Influence of Hyperon on $^1S_0$ Superfluidity of Nucleons in Neutron Star Matter *

CHEN Wei(陈伟)[1**], LAM Yu-Yiu(林汝耀)[1], WEN De-Hua(文德华)[3], LIU Liang-Gang(刘良钢)[2]

[1]*Department of Physics, Jinan University, Guangzhou 510632*
[2]*Department of Physics, Zhongshan University, Guangzhou 510275*
[3]*Department of Physics, South China University of Technology, Guangzhou 510641*

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We investigate the property of $^1S_0$ superfluidity of neutrons and protons in the neutron star matter. On the basis of the result, we study the effects of hyperons on the $^1S_0$ pairing gaps of the two species of the particles. The parameter sets we use are for the Hartree approximation of the relativistic $\sigma-\omega$ model and the mean field approximation of the Walecka model, respectively. The predicted domain of superfluidity is very close to other works, whereas differences appear in the predicted value of the maximum gap. It is found that Σ^-, Λ and Ξ^- have some influences, the scales of which depend on hyperon-meson coupling constants we use besides other factors, on the $^1S_0$ superfluidity of protons in some density range, and do not have influence on superfluidity of neutrons. Other hyperons have no influence on the $^1S_0$ superfluidity of neutron and proton due to these hyperons appearing after $^1S_0$ neutron and proton pairs disappear.

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The interaction of neutrons in the highly compressed matter inside neutron stars is possible to lead to superfluidity. Experimental studies of atomic nuclei reveal that nuclear forces between like-particles are attractive. This attraction helps neutron pair up through the well-known Bardeen–Cooper–Schrieffer (BCS) theory. The same effect probably occurs in a homogeneous degenerate nuclear fluid. In neutron matter, superfluidity may exist at temperatures below $\sim 1\,\mathrm{MeV} = 10^{10}\,\mathrm{K}$. The same reasoning leads to the prediction that the protons in nuclear matter (whose number is of the order of a few percent of all baryons) will be paired, and will give rise to the phenomenon of superconductivity.

The region of superfluidity or superconductivity in neutron star is that in the inner crust region, neutrons will form the $^1S_0$ pairs[1−3] at the corresponding baryon density $10^{-3}\rho_0 \leq \rho_B \leq 0.7\rho_0$, where $\rho_0 \simeq 0.16\,\mathrm{fm}^{-3}$ is the saturation density of symmetric nuclear matter, the $^1S_0$ partial wave of the nucleon–nucleon (NN) interaction is attractive. In the core region $\rho_B \geq 0.7\rho_0$, the $^3P_2$ neutron pair may also appear since the $^3P_2$ partial wave of the NN interaction becomes attractive enough.[1,4] In contrast, the $^1S_0$ partial wave would become repulsive there so that the neutrons would cease to pair in the $^1S_0$ state. Instead, the $^1S_0$ proton pairs are predicted to appear owing to its fraction smaller than neutrons.[1,3] At much higher baryon density $\rho_B \geq 2\rho_0$, various hyperons may emerge; some kinds of them possibly form pairs in the same way as nucleons.[5−7]

The anticipated influences of superfluidity of neutron star matter on the astrophysical behaviour of neutron stars can be classified to be thermal, magnetic, and hydrodynamical.

To begin with, the heat capacity of a superfluid or superconductor is smaller than the heat capacity of matter in a normal Fermi-gas or Fermi-fluid state.[8,9] The reason is obvious, i.e., one must first break the pair to excite baryons; there is an energy gap Δ, and the heat content at low temperatures will be proportional to $e^{-\Delta k/T}$.[10] On the other hand, the rate of heat losses due to neutrino emission by the URCA process (i.e., by the interplay between $p+e^- \to n+\nu$ and $n \to p+e^-+\bar{\nu}$) is decelerated by the pairing of neutrons and protons.[11−14]

In a strong magnetic field the transition to superfluidity can be delayed by magnetic field decay,[15,16] and the anisotropic ($^3P_2$) neutron pairing state has a angle-dependent energy gap with a node along the field direction.[17,18] The proton superconductor is generally assumed to be type-Ⅱ, which means that the magnetic field is confined to flux tubes,[19,20] whose radial dimension is given by the London penetration depth, $\lambda \sim 40-100\,\mathrm{fm}$. Thus it supports a stable lattice of magnetic flux tubes in the presence of magnetic field. Recently, this assumption is however found to be inconsistent with the phenomenon of long period precession in isolated pulsars.[20,21]

The superfluid is like an ideal liquid only on the microscopic scale. One essential property of a superfluid is that its flow is irrotational. The way the superfluid solves the contradiction between irrotationality and rotation of the star is, like superfluid Helium in laboratories, via the creation of quantized vortices that carry all the angular momentum of the superfluid. These discrete, quantized vortex lines distributed in the liquid make the macroscopic behaviour similar

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** Email: tchenw@jnu.edu.cn

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to that of a normal, viscous liquid.[22] They together with flux tubes may be related to the phenomenon of glitch.[23−25]

Superfluidity properties of neutron star matter have been studied for a long time, with various realistic interactions