of the c.m. motion may be expressed by $\boldsymbol{A}^+$ and $\boldsymbol{A}$,[20]

$$i\boldsymbol{k}_j\cdot\boldsymbol{r} = i\boldsymbol{\xi}_j\cdot(\boldsymbol{A}^+ + \boldsymbol{A}), \quad \boldsymbol{\xi}_j = \frac{\sqrt{\hbar}\boldsymbol{k}_j}{\sqrt{2m\Omega}}, \tag{2}$$

where the crucial parameter $|\boldsymbol{\xi}_j|^2$ is the ratio of the atom recoil energy due to the jth mode electromagnetic field to the energy of a single excitation of the c.m. motion state in the trap.

The state vector of the atom–trap system may be written as

$$\begin{aligned} |\psi(t)\rangle = &a_N(t)|N,1,0\rangle + \sum_{M,j} b_{M,j}(t)|M,2,j\rangle \\ &+\sum_{L,j} c_{L,j}(t)|L,3,j\rangle, \end{aligned} \tag{3}$$

where $|N\rangle$ represents the initial centre-of-mass state; $|M\rangle$ and $|L\rangle$ represent the final c.m. states, and $\sum_M |M\rangle\langle M| = 1$, $\sum_L |L\rangle\langle L| = 1$; $|1\rangle$, $|2\rangle$ and $|3\rangle$ represent the three levels of the three-level atom; $|0\rangle$ and $|j\rangle$ refer to the photon vacuum and the single photon states, respectively.

The evolution of the state vector in the interaction picture obeys the Schrödinger equation

$$i\hbar\frac{\partial|\psi(t)\rangle}{\partial t} = H_I|\psi(t)\rangle. \tag{4}$$

Substituting Eqs. (1) and (3) into Eq. (4), and multiplying the left by $\langle 0,1,N|$, $\langle j,2,M|$ and $\langle j,3,L|$ respectively and then taking the inner product, we obtain

$$\dot{a}_N(t) = -i\sum_{M,j} F_{M,j}(t)b_{M,j}(t) - i\sum_{L,j} G_{L,j}(t)c_{L,j}, \tag{5a}$$

$$\dot{b}_{M,j}(t) = -iF^*_{M,j}(t)a_N(t), \tag{5b}$$

$$\dot{c}_{L,j}(t) = -iG^*_{L,j}(t)a_N(t), \tag{5c}$$

where

$$\begin{aligned} F_{M,j}(t) = &g_{j1}\exp[i(\omega_{01}-\omega_j)t] \\ &\cdot\langle N|\exp[i\boldsymbol{\xi}_j\cdot(\boldsymbol{A}^+e^{i\Omega t}+\boldsymbol{A}e^{-i\Omega t})]M\rangle, \end{aligned} \tag{6a}$$

$$\begin{aligned} G_{L,j}(t) = &g_{j2}\exp[i(\omega_{02}-\omega_j)t] \\ &\cdot\langle N|\exp[i\boldsymbol{\xi}_j\cdot(\boldsymbol{A}^+e^{i\Omega t}+\boldsymbol{A}e^{-i\Omega t})]L\rangle. \end{aligned} \tag{6b}$$

Equation (5) may be solved by the time-dependent perturbation theory. We first assume that $a_N(t)$ varies so slowly that it can be taken out of the integral of Eqs. (5b) and (5c). Then substituting the solutions of Eqs. (5b) and (5c) into Eq. (5a), we obtain the first-order probability amplitude of the $|1\rangle$ state in the initial condition of ground c.m. state and the excited atomic state

$$a^{\mathrm{I}}(t) = \exp(-\gamma t/2), \tag{7}$$

where $\gamma = \gamma_1 + \gamma_2$, and

$$\begin{aligned} \gamma_1 = &\sum_j \gamma_{j1} = 2\pi\sum_j g^2_{j1}\exp[-|\boldsymbol{\xi}_j|^2] \\ &\cdot\sum_n \frac{(|\boldsymbol{\xi}_j|^2)^n}{n!}\delta(\omega_j-\omega_{01}+n\Omega), \end{aligned} \tag{8a}$$

