Stopband Phenomena in the Passband of Left-Handed Metamaterials *

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(Received 22 June 2005)

Stopband phenomena are reported in the passband of left-handed metamaterials. The samples with linear defect are designed by removing one layer of split ring resonators (SRRs). It is shown that the left-handed transmission peaks have a distinct transform with the relative deviation of the SRRs centre from the wire centre δ, from a single left-handed peak, double left-handed peaks with different magnitude to no transmission peak, i.e. left-handed properties of metamaterials disappear. Numerical simulation shows that the change of δ makes the effective permeability shift at a frequency range, where stopband occurs. It is thought that the stopband in left-handed passband is due to the symmetry breaking between SRRs and wires in the metamaterials.

PACS: 41. 20. Jb, 73. 20. Mf, 42. 60. Da, 42. 70. Qs

Recently, a new area of research, called left-handed metamaterials (LHMs) has attracted considerable interests.[1−8] LHMs are characterized by a negative effective permittivity and simultaneously a negative effective permeability, and present a variety of unusual electromagnetic and optical properties, such as perfect lens,[1−4] reversed Doppler shift,[5] Cherenkov radiation,[6] and Snell effect.[7,8] The composite mediums made use of an array of wires to create a frequency region with $\varepsilon_{\rm eff} < 0$, interspersed with an array of split ring resonators (SRRs) having a frequency region with $\varepsilon_{\rm eff} > 0$, both introduced by Pendry *et al.*,[9,10] have been extensively studied previously.[11−13] Another metamaterial, photonic crystals are a periodic array of dielectric scatters for which the solution of Maxwell's equations over some frequency range consists of evanescent propagation vectors for all possible direction in space, i.e. a stopband occurs, which may lead to the inhibition of spontaneous emission.[14] John[15] theoretically suggests that photonic superlattices with moderated disorder may possess strong localization of photons. Experiments show that when the periodicity is broken by introducing a defect into a conventional photonic crystal, a localized defect mode, which is a high-Q transmission peak, will appear inside the Bragg gap due to change of the interference behaviour of light.[16,17] Defect modes in photonic crystals open up many potential applications, such as thresholdless lasers, high-Q cavities and optical filters, etc.[18,19] Although physical principles of LHMs are different from that of photonic crystals, defects in LHMs may also be of theoretical and experimental interest. One of the main characteristics of LHMs is the existence of left-handed transmission peak. Point and linear defects in LHMs introduced or replaced by different size SRRs have been experimentally studied and shown that the left-handed transmission peak can be adjusted.[20−22] The measured transmission of the SRRs medium in free space shows several passbands and stopbands.[23] However, in these frequency bands, it does not show left-handed properties. Whether stopband can occur in the left-handed passband has not been theoretically or experimentally reported in literature. In this Letter, the stopband in left-handed passband is firstly studied, which can be analogue to the passband in the stopband of photonic crystals. A method is suggested to create stopband in the passband of LHMs, by changing the symmetry of the LHMs.

Two kinds of samples are fabricated using a shadow mask/etching technique. The copper hexagonal SRRs and wire strips are etched on the opposite sides of a teflon circuit board to obtain sample 1, or etched on the separate fiberglass circuit boards and stacked together to obtain sample 2. The circuit boards are 1.50 mm thick and have the relative permittivities of 2.65 for teflon and 4.60 for fiberglass. The dimensions of an individual SRRs are as follows: $r = 1.00$ mm, $d = 0.20$ mm, $c = 0.40$ mm, and $g = 0.25$ mm (Fig. 1(a)). The strips composing SRRs and wires are 0.02 mm thick, and the dimensions of wires are 0.50 mm wide and 9.90 mm long. The normal LHM samples consist of three layers of SRRs (Fig. 1(b)), and the designed samples with linear defect are obtained by removing one layer of SRRs (Fig. 1(c)). The parameter δ is the relative deviation of the centre of two layers of SRRs from the wire centre, and is defined as positive when the SRRs deviate towards the top of the wires, i.e. the positive direction of the x axis. The distances between

* Supported by the National Natural Science Foundation of China for Distinguished Young Scholar of China under Grant No 50025207, and the National Basic Research Programme of China under Grant No 2004CB719800.

