Generation of Three-Dimensional Entangled States for Two Atoms Trapped in Different Cavities *

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We propose a scheme for generating three-dimensional entangled states for two atoms trapped in two separate cavities. The scheme is based on the detection of photons leaking from the cavities after the atom-cavity interaction. The scheme is useful for the test of quantum nonlocality and quantum information processing.

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Entanglement of two particles is one of the most striking features of quantum mechanics. It is not only of significance for test of quantum mechanics against local hidden theory,[1] but also useful in the field of quantum information, such as quantum cryptography[2] and quantum teleportation.[3] Most of the research in quantum entanglement focuses on particles belonging to two-dimensional Hilbert space. Two-dimensional entangled states for two particles have been observed for photons,[4] atoms in cavity QED,[5,6] and ions in a trap.[7] Schemes have been proposed for the generation of two-dimensional entangled states for two atoms trapped in two separate optical cavities.[8−11] On the other hand, it has been shown that violations of local realism by two entangled N-dimensional systems are stronger than those for two qubits.[12] Furthermore, it has been shown that quantum cryptography based on entangled qutrits is more secure than that based on entangled qubits.[13] Schemes have been proposed for the generation of entangled states for two or more multilevel atoms in a cavity.[14,15]

In this Letter, we propose a scheme for generating three-dimensional entangled states for two atoms trapped in two separate cavities. In the scheme, each atom and the respective cavity are first prepared in an appropriately entangled state after an interaction. Then a joint detection of photons leaking from the cavities results in two-dimensional nonmaximally entanglement between the two atoms since one can not distinguish which cavity the photon comes from due to interference. After a local transformation is performed on each atom, the interaction-detection procedure is repeated. The detection of a single photon may project the two atoms to a three-dimensional entangled state. As far as we know, this is the first scheme for the generation of three-dimensional entangled states for two atoms each trapped in a cavity.

The atoms have one excited state $|r\rangle$ and three ground states $|e\rangle$, $|g\rangle$, and $|f\rangle$. The transition $|g\rangle \rightarrow |r\rangle$ is coupled to the cavity mode with the coupling constant g. We first drive the transition $|e\rangle \rightarrow |r\rangle$ by a classical laser field with a Rabi frequency Ω in the respective cavities. Assume that the classical field and cavity mode are detuned from the respective transitions by Δ, as shown in Fig. 1. Two distant atoms are trapped in two single-mode optical cavities, respectively. Photons are allowed to leak from both cavities and are then mixed on a beam-splitter, which destroys the path information, as shown in Fig. 2. The photons are then detected by two photodetectors. Assume that the two atoms are initially prepared in the state $|e_1\rangle|e_2\rangle$. Both the two cavities are initially in the vacuum states.

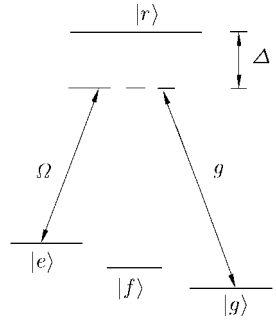


Fig. 1. The level configuration of the atoms. The transition $|g\rangle \rightarrow |r\rangle$ is coupled to the cavity mode, while the transition $|e\rangle \rightarrow |r\rangle$ is coupled to a classical field. Both the cavity mode and classical field are detuned from their respective transitions by Δ.

In the case $\Delta \gg \Omega, g$ the upper level $|r\rangle$ can be adiabatically eliminated. When $\Delta \gg \gamma$, where γ is the spontaneous decay rate, the spontaneous emission can be neglected. This leads to the interaction Hamiltonian between the jth atom and the respective cavity[16−18]

$$H_j = \Big[\frac{\Omega^2}{\Delta}|e_j\rangle\langle e_j| + \frac{g^2}{\Delta}a_j^+ a_j|g_j\rangle\langle g_j| + \lambda(a_j S_j^+ + a_j^+ S_j^-)\Big], \quad (1)$$

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where a_j is the annihilation operator of the jth cavity mode, $\lambda = \frac{\Omega g}{\Delta}$, $S_j^+ = |e_j\rangle\langle g_j|$ and $S_j^- = |g_j\rangle\langle e_j|$. The first and second terms describe the Stark shifts for the states $|e_j\rangle$ and $|g_j\rangle$, induced by the classical field and cavity fields, respectively. The third term describes the Raman coupling of the two ground states. The Hamiltonian can be switched on or off by switching on or off the classical field.