$$\begin{aligned} \gamma_2 = &\sum_j \gamma_{j2} = 2\pi\sum_j g^2_{j2}\exp[-|\boldsymbol{\xi}_j|^2] \\ &\cdot\sum_n \frac{(|\boldsymbol{\xi}_j|^2)^n}{n!}\delta(\omega_j-\omega_{02}+n\Omega), \end{aligned} \tag{8b}$$

where γ_1 (γ_2) is the decay rates from level $|1\rangle$ to level $|2\rangle$ ($|3\rangle$) and it is the same as the result from the two level atomic model in Ref. [20].

To obtain the second-order solution, we first substitute Eq. (7) into Eqs. (5b) and (5c) and integrate them, and then substitute their solutions into Eq. (5a), finally we integrate and square Eq. (5a) and derive the second-order probability of the level $|1\rangle$,

$$|a^{II}_j(t)|^2 = \Big|\sum_n g^2_{j1}\exp[-|\boldsymbol{\xi}_j|^2]\frac{(|\boldsymbol{\xi}_j|^2)^n}{n!}$$

$$\cdot \frac{2e^{-\frac{\gamma}{2}t}\cos(\omega_j-\omega_{01}+n\Omega)t-e^{-\gamma t}}{[(\omega_j-\omega_{01}+n\Omega)^2+(\gamma/2)^2]}$$
$$+\sum_m g_{j2}^2\exp[-|\boldsymbol{\xi}_j|^2]\frac{(|\boldsymbol{\xi}_j|^2)^m}{m!}$$
$$\cdot \frac{2e^{-\frac{\gamma}{2}t}\cos(\omega_j-\omega_{02}+m\Omega)t-e^{-\gamma t}}{[(\omega_j-\omega_{02}+m\Omega)^2+(\gamma/2)^2]}$$
$$+P_{\rm int}\Big|, \tag{9}$$

where the interference term $P_{\rm int}$ coming from the square of the sum of two parts of $a_j^{\Pi}(t)$ is given by

$$P_{\rm int}=\sum_{n,m}g_{j1}g_{j2}\exp[-|\boldsymbol{\xi}_j|^2]\times\sum_{n,m}\frac{(|\boldsymbol{\xi}_j|^2)^{n+m}}{n!m!}$$
$$\cdot\{2[\gamma^2/4+(\omega_j-\omega_{01}+n\Omega)(\omega_j-\omega_{02}+m\Omega)]A$$
$$-\gamma(\omega_{02}-\omega_{01}+n\Omega-m\Omega)B\}$$
$$\cdot\left\{\left[(\omega_j-\omega_{01}+n\Omega)^2+\left(\frac{\gamma}{2}\right)^2\right]\right.$$
$$\left.\cdot\left[(\omega_j-\omega_{02}+m\Omega)^2+\left(\frac{\gamma}{2}\right)^2\right]\right\}^{-1}, \tag{10}$$

where

$$A=1-e^{-\frac{1}{2}\gamma t}[\cos(\omega_j-\omega_{01}+n\Omega)t$$
$$+\cos(\omega_j-\omega_{02}+m\Omega)t]$$
$$+e^{-\gamma t}\cos(\omega_{02}-\omega_{01}+n\Omega-m\Omega)t, \tag{11a}$$

$$B=e^{-\frac{1}{2}\gamma t}[\sin(\omega_j-\omega_{01}+n\Omega)t$$
$$-\sin(\omega_j-\omega_{02}+m\Omega)t]$$
$$-e^{-\gamma t}\sin(\omega_{02}-\omega_{01}+n\Omega-m\Omega)t. \tag{11b}$$

