Gravitational Faraday Rotation of the Earth and Its Possible Test *

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It is shown that the rotation of the polarization plane of rays induced by a rotating body can be accumulated by means of a long baseline optical cavity. Theoretical analysis shows that the presently proposal experimental scheme is possible to test this gravitational Faraday rotation effect on the Earth, especially including how to effectively suppress the dominant part of the Sagnac effect due to the rotation of the Earth with a reasonable experimental configuration.

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There are two kinds of general relativistic effects for electromagnetic theory in a curved spacetime. One is the geodesic effect, in which a ray follows a null geodesic regardless of its polarization state. The other is the gravitational Faraday rotation effect, in which the polarization vector for linearly polarized light ray is parallelly transported along the ray.[1−5] The geodesic effect has been measured in some experiments, such as the light bending and time delay of electromagnetic waves,[6,7] but the gravitational Faraday rotation has not been detected up to date.

The gravitational effect on the polarization of rays was first investigated by Skrotskii[8] and Rytov,[9] which is also known as the Skrotskii or Rytov effect. Following them, several authors have discussed the effect in detail.[10−14] So far, we have known that the integral form of this effect in the weak field approximation, which is similar to the usual Faraday effect,[15,16] can be expressed as[4]

$$\Omega^{sk} = -\frac{1}{2}\int_{sou}^{obs} \sqrt{g_{00}}\boldsymbol{B}_g \cdot d\boldsymbol{l}, \qquad (1)$$

where Ω^{sk} is the net angle of rotation of polarized vector around the propagating direction as shown in Fig. 1; $\boldsymbol{B}_g$ is gravitomagnetic field; g_{00} is the zero-zero component of the metric, and $d\boldsymbol{l}$ is an element displacement of the path of the light ray propagating from the light source to the observer. This effect is also called the "gravitational Faraday rotation,"[17,4] and it is evidently proportional to the integral of the component of the gravitomagnetic field along the propagation path. Meanwhile this effect is purely geometrical effect and it does not depend on the frequency of the light ray. However, it is much smaller than the effect of the bending of light, since the effect of the gravitomagnetic field is usually much smaller than of the gravitoelectric field.

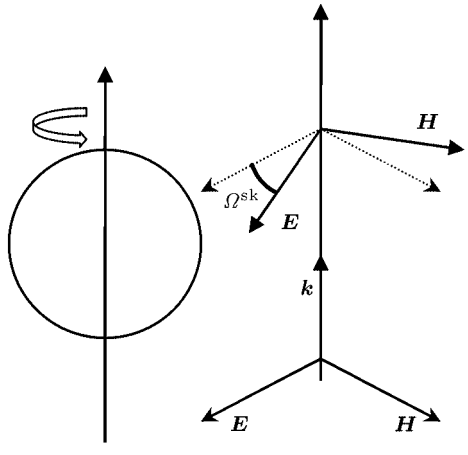


Fig. 1. Gravitational Faraday rotation. The polarization vector of a linearly polarized light is dragged due to the gravitomagnetic field of a rotating body.

In this Letter, we have shown that the gravitational Faraday rotation is an accumulated effect for a round-trip ray. Therefore, we propose a possible experimental scheme on the Earth to test this accumulated effect of a light ray in an optical cavity induced due to the rotation of Earth. Finally we consider the influence of the Sagnac effect,[18] including how to suppress it.

Following Mach's principle, the rotation of the polarization plane of the light ray can be expressed using the dragging of inertial frames.[12] Due to the gravitomagnetic field, inertial frames near the massive moving source should be dragged in the direction of motion of the source of the gravitational field. The angular velocity with which a local inertial frame, instantaneously at rest in a stationary frame, rotates with respect to the stationary frame can be expressed as $-\sqrt{g_{00}}\nabla \times \boldsymbol{A}/2$ with $\boldsymbol{A} = -g_{0a}/g_{00}$.[19,2] Therefore, the total rotation of the polarization vector around the wave vector $\boldsymbol{k}$ of a ray along the path from the