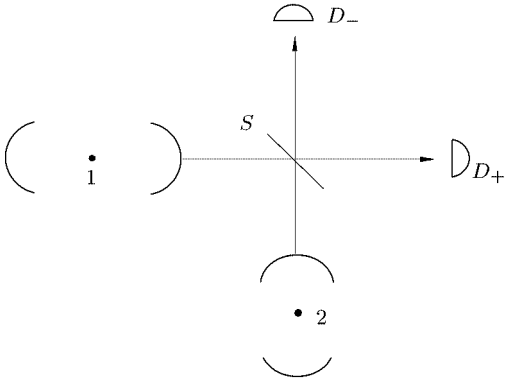


Fig. 2. The experimental setup. Two distant atoms 1 and 2 are trapped in separated cavities. Photons can leak through the sides of the cavities facing the beam-splitter S and then detected by the photon-detectors D_+ and D_-.

Consider the strong coupling regime, i.e, $\lambda \gg \kappa$, where κ is the cavity decay rate. In this case the cavity decay can be neglected during the atom-cavity interaction. Assume that $\Omega = g$. Then we obtain the evolution

$$|e_j\rangle|0_j\rangle \rightarrow \cos\lambda t|e_j\rangle|0_j\rangle - i\sin\lambda t|g_j\rangle|1_j\rangle. \quad (2)$$

On the other hand, the state $|g_j\rangle|0_j\rangle$ and $|f_j\rangle|0_j\rangle$ undergoes no transitions. Here we have discarded the common phase factor $e^{-i\lambda t}$. We first switch on the Hamiltonian of Eq. (1) in the first cavity by turning on the respective classical field. The interaction time t_1 satisfies $\lambda t_1 = \pi/4$. Then the first atom and cavity is in the state

$$\frac{1}{\sqrt{2}}(|e_1\rangle|0_1\rangle - i|g_1\rangle|1_1\rangle). \quad (3)$$

Meanwhile, we switch on the Hamiltonian in the second cavity for a time t_1' given by $sin\lambda t_1' = 1/\sqrt{5}$. This leads to

$$\frac{2}{\sqrt{5}}|e_2\rangle|0_2\rangle - i\frac{1}{\sqrt{5}}|g_2\rangle|1_2\rangle. \quad (4)$$

The state of the whole system is

$$\frac{1}{\sqrt{2}}(|e_1\rangle|0_1\rangle - i|g_1\rangle|1_1\rangle)$$
$$\Big(\frac{2}{\sqrt{5}}|e_2\rangle|0_2\rangle - i\frac{1}{\sqrt{5}}|g_2\rangle|1_2\rangle\Big). \quad (5)$$

Now we let the classical field off and wait the detectors to click. Due to the cavity decay during the waiting stage the system is governed by the non-Hermitian Hamiltonian $\sum_{j=1}^{2} -i\kappa a_j^+ a_j$.[16] This leads to

$$\sqrt{\frac{1}{(1+e^{-2\kappa t})\Big(\frac{4}{5}+\frac{1}{5}e^{-2\kappa t}\Big)}}(|e_1\rangle|0_1\rangle - ie^{-\kappa t}|g_1\rangle|1_1\rangle)$$
$$\cdot\Big(\frac{2}{\sqrt{5}}|e_2\rangle|0_2\rangle - i\frac{1}{\sqrt{5}}e^{-\kappa t}|g_2\rangle|1_2\rangle\Big). \quad (6)$$