With time evolution to infinity, $|a_j^{\Pi}(t)|^2$ becomes the second-order probability spectrum in the level $|1\rangle$, which is written as

$$|a_j^{\Pi}(\infty)|^2=\Big|\sum_{n,m}g_{j1}g_{j2}\exp[-|\boldsymbol{\xi}_j|^2]$$
$$\cdot\sum_{n,m}\frac{(|\boldsymbol{\xi}|^2)^{n+m}}{n!m!}\{2[\gamma^2/4+(\omega_j-\omega_{01}+n\Omega)$$
$$\cdot(\omega_j-\omega_{02}+m\Omega)]\}\Big\{\Big[(\omega_j-\omega_{01}+n\Omega)^2$$
$$+\left(\frac{\gamma}{2}\right)^2\Big]\cdot\Big[(\omega_j-\omega_{02}+m\Omega)^2$$
$$+\left(\frac{\gamma}{2}\right)^2\Big]\Big\}^{-1}\Big|. \tag{12}$$

The second-order probability $|a_j^{\Pi}(t)|^2$ of the level $|1\rangle$, as shown in Eq. (9), will be decreased with the time increasing and modified by the decay rate γ, the oscillation frequency Ω and the parameter $|\boldsymbol{\xi}_j|^2$. When $t>0$, the emitted light is composed of the contributions of decays from the level $|1\rangle$ to the levels $|2\rangle$ and $|3\rangle$, which leads to an interference pattern characterized by the dark lines in the second-order probability spectrum (see below) when the natural width γ is the order of the frequency different Δ between the two lower levels and the oscillation frequency Ω is much larger than the natural width γ. That is, the condition to generate the quantum interference effect is given by

$$\gamma\sim\Delta,$$
$$\gamma\ll\Omega. \tag{13}$$

Here the spontaneously generated interference pattern results from the same vacuum mode in the decay processes from the upper level $|1\rangle$ to the two near-by lower levels $|2\rangle$ and $|3\rangle$, which is similar to the case in Refs. [13,21].

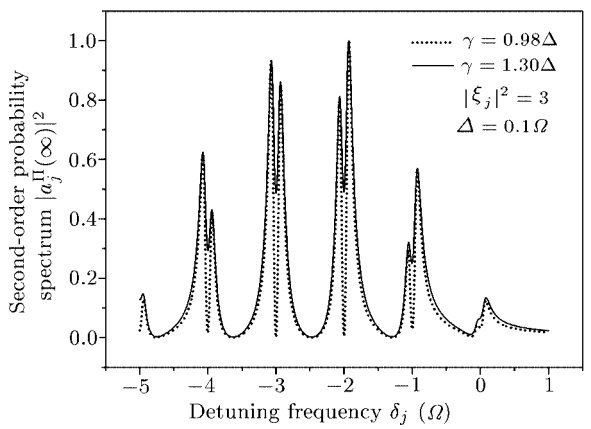

Fig. 2. Dependence of the second-order probability spectrum $|a_j^{\Pi}(\infty)|^2$ on the detuning $\delta_j=\omega_j-(\omega_{01}+\omega_{02})/2$ as $\Delta=0.1\Omega$, $|\boldsymbol{\xi}_j|^2=3$, $\gamma=0.98\Delta$ and $\gamma=1.3\Delta$.

From the second-order probability spectrum of the level $|1\rangle$, as shown in Eq. (12), we know that the probabilities of some radiation electromagnetic field modes in the level $|1\rangle$ initially will not be decayed to zero eventually (but the total population in the level $|1\rangle$ will be decayed to zero eventually if a radiation electromagnetic field mode will be zero eventually), which is also caused by the quantum interference effect in this atom. The spontaneous emissions from the level $|1\rangle$ to the two lower levels with a frequency different Δ ($\sim\gamma$) coupled by the same vacuum mode produce quantum destructive interference effect, and then make the second-order probability spectrum appear multiple dark lines, which is also related with the parameters γ, Δ, Ω and $|\boldsymbol{\xi}_j|^2$. From Eq. (12), we obtain the dependence of the second-order probability spectrum $|a_j^{\Pi}(\infty)|^2$ on $\delta_j=\omega_j-(\omega_{01}+\omega_{02})/2$, as shown in Fig. 2, with $\Delta=0.1\Omega$, $|\boldsymbol{\xi}_j|^2=3$, and $\gamma=0.98\Delta$, 1.3Δ. We can see from Fig. 2 that there are a series of dark lines in the second-order probability spectrum, which results from the vacuum-induced quantum in-