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unit cells are $a = 5.00\,\mathrm{mm}$ and $b = 3.25\,\mathrm{mm}$. The microwave transmission spectra of LHM samples are obtained in a rectangular waveguide with cross section $22.86 \times 10.16\,\mathrm{mm}^2$. The scattering parameters are measured using an automatic network analyser AV3618 made in China. The waveguide is excited with a TE_{10} mode. For all measurements, microwave propagates along the z axis, and the samples are vertically placed in the waveguide along the propagation direction of microwave with the electric field E parallel to the x axis and magnetic field H parallel to the y axis.

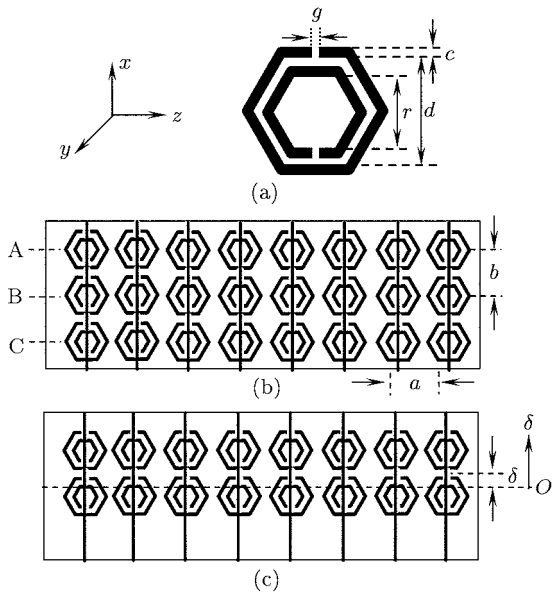


Fig. 1. Schematic of metamaterials used in the experiment. (a) The dimensions of a single copper hexagonal split ring resonators; $r = 1.00\,\mathrm{mm}$, $c = 0.40\,\mathrm{mm}$, $d = 0.20\,\mathrm{mm}$, and $g = 0.25\,\mathrm{mm}$. (b) The normal LHMs consisting of three layers of SRRs on the one side of a board and wires on the other. (c) LHMs with linear defect consisting of two layers of SRRs. The parameter δ is the relative deviation of the centre of two layers of SRRs from the wire centre and is defined as positive when the SRRs deviate towards the top of the wires, i.e., the positive direction of the x axis.

The transmissions versus frequency of normal LHMs with three layers of SRRs and LHMs with linear defect fabricated by removing the bottom layer of SRRs are measured in a rectangular waveguide, and the results are shown in Fig. 2.

It can be seen that the normal LHMs present only one left-handed transmission peak with the magnitude of $-17\,\mathrm{dB}$ between 10.30 GHz and 11.10 GHz, which suggests (but does not prove) left-handed properties. For the LHMs with linear defect ($\delta = +1.4\,\mathrm{mm}$), a stopband with the central frequency of 10.90 GHz occurs in the left-handed passband, from 10.30 GHz to 11.30 GHz. The stopband divides the left-handed transmission peak into two peaks, from 10.30 GHz to 10.90 GHz and from 10.90 GHz to 11.30 GHz, yielding a total bandwidth of the two left-handed passbands of 1.00 GHz. The magnitude of transmission of LHMs with linear defect is enhanced from $-17\,\mathrm{dB}$ to $-12\,\mathrm{dB}$ and transmission passband is widened from 0.80 GHz to 1.00 GHz with respect to that of normal LHMs. The combination of layer-A and layer-B SRRs presents only one resonance peak at 10.84 GHz. Thus the split of left-handed transmission peak is not because of the multi-resonance behaviour of SRRs, but a nonlinear electromagnetic coupling interaction. The corresponding transmission phase versus frequency is shown in Fig. 3. It is seen that the phase monotonously decreases with the frequency at the passband, that is, the curve has a negative slope ($d\Phi/d\omega < 0$) at the passband. It is known that the phase angle Φ is linearly proportional to the wave number, $k = \pi\Phi/180^\circ a$, where a is the lattice constant. Thus the group velocity ($d\omega/dk$) has the same sign as $d\Phi/d\omega$, which is negative at the passband and is one of the characteristic signature of the LHMs. The transmission phase has a jump from -137° to $+28.7^\circ$ at the frequency of 10.86 GHz, where stopband occurs. Phase angles remain unchanged with respect to that of the incident wave (no phase shift occurs) near the maximum of the two peaks.

