Global Chaos Synchronization between Two New Different Chaotic Systems via Active Control *

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We present chaos synchronization between two new different chaotic systems by using active control. The proposed controller ensures that the states of the controlled chaotic response system asymptotically synchronizes the states of the drive system. Numerical simulations are shown to verify the result.

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Chaos is very interesting nonlinear phenomenon and has been intensively studied in the last three decades.[1−11] In particular, the subject of chaotic synchronization has received considerable attention since 1990. Also, numerous applications to synchronization have been widely explored in a variety of fields including physical, chemical, and ecological system, etc. Thus a wide variety of approaches have been proposed for the synchronization of chaotic systems which include linear and nonlinear feedback control, adaptive control, time-delay feedback control, etc. Most of the above-mentioned methods synchronize two identical chaotic systems. However, more and more applications of chaos synchronization in secure communications make it much more important to synchronize two new different chaotic systems, however the method of the synchronization of two different chaotic systems is far from straightforward. In this Letter, we apply active control theory to synchronize two different chaotic systems.

Recently, Lü, Chen and Cheng introduced a new chaotic system of three-dimensional quadratic autonomous ordinary differential equations, which can display two 1-scroll chaotic attractors simultaneously with only three equilibria and two 2-scroll chaotic attractors simultaneously with five equilibria.

$$\dot{x} = -\frac{ab}{a+b}x - yz + c, \quad \dot{y} = ay + xz,$$
$$\dot{z} = bz + xy. \tag{1}$$

where a, b, and c are real constants; and x, y, and z are status variables. This system is found to be chaotic in a wide parameter range and has many interesting complex dynamical characteristics. The system is chaotic for the parameters $a = -10$, $b = -4$ and $|c| < 19.2$, it displays the chaotic attractor as shown in Fig. 1. Detailed dynamic properties of this system can be found in Ref. [1].

The second system which can display two 2-scroll chaotic attractors and two 4-scroll chaotic attractors can be described as follows:

$$\dot{x} = px - yz, \quad \dot{y} = -vy + xz, \quad \dot{z} = -uz + xy, \tag{2}$$

where p, v, and u are positive control parameters. This system exhibits a chaotic attractor at the parameter values $p = 0.4$, $v = 12$ and $u = 5$, it displays the chaotic attractor as shown in Fig. 2. This system bridges the gap between the Lorenz and Chen Attractors. Details of other dynamic properties of the system can be found in Ref. [2].

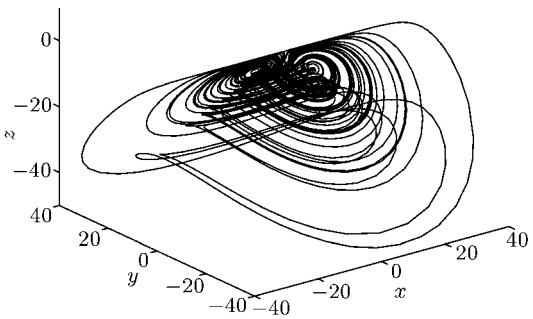

Fig. 1. Chaotic attractor ($a = -10$, $b = -4$, $c = 0$).

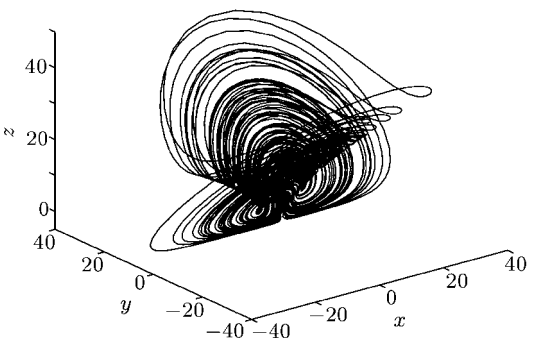

Fig. 2. The two-scroll chaotic attractor with parameters: $p = 4.5$, $v = 12$ and $u = 5$.

At the same time, the two different systems are chaotic. For instance, when the parameters are $a = -10$, $b = -4$, $c = 0$ and $p = 4.5$, $v = 12$ and

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$u = 5$,the orbits of three state of the two systems for initial conditions $(x(0), y(0), z(0)) = (3, -4, 2)$ and $(x(0), y(0), z(0)) = (1, 1, 1)$ are presented in Fig. 3. In order to observe the synchronization behaviour between the above systems, we assume that Lü–Chen–Cheng system drives the second system. Therefore, we define the drive and response systems as follows:

$$\begin{aligned} \dot{x_1} &= -\frac{ab}{a+b}x_1 - y_1 z_1 + c, \\ \dot{y_1} &= a y_1 + x_1 z_1, \\ \dot{z_1} &= b z_1 + x_1 y_1, \end{aligned} \tag{3}$$

and

$$\begin{aligned} \dot{x_2} &= p x_2 - y_2 z_2 + u_1(t), \\ \dot{y_2} &= -v y_2 + x_2 z_2 + u_2(t), \\ \dot{z_2} &= -u z_2 + x_2 y_2 + u_3(t). \end{aligned} \tag{4}$$