terference effect.[13,21] From Eq. (12), in fact, we know that if $\gamma = 0.98\Delta$, there are a series of the deepest dark lines at $\delta_j = 0, -\Omega, -2\Omega, \cdots$, as shown by the dotted line in Fig. 2, which is produced by the complete destructive quantum interference in the process of decay from the upper level to two lower levels. If γ is closed to 0.98Δ, we find that there are a series of shallower dark lines in the spectrum, as shown by the solid line in Fig. 2, which stems from the partial destructive quantum interference in the spontaneous-decay process from the excited state $|1\rangle$ to two lower levels $|2\rangle$ and $|3\rangle$. If γ is far away from 0.98Δ, we find that there are no dark lines in the spectrum, and the quantum interference effect will be disappeared.

To study quantitatively this quantum interference effect, we define a normalized interference degree that is given by the ratio of the difference between the maximum value around $\delta_j = -|\boldsymbol{\xi}_j|^2$ and the value at $\delta_j = -|\boldsymbol{\xi}_j|^2$ to the maximum value around $\delta_j = -|\boldsymbol{\xi}_j|^2$ in the second-order probability spectrum, by which we calculate the dependence of the interference degree on γ/Δ as $\Delta = 0.1\Omega$ (0.2Ω), and the result is shown in Fig. 3. We can see from Fig. 3 that the maximum interference degree will appear at $\gamma = 0.98\Delta$ (0.925Δ) as $\Delta = 0.1\Omega$ (0.2Ω), and the interference degree will be rapidly decreased as γ/Δ leaving the position of maximum interference degree.

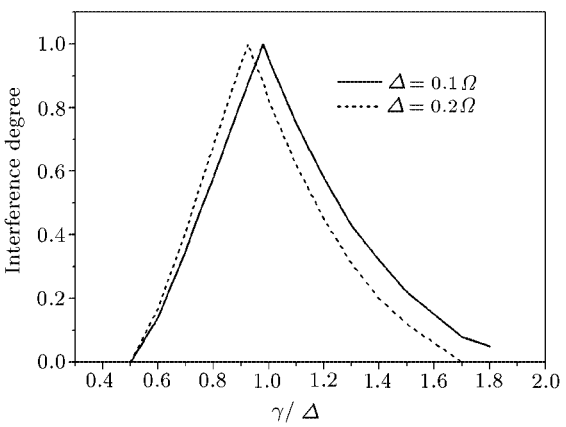

Fig. 3. Dependence of the interference degree on γ/Δ as $\Delta = 0.1\Omega$, 0.2Ω.

In conclusion, the dynamic equation of a Λ-configuration three-level atom confined in an approximately harmonic trapping potential has been solved by the time-dependent perturbation theory. In consideration of quantization of c.m. motion in the trap, we have derived the second-order solutions in the case of the trapped atom in the excited state and the ground c.m. oscillation state initially, and found that there are a series of dark lines in the second-order probability spectrum with multi-peak structures, which results from the vacuum-induced quantum interference in the process of spontaneous emission in this system, and can be modified by the oscillation frequency Ω of atom in the trap and the frequency difference Δ between two lower levels. Thus this quantum interference effect may develop a new method to measure Ω or Δ even γ. In the future, we will analyse the effect of the quantization of c.m. motion on the special quantum interference effects such as LWI, EIT, and so on.

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