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source to the observer is

$$\Omega = -\frac{1}{2}\int_{sou}^{obs}\sqrt{g_{00}}\nabla\times\boldsymbol{A}\cdot d\boldsymbol{l} = -\frac{1}{2}\int_{sou}^{obs}\sqrt{g_{00}}\boldsymbol{B}_g\cdot d\boldsymbol{l}, \tag{2}$$

where we identify gravitomagnetic field $\boldsymbol{B}_g = \nabla\times\boldsymbol{A}$ and $d\boldsymbol{l} = \hat{k}dl$. In Eq. (2), the observer should be at rest in the stationary frame, i.e., only the zeroth component of the four-velocity is nonvanishing. Thus, the local orthonormal tetrads $\omega^{\alpha}_{(\beta)}$ are defined by $\omega^{\alpha}_{(0)} = (1 - h_{00}/2, 0, 0, 0)$, $\omega^{\alpha}_{(1)} = (-h_{0j}a^j, a^i + h_{ij}a^j/2)$, $\omega^{\alpha}_{(2)} = (-h_{0j}b^j, b^i + h_{ij}b^j/2)$ and $\omega^{\alpha}_{(3)} = (-h_{0j}k^j, k^i + h_{ij}k^j/2)$ with $g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$. The spatial vectors $\boldsymbol{a} = (a^1, a^2, a^3)$, $b = (b^1, b^2, b^3)$ and $\boldsymbol{k} = (k^1, k^2, k^3)$ are orthonormal in the Euclidean sense and the vector $\boldsymbol{k}$ usually defines the spatial direction of propagation of the light ray.[5] From Eq. (2), we conclude that the gravitational Faraday rotation is only a gravitomagnetic effect or a pure geometrical effect. Now we consider the case that the light ray travels along a path and then is reflected by a mirror. If the path is chosen in the x axis, the total rotation angle of the polarization vector for the round-trip light can be expressed as

$$\begin{aligned}\Omega^{sk}_{\text{round-trip}} =& -\frac{1}{2}\int_0^l\sqrt{g_{00}}B_x dl \\ &+(-1)\Big(-\frac{1}{2}\int_l^0\sqrt{g_{00}}B_x dl\Big) \\ =& -\int_0^l\sqrt{g_{00}}B_x dl,\end{aligned} \tag{3}$$

where $B_x = \boldsymbol{B}_g\cdot\boldsymbol{e}_x$ and the factor (-1) means that the wave vector of the reflected light is opposite to that of the incident light. From Eq. (3), it is concluded that the gravitational Faraday rotation is an accumulated effect for a round-trip light by analogy with the usual Faraday rotation, and the rotation angle is proportional to the optical length of the light ray.

Hence, in order to increase the effect we can make the light ray reflected many times between an optical cavity as possible, When the light ray in the cavity comes and goes n times, the total deflection angle is equal to $n\Omega^{sk}_{\text{round-trip}}$.

If we want to perform such experiment on the surface of the rotating source, for example, on the surface of the Earth, the above result should be transformed from a static frame to a co-rotating one. For simplicity, we assume that a light ray propagates in a weak gravitational field. Generally, the external gravitational field for a rotation spherical symmetric body described by the Kerr metric is

$$\begin{aligned}ds^2 =& \Big(1-\frac{2GM}{c^2r}\Big)c^2dt^2 + 4\frac{GJ}{c^3r}\sin^2\theta d\varphi c dt \\ &-\Big(1+\frac{2GM}{c^2r}\Big)[dr^2 + r^2(d\theta^2+\sin^2\theta d\varphi^2)],\end{aligned} \tag{4}$$

where M and J refer to the mass and the angular momentum of the rotating source, respectively. We transform the Kerr metric to the surface of the rotating source with an angular velocity of ω, for example, the angular velocity of the Earth. Substituting $d\varphi = d\tilde{\varphi} - \omega dt$ into Eq. (4), one finds that the Kerr metric can be rewritten as

$$\begin{aligned}ds^2 =& \Big[\Big(1-\frac{2GM}{c^2r}\Big) - 4\frac{GJ\omega}{c^3r}\sin^2\theta \\ &-\Big(1+\frac{2GM}{c^2r}\Big)\frac{r^2\omega^2\sin^2\theta}{c^2}\Big]c^2dt^2 \\ &+\Big[4\frac{GJ}{c^3r}\sin^2\theta + 2\Big(1+\frac{2GM}{c^2r}\Big)\frac{\omega r^2\sin^2\theta}{c}\Big]d\varphi c dt \\ &-\Big(1+\frac{2GM}{c^2r}\Big)[dr^2 + r^2(d\theta^2+\sin^2\theta d\varphi^2)],\end{aligned} \tag{5}$$

where we have ignored the wave mark of $\tilde{\varphi}$. By transforming the spherical coordinates (r, θ, φ) to Cartesian coordinates (x, y, z) in the usual manner, $x = r\cos\varphi\sin\theta$, $y = r\sin\varphi\sin\theta$ and $z = r\cos\theta$, the above equation can be expressed as