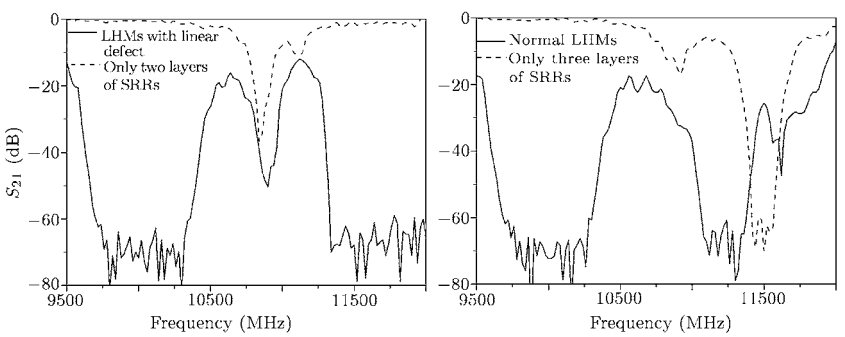


Fig. 2. Transmission versus frequency for LHM samples using teflon circuit board (i.e. sample 1). (a) LHMs samples with linear defect ($\delta = +1.4\,\mathrm{mm}$) (solid line) and the corresponding two layers of SRRs only (dashed line). (b) Normal LHMs samples (solid line), and the corresponding three layers of SRRs only (dashed line). The stopband at 10.90 GHz occurs in the left-handed range from 10.30 to 11.30 GHz for samples with linear defect, but no stopband occurs for normal LHMs.

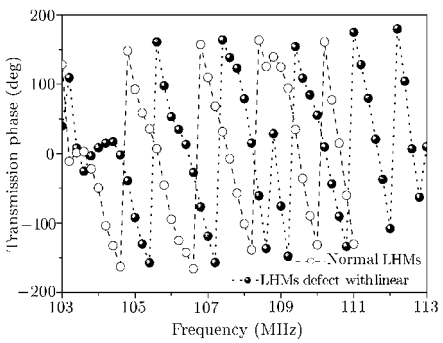


Fig. 3. Transmission phase versus frequency for LHM samples with linear defect (solid line), and normal LHMs (dashed line). The phase has a transition from -137° to $+28.7^\circ$ at the frequency of 10.86 GHz, where stopband occurs for LHMs with linear defect.

Furthermore, it is found that the LHMs consisting of the combination of layer-A and layer-C SRRs, and the combination of layer-B and layer-C SRRs do not present stopband in left-handed passband (data is not shown in the figures), which indicate that the left-handed stopband properties are strongly affected by the inter-layer distance of SRRs and relative deviation of the SRRs centre from the wire centre δ. The transmission spectra of LHMs with linear defect for different δ are shown in Fig. 4.