The registering of a click at one of the photon-detectors at moment t_2 corresponds to the action of the jump operators $(a_1 \pm a_2)/\sqrt{2}$ on the state. Thus the system is projected to

$$\sqrt{\frac{1}{1+\frac{1}{5}e^{-2\kappa t_2}}}\Big[\Big(-i\frac{2}{\sqrt{5}}|g_1\rangle|e_2\rangle \mp i\frac{1}{\sqrt{5}}|e_1\rangle|g_2\rangle\Big)|0_1\rangle|0_2\rangle$$
$$-\frac{1}{\sqrt{5}}e^{-\kappa t_2}|g_1\rangle|g_2\rangle(|0_1\rangle|1_2\rangle \pm |1_1\rangle|0_2\rangle)\Big]. \quad (7)$$

Then we wait for another time t_2'. If no photon decays from either cavity during this period the system evolves to

$$\sqrt{\frac{1}{1+\frac{1}{5}e^{-2\kappa(t_2+t_2')}}}\Big[\Big(-i\frac{2}{\sqrt{5}}|g_1\rangle|e_2\rangle$$
$$\mp i\frac{1}{\sqrt{5}}|e_1\rangle|g_2\rangle\Big)|0_1\rangle|0_2\rangle$$
$$-\frac{1}{\sqrt{5}}e^{-\kappa t_2}e^{-\kappa t_2'}|g_1\rangle|g_2\rangle(|0_1\rangle|1_2\rangle \pm |1_1\rangle|0_2\rangle)\Big]. \quad (8)$$

In the case that $\kappa(t_2+t_2') \gg 1$ we have $e^{-2\kappa(t_2+t_2')} \ll 1$. Then the two atoms approximate the nonmaximally entangled state

$$\frac{2}{\sqrt{5}}|g_1\rangle|e_2\rangle \pm \frac{1}{\sqrt{5}}|e_1\rangle|g_2\rangle. \quad (9)$$

Here we have discarded the common phase factor $-i$.

Now the local transformation $|g_1\rangle \longrightarrow |f_1\rangle$ and $|g_2\rangle \longrightarrow |f_2\rangle$ are performed. In order to perform the transformation $|g_j\rangle \longrightarrow |f_j\rangle$ we excite the jth atom with a pair of off-resonant classical fields driving the transitions $|g_j\rangle \longrightarrow |r_j\rangle$ and $|f_j\rangle \longrightarrow |r_j\rangle$, respectively. The two classical fields are detuned from the respective transitions by the same amount δ. In the case that the detuning δ is much larger than the respective coupling strength the upper level $|r_j\rangle$ can be adiabatically eliminated and the two classical fields just induce the Raman transition between the states $|g_j\rangle$ and $|f_j\rangle$.[19] We can obtain required transformations by controlling the phases of the classical fields and interaction time appropriately. This leads to

$$\frac{2}{\sqrt{5}}|f_1\rangle|e_2\rangle \pm \frac{1}{\sqrt{5}}|e_1\rangle|f_2\rangle. \quad (10)$$

We switch on the Hamiltonian in the first cavity for a

time $\pi/2\lambda$, resulting in

$$\frac{2}{\sqrt{5}}|f_1\rangle|e_2\rangle|0_1\rangle|0_2\rangle \mp i\frac{1}{\sqrt{5}}|g_1\rangle|f_2\rangle|1_1\rangle|0_2\rangle. \quad (11)$$

The transformation $|f_1\rangle \longrightarrow |e_1\rangle$ leads to

$$\frac{2}{\sqrt{5}}|e_1\rangle|e_2\rangle|0_1\rangle|0_2\rangle \mp i\frac{1}{\sqrt{5}}|g_1\rangle|f_2\rangle|1_1\rangle|0_2\rangle. \quad (12)$$