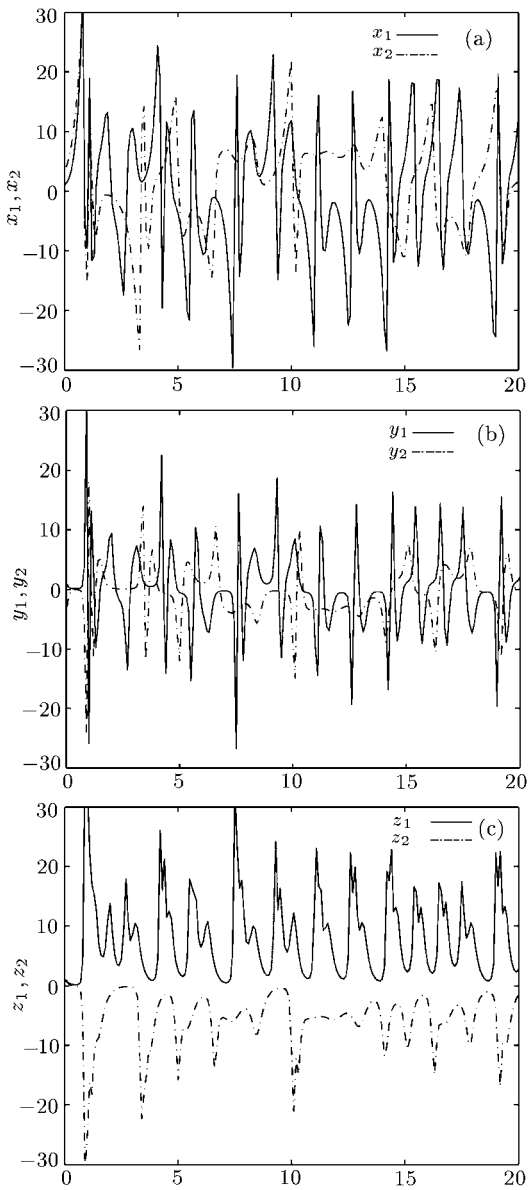

Fig. 3. Chaotic response.

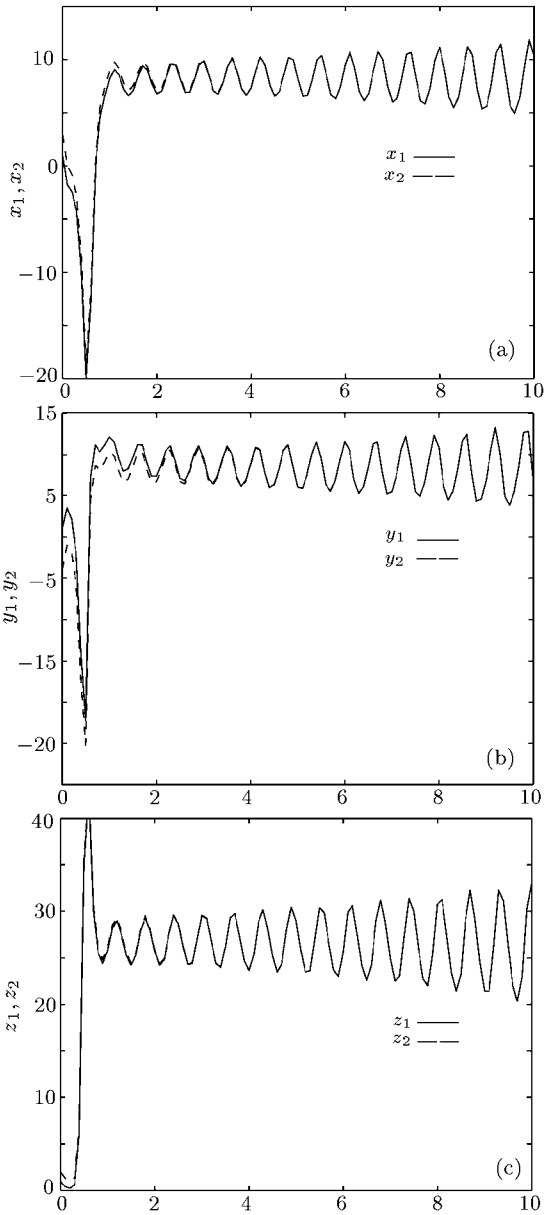

Fig. 4. The orbits of three states of drive and response systems.