It can be seen that when two layers of SRRs locate at the top of the wires ($\delta = +1.6$ mm), only one left-handed transmission peak occurs at 9.04 GHz, and its bandwidth is 0.64 GHz, which is similar to the transmission properties of normal LHMs (shown in Fig. 1(b)). For $\delta = +1.0$ mm, the stopband with the frequency of 9.44 GHz occurs in the left-handed passband, from 8.92 GHz to 9.94 GHz. The left-handed transmission peak is divided into two peaks by the stopband, from 8.92 GHz to 9.44 GHz and from 9.44 GHz to 9.94 GHz, yielding a total bandwidth of the two left-handed passbands of 1.02 GHz. The magnitude of the left peak is smaller than that of right peak. When two layers of SRRs locate at the middle of the wires ($\delta = 0$ mm), the left and right peaks have the same magnitude value. When $\delta = -1.0$ mm, the magnitude of the left peak becomes larger than that of the right peak, and the left-handed transmission peak disappears when two layers of SRRs locate at the bottom of the wires ($\delta = -1.6$ mm), where the metamaterial has no left-handed properties.

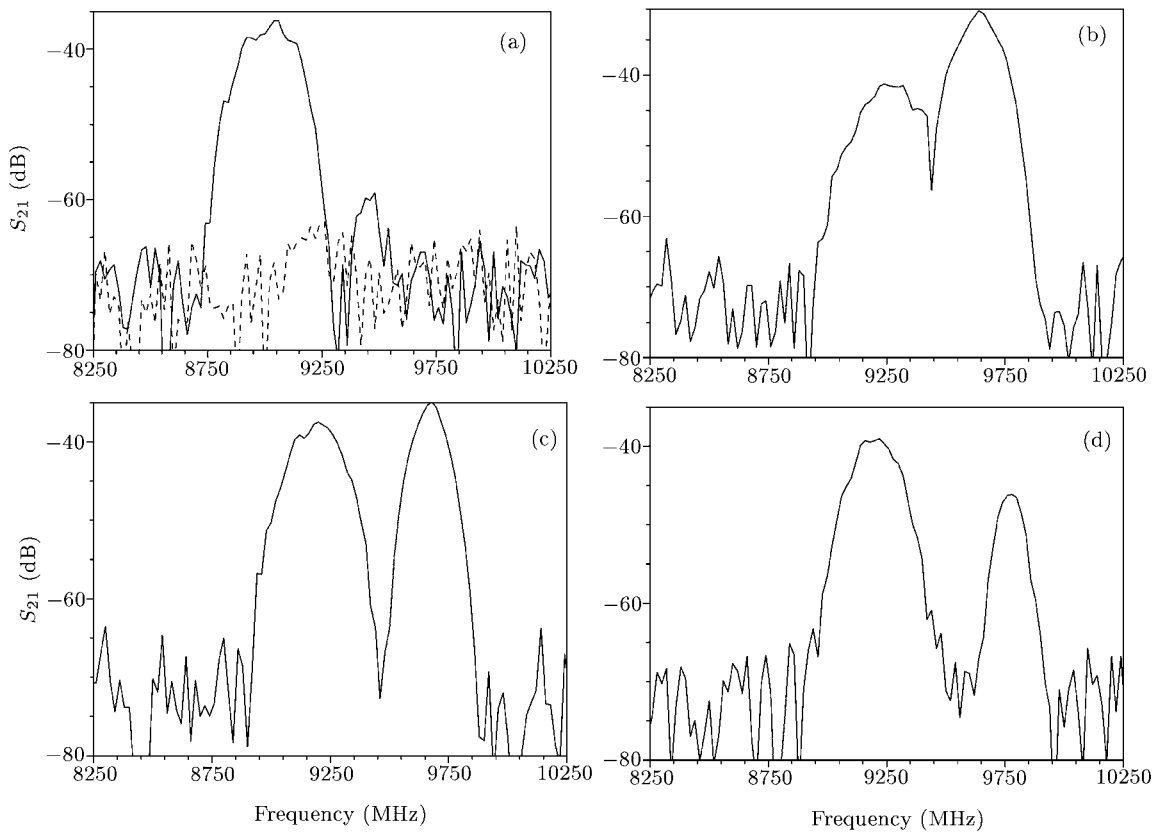


Fig. 4. Transmission versus frequency for LHM samples with linear defect using fiberglass circuit board (i.e. sample 2) for various δ: (a) $\delta = +1.6$ mm (solid curve) and $\delta = 1.6$ mm (dashed curve); (b) $\delta = +1.0$ mm; (c) $\delta = 0$ mm; (d) $\delta = 1.0$ mm.