$$\begin{aligned}ds^2 =& \Big[\Big(1-\frac{2GM}{c^2r}\Big) - \frac{\omega^2(x^2+y^2)}{c^2}\Big]c^2dt^2 \\ &+2\Big[2\frac{GJ}{c^3r^3} + \Big(1+\frac{2GM}{c^2r}\Big)\frac{\omega}{c}\Big][-ydxcdt + xdycdt] \\ &-\Big(1+\frac{2GM}{c^2r}\Big)(dx^2+dy^2+dz^2)\end{aligned} \tag{6}$$

in the post-Newtonian approximation. We also note that the element of a stationary spacetime is

$$\begin{aligned}ds^2 =& g_{\mu\nu}dx^\mu dx^\nu \\ =& g_{00}c^2dt^2 + 2g_{0a}cdtdx^a + g_{ab}dx^a dx^b,\end{aligned} \tag{7}$$

where g_{00} is independent of time t. Comparing Eq. (6) with Eq. (7), we can obtain the expressions of the metric tensors as follows:

$$\begin{aligned}g_{00} =& 1 - \frac{2GM}{c^2r} - \frac{\omega^2(x^2+y^2)}{c^2}, \\ g_{0a} =& 2\frac{G}{c^3}\frac{\boldsymbol{J}\times\boldsymbol{r}}{r^3} + \Big(1+\frac{2GM}{c^2r}\Big)\frac{\boldsymbol{\omega}\times\boldsymbol{r}}{c}.\end{aligned} \tag{8}$$

Obviously, Eq. (6) describes a stationary spacetime and the observer on the surface of the Earth is at rest in this stationary frame, whose world line is the x^0-coordinate line. Thus, we can use Eq. (2) to discuss the gravitational Faraday rotation. Substituting Eq. (8) into Eq. (2), the rotation angle of a polarized light ray on the surface of the Earth is obtained to be

$$\begin{aligned}\Omega =& \frac{1}{2}\int_{sou}^{obs}\Big[\frac{2G}{c^3}\cdot\frac{3(\hat{r}\cdot\boldsymbol{J})\hat{r} - \boldsymbol{J}}{r^3} \\ &+\frac{2\boldsymbol{\omega}}{c} + \frac{2GM}{c^3}\cdot\frac{\boldsymbol{\omega} + (\hat{r}\cdot\boldsymbol{\omega})\hat{r}}{r}\Big]\cdot d\boldsymbol{l}.\end{aligned} \tag{9}$$

The above equation consists of three parts. The first term represents the gravitational Faraday rotation due to the gravitomagnetic field, which is also known as the Lense–Thirring effect generated by the spin of the Earth.[20] The second term represents the Sagnac effect attributed to the co-rotating of the apparatus on the surface of the Earth, and the third one represents the general relativity correction to the Sagnac effect. Both the first and third term are general relativity effects. Here we only focus on the test of the first term, the gravitational Faraday rotation, by means of a ground-based experiment.

We first assume that the light ray travels parallelly with the rotation axis of the Earth and the length of baseline is 1 km. If such experiment was performed near the equator, the gravitational Faraday rotation would be about 6.7×10^{-20} rad, the Sagnac effect of the second term 2.4×10^{-10} rad, and the effect of the third term in Eq. (9) 1.7×10^{-19} rad. Here the relative parameters were approximately estimated to be $r = 6.4 \times 10^6$ m, $M = 6 \times 10^{27}$ g, $J = 2Mr^2\omega/5$ and $\omega = 7.3 \times 10^{-5}$ rad/s. It is noted that the gravitational Faraday rotation is about 10 orders smaller than the Sagnac effect, which implies that the detection of the gravitational Faraday rotation with such configuration is very difficult from the observational viewpoint.

In order to observe the effect of the gravitational Faraday rotation, the dominant part of the Sagnac effect, i.e. the second term in Eq. (9), should be suppressed effectively. To do so, we assume that the light ray travels perpendicular to the rotation axis of the Earth but not in the equatorial plane. It is clear that the direction of the angular momentum of the Earth $\boldsymbol{J}$ is orthogonal to the position vector $\boldsymbol{r}$ in the equatorial plane and the angle of rotation including all the three terms in Eq. (9) will be zero.

According to the above analysis we can assume that the geographical latitude of the laboratory is λ in the north hemisphere, which means that the angle between the direction of the angular momentum of the Earth and the position vector of the experimental location is $\theta = 90^\circ - \lambda$. Let us consider a linearly polarized light ray propagating perpendicular to $\boldsymbol{J}$ in the meridian plane in an optical cavity as shown in Fig. 2, which means $\boldsymbol{J} \cdot d\boldsymbol{l} = \boldsymbol{\omega} \cdot d\boldsymbol{l} = 0$, Eq. (9) will become

$$\begin{aligned}\Omega &= \frac{1}{2}\int_{sou}^{obs}\Big[\frac{12GM}{5c^3}\cdot\frac{(\hat{r}\cdot\boldsymbol{\omega})\hat{r}}{r} + \frac{2GM}{c^3}\cdot\frac{(\hat{r}\cdot\boldsymbol{\omega})\hat{r}}{r}\Big]\cdot d\boldsymbol{l} \\ &= \frac{1}{2}\frac{\sin 2\theta}{2}\Big[\frac{12GM\omega}{5c^3 r} + \frac{2GM\omega}{c^3 r}\Big]\cdot L, \qquad (10)\end{aligned}$$

where $J = 2Mr^2\omega/5$ is used in the above equation, and L is the integral length. In fact, the light ray follows a null geodesic and its three-dimensional wave vector $\boldsymbol{k}$ is variational. However, to the lowest order, the spacetime is approximately flat, since $\omega r \ll c$. Therefore, the wave vector $\boldsymbol{k}$ can be considered as a constant. It is rational to assume that the spatial direction of propagation of the light ray is perpendicular to $\boldsymbol{J}$.

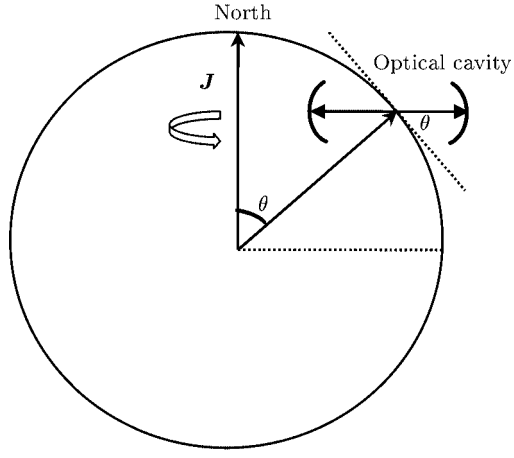


Fig. 2. Schematic experiment for test of gravitational Faraday rotation. The geographical colatitude of the experiment location is θ. A linearly polarized light ray propagates perpendicular to $\boldsymbol{J}$ in the meridian plane in an optical cavity. The baseline of the optical cavity lies in the south-north direction but with an elevation of θ degrees.

The first term of the square brackets in the above equation represents the gravitational Faraday rotation, which is 6/5 times to the second term, the general relativity correction to the Sagnac effect. Therefore, although both of the two effects are measured simultaneously, their contributions can be individually determined. From Eq. (10) we know that the optimal experimental location should be $\theta = 45^\circ$. If the integral length is 10^9 m, for example, the light ray is reflected 10^6 times in the optical cavity of 1-km baseline, the total rotation angle will be 1.9×10^{-13} rad. This size of the angle deflection can be detected in principle. However, in the experiment there will be energy loss when the light ray is reflected by the mirrors. If we require that the light can be reflected 10^6 times, the reflectivity of the mirrors should be better than $(1\text{–}10^{-6})$. At the same time, in order to suppress the Sagnac effect below the gravitational Faraday effect, the direction of light ray travel should be strictly perpendicular to $\boldsymbol{J}$ with the angular error in 10^{-10} rad. If the arm length of optical cavity is 1km, the corresponding perpendicular error must be less than 0.1 μm.

The test of the rotation of the polarized plane can be also considered as the test of the gravitomagnetic field $\boldsymbol{B}_g$ with

$$\begin{aligned}\boldsymbol{B}_g &= \frac{2G}{c^3}\cdot\frac{\boldsymbol{J} - 3(\hat{r}\cdot\boldsymbol{J})\hat{r}}{r^3} = B_{\parallel}\boldsymbol{e}_k + B_{\perp}\boldsymbol{e}_J \\ &= -\frac{12\Omega}{11L}\Big(\boldsymbol{e}_k + \frac{1+3\cos 2\theta}{3\sin 2\theta}\boldsymbol{e}_J\Big), \qquad (11)\end{aligned}$$

where $B_{\parallel}$ is the component of $\boldsymbol{B}_g$ along the line of

ray $\boldsymbol{e}_k$, and $B_\perp$ is the component of $\boldsymbol{B}_g$ along the rotation axis of Earth $E\boldsymbol{e}_J$. Obviously, the gravitomagnetic field $\boldsymbol{B}_g$ can be correspondingly determined with the measurement of the angle deflection Ω.

In summary, we have discussed the accumulated effect of the gravitational Faraday rotation and proposed a feasible scheme to detect the rotation angle of the polarization plane on the surface of Earth. By making the light ray to travel perpendicularly to the rotating axis of the Earth but not in the equatorial plane, we can effectively suppress the main part of the Sagnac effect, and then measure the gravitational Faraday rotation effect accordingly.

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