We have introduced three control function $u_1(t)$, $u_2(t)$ and $u_3(t)$ in Eq. (4). Our goal is to determine the control function $u_1(t)$, $u_2(t)$, and $u_3(t)$. In order to estimate the control function, we subtract Eq. (3) from Eq. (4). We define the error system as the differences between the system (3) and the controlled system (4). Let us define the state errors between the response system (4) that is to be controlled and the controlling system (3) as

$$\begin{aligned} e_1 &= x_2 - x_1, \\ e_2 &= y_2 - y_1, \\ e_3 &= z_2 - z_1. \end{aligned}$$

Subtracting Eq. (3) from Eq. (4) and using the nota-

tion of Eq. (5) yields

$$\begin{aligned}
\dot{e_1} &= pe_1 + \Big(p + \frac{ab}{a+b}\Big)x_1 - y_2z_2 + y_1z_1 + u_1(t),\\
\dot{e_2} &= -ve_2 - (v+a)y_1 + x_2z_2 - x_1z_1 + u_2(t),\\
\dot{e_3} &= -ue_3 - (u+b)z_1 + x_2y_2 - x_1y_1 + u_3(t).
\end{aligned} \tag{5}$$

Then we define the control function $u_1(t)$, $u_2(t)$, and $u_3(t)$ as follows:

$$\begin{aligned}
u_1(t) &= V_1(t) - \Big(p + \frac{ab}{a+b}\Big)x_1 + y_2z_2 - y_1z_1,\\
u_2(t) &= V_2(t) + (v+a)y_1 - x_2z_2 + x_1z_1,\\
u_3(t) &= V_3(t) + (u+b)z_1 - x_2y_2 + x_1y_1.
\end{aligned} \tag{6}$$

Hence the error system (6) becomes

$$\begin{aligned}
&\dot{e_1} = pe_1 + V_1(t) \quad \dot{e_2} = -ve_2 + V_2(t),\\
&\dot{e_3} = -ue_3 + V_3(t).
\end{aligned} \tag{7}$$

The error system (8) to be controlled is a linear system with a control input $V_1(t), V_2(t)$ and $V_3(t)$ as a function of the error states e_1, e_2 and e_3. The system e_1, e_2 and e_3 converge to zero as time t tends to infinity, this implies that the Lü–Chen–Cheng system and the second system are synchronization with active control. There are many possible choices for the control $V_1(t), V_2(t)$ and $V_3(t)$. We choose

$$\begin{pmatrix} V_1(t) \\ V_2(t) \\ V_3(t) \end{pmatrix} = A \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} \tag{8}$$

where A is a 3×3 constant matrix. In order to make the error system stable, we must choose the proper matrix A whose all eigenvalues have negative real parts. Let the matrix A is chosen in the following form:

$$A = \begin{pmatrix} -1-p & 0 & 0 \\ 0 & v-1 & 0 \\ 0 & 0 & u-1 \end{pmatrix} \tag{9}$$

In this particular choice, the error system (8) has the eigenvalues $-1, -1$ and -1. This choice will lead to the error states e_1, e_2 and e_3 converge to zero as time t tends to infinity and hence the synchronization between the two different chaotic systems is achieved.

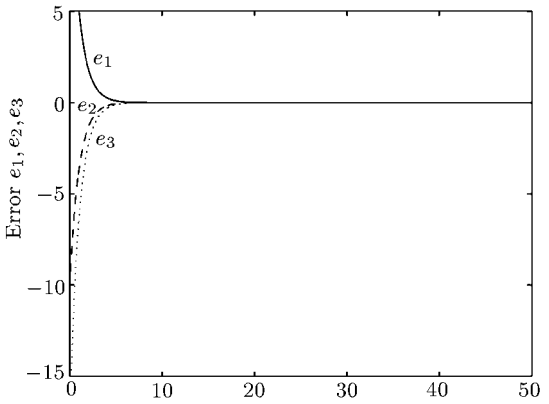

Fig. 5. Synchronization errors.

In the numerical simulations, the fourth-order Runge–Kutta method is used to solve the system with time step 0.1. For this numerical simulation, we assume that the initial condition $(x_1(0), y_1(0), z_1(0)) = (3, -4, 2)$ and $(x_2(0), y_2(0), z_2(0)) = (1, 1, 1)$ is employed. Figures 4 and 5 display the synchronization errors of systems (1) and (2). It can be seen that the synchronization errors converge to zero rapidly.

In summary, we have demonstrated that chaos synchronization between two different chaotic systems using active control is achieved. The second system is controlled to be the Lü–Chen–Cheng system. We can use active control theory to synchronize other two different chaotic systems.

References

[1] Lü J, Cheng G and Chen D 2004 *Int. J. Bifurcat. Chaos* 1507
[2] Yassen M T 2005 *Chaos, Solitons & Fractals* **26** 913
[3] Guan X P and He Y H 2004 *Chin. Phys. Lett* **21** 227
[4] Yassen M T 2005 *Chaos, Solitons & Fractals* **23** 131
[5] Sun H H, Onaral B and Tsao Y 1984 *IEEE Trans. Biomed. Eng.* **31** 664
[6] Lü J, Zhou T S and Zhang S C 2002 *Chaos, Solitons & Fractals* **14** 529
[7] Zhou T S, Lü J, Chen G and Tang Y 2002 *Phys. Lett.* A **301** 231
[8] Park J H 2006 *Chaos, Solitons & Fractals* **27** 549
[9] Pecora L M and Carroll T L 1990 *Phys. Rev. Lett.* **64** 821
[10] Deng W H and Li C P 2005 *Physica* A **353** 61
[11] Shi X and Lu Q S 2004 *Chin. Phys. Lett* **21** 1695