Therefore, the transmission spectra are distinct with the variation of δ from a single left-handed peak, double left-handed peaks with different magnitudes to no transmission peak, i.e. left-handed properties of metamaterials disappear. The parameter δ determines the appearance of the stopband in the left-handed passband, which results from the change of macroscopic electromagnetic parameters of metamaterials (effective permeability and permittivity). It is known that the relative permittivity of the metamaterials is in the form[11,12]

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\gamma)}, \tag{1}$$

and the relative permeability is

$$\mu(\omega) = 1 - \frac{F\omega_0^2}{\omega^2 - \omega_0^2 + i\Gamma\omega}. \tag{2}$$

By choosing appropriate parameters in Eqs. (1)

and (2), the numerical simulated transmission can be well fit to the experimental data, and the corresponding real part of relative permeability $\mu_{\rm real}$ versus the frequency for different δ can be obtained and shown in Fig. 5 (the relative permittivity remains nearly unaffected with δ in the metamaterials).

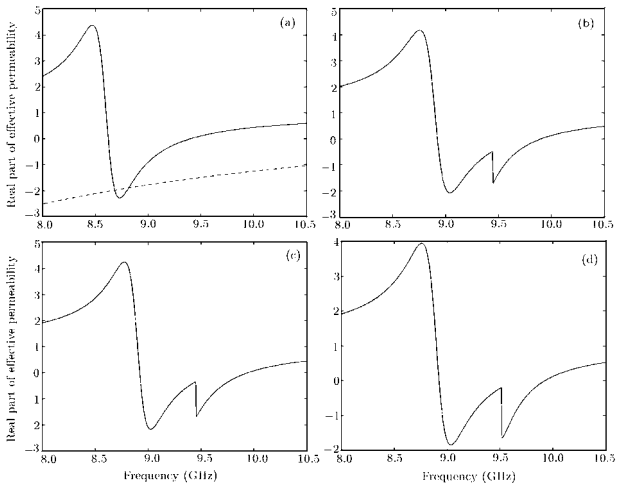


Fig. 5. Simulated effective permeability as a function of frequency for various δ: (a) $\delta = +1.6$ mm; (b) $\delta = +1.0$ mm; (c) $\delta = 0$ mm; (d) $\delta = 1.0$ mm. The dashed curve shows the real part of effective permittivity versus frequency. The real part of effective permeability has a transition when the stopband occurs.

It can be seen that when SRRs locate at the top of the wires ($\delta = +1.6$ mm), the curve of $\mu_{\rm real}$ versus frequency is smooth and there is a frequency range where $\mu_{\rm real}$ and $\varepsilon_{\rm real}$ are simultaneously less than zero. For $\delta = +1.0$, 0, and -1.0 mm, $\mu_{\rm real}$ has a jump at the stopband frequency from -0.478 to -1.755, -0.340 to -1.729, and -0.195 to -1.681, respectively. At the same time, $\mu_{\rm real}$ is less than zero at the stopband, i.e. metamaterials also possess left-handed properties in the stopband. The frequency range with negative $\mu_{\rm real}$ is widen with respect to that of $\delta = +1.6$ mm. For $\delta = -1.6$ mm, $\mu_{\rm real}$ is larger than zero, so there is no left-handed properties and it presents plasmon-like transmission properties (not shown in the figures). Therefore, the stopband in the left-handed passband can appear by adjusting the symmetry of LHMs, and correspondingly the permeability is adjusted. The stopband in the left-handed passband can be analogue to defect mode in stopband of photonic crystals. The electromagnetic wave in the stopband of LHMs can not travel through it and may be considered as a localized photonic well. These two kinds of metamaterials have reversed transmission properties, transmitted mode in stopband for photonic crystals, but inhibited mode in passband for LHMs. It is thought that the symmetry breaking of electromagnetic interaction between SRRs and wires changes the effective permeability of metamaterials, which is the origin of the stopband in the left-handed passband.