The transformation $|f_1\rangle \longrightarrow |e_1\rangle$ can be achieved by exciting the first atom with another pair of off-resonant classical fields driving the transitions $|f_1\rangle \longrightarrow |r_1\rangle$ and $|e_1\rangle \longrightarrow |r_1\rangle$, respectively. Suppose that the two classical fields are detuned from the respective transitions by the same amount δ' which is much larger than the respective coupling strength. In this case the state $|r_1\rangle$ can be adiabatically eliminated and the two classical fields just induce the Raman transition between the states $|f_1\rangle$ and $|e_1\rangle$. Then the transformation $|g_1\rangle \longrightarrow |f_1\rangle$ results in

$$\frac{2}{\sqrt{5}}|e_1\rangle|e_2\rangle|0_1\rangle|0_2\rangle \mp i\frac{1}{\sqrt{5}}|f_1\rangle|f_2\rangle|1_1\rangle|0_2\rangle. \quad (13)$$

We now simultaneously switch on the Hamiltonians in both the cavities for a time $\pi/4\lambda$. This leads to

$$\frac{1}{\sqrt{5}}(|e_1\rangle|0_1\rangle - i|g_1\rangle|1_1\rangle)(|e_2\rangle|0_2\rangle - i|g_2\rangle|1_2\rangle)$$
$$\mp i\frac{1}{\sqrt{5}}|f_1\rangle|f_2\rangle|1_1\rangle|0_2\rangle. \quad (14)$$

Now we wait for the detectors to click. During the waiting period the system evolves to

$$\sqrt{\frac{1}{(1+e^{-2\kappa t})^2+e^{-2\kappa t}}}[(|e_1\rangle|0_1\rangle - ie^{-\kappa t} \cdot |g_1\rangle|1_1\rangle)(|e_2\rangle|0_2\rangle - ie^{-\kappa t}|g_2\rangle|1_2\rangle) \mp ie^{-\kappa t}|f_1\rangle|f_2\rangle|1_1\rangle|0_2\rangle]. \quad (15)$$

The detection of a single resister after a time t_3 leads to

$$\frac{1}{\sqrt{3+2e^{-2\kappa t_3}}}[(-i|g_1\rangle|e_2\rangle \mp i|e_1\rangle \cdot |g_2\rangle \mp i|f_1\rangle|f_2\rangle)|0_1\rangle|0_2\rangle - e^{-\kappa t_3}|g_1\rangle|g_2\rangle(|0_1\rangle|1_2\rangle \pm |1_1\rangle|0_2\rangle)]. \quad (16)$$

Then we wait for another time t_3'. If no photon decays from either cavity during this period the state is

$$\frac{1}{\sqrt{3+2e^{-2\kappa(t_3+t_3')}}}[(-i|g_1\rangle|e_2\rangle \mp i|e_1\rangle \cdot |g_2\rangle \mp i|f_1\rangle|f_2\rangle)|0_1\rangle|0_2\rangle - e^{-\kappa(t_3+t_3')}|g_1\rangle|g_2\rangle(|0_1\rangle|1_2\rangle \pm |1_1\rangle|0_2\rangle)]. \quad (17)$$

In the case $e^{-\kappa(t_3+t_3')} \ll 1$ the system approximates the three-dimensional maximally entangled state

$$\frac{1}{\sqrt{3}}(|g_1\rangle|e_2\rangle \pm |e_1\rangle|g_2\rangle \pm |f_1\rangle|f_2\rangle), \quad (18)$$

with the cavities left in the vacuum states. The probability of success is

$$P = P_1P_2 = 0.3, \quad (19)$$

where $P_1 = 1/2$ and $P_2 = 3/5$ are the probabilities of success during the two detection stages, respectively.

In summary, we have presented a scheme for the generation of three-dimensional entangled states for two atoms trapped in two separate cavities based on the interference between two paths from which the photon comes from. The scheme involves the atom-cavity interactions, joint photon detections, and local transformations on the atoms. The scheme not only is useful for the test of quantum mechanics against local hidden variable theory, but also has practical applications in quantum information processing.

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