In summary, it has been shown that the stopband in the left-handed passband may occur in the LHMs with linear defects, which consists of two layers of split ring resonators and one layer of vacancy on the one side of the substrate, and wires on the other. The transmission spectra of the corresponding SRRs present only one resonance peak, which indicates that stopband phenomena are a nonlinear electromagnetic coupling interaction between SRRs and wires. The properties of the stopband are affected by relative deviation of the SRRs centre from the wire centre δ, which can be used to adjust the effective permeability of the metamaterials. It is thought that the stopband in the left-handed passband is due to the symmetry breaking between split-ring resonators and wires in the metamaterials. Such stopband in LHMs may be expected to lead to interesting devices for telecommunication filters as well as optical negative refractive index sensing.

References

[1] Pendry J B 2000 *Phys. Rev. Lett.* **85** 3966
[2] Patanjali V P, Wentao T L, Plarenta V and Srinivas S 2003 *Nature* **426** 404
[3] Wiltshire M C K, Pendry J B, Young I R, Larkman D J, Gilderdale D J and Hajnal J V 2001 *Science* **291** 849
[4] Shen J T and Platzman P M 2002 *Appl. Phys. Lett.* **80** 3286
[5] Seddon N and Bearpark T 2003 *Science* **302** 1537
[6] Luo C Y, Ibanescu M, Johnson S G et al 2003 *Science* **299** 368
[7] Houck A A, Brock J B and Chuang I L 2003 *Phys. Rev. Lett.* **90** 137401
[8] Parazzoli C G, Greegor R B, Li K, Koltenbah B E C and Tanielian M 2003 *Phys. Rev. Lett.* **90** 107401
[9] Pendry J B, Holden A J, Stewart W J and Youngs I 1996 *Phys. Rev. Lett.* **76** 4773
[10] Pendry J B, Holden A J, Robbins D J and Stewart W J 1999 *IEEE Trans. Microwave Theory Tech.* **47** 2075
[11] Shelby R, Smith D R and Schultz S 2001 *Science* **292** 77
[12] Smith D R, Padilla W J, Vier D C, Nemat-Nasser S C and Schultz S 2000 *Phys. Rev. Lett.* **84** 4184
[13] Smith D R and Kroll N 2000 *Phys. Rev. Lett.* **85** 2933
[14] Yablonovitch E 1987 *Phys. Rev. Lett.* **58** 2059
[15] John S 1987 *Phys. Rev. Lett.* **58** 2486
[16] Smith D R, Dalichaouch R, Kroll N, Schultz S, McCall S L and Platzman P M 1993 *J. Opt. Soc. Am.* B **10** 314
[17] Povinelli M L, Johnson S G, Joannopoulos J D and Pendry J B 2003 *Appl. Phys. Lett.* **82** 1069
[18] Painter O, Lee R K, Scherer A, Yariv A, O 'Brien J D, Dapkus P D and Kim I 1999 *Science* **284** 1819
[19] Yokoyama H 1992 *Science* **256** 66
[20] Zhao Q, Zhao X P, Kang L, Zhang F L, Liu Y H and Luo C R 2004 *Acta Phys. Sin.* **53** 2206
[21] Kang L, Zhao Q and Zhao X P 2004 *Acta Phys. Sin.* **53** 3379
[22] Zhang F L, Zhao Q, Liu Y H, Luo C R and Zhao X P 2004 *Chin. Phys. Lett.* **21** 1330
[23] Ozbay E, Aydin K, Cubukcu E and Bayindir M 2003 *IEEE Trans. Antennas Properg.* **51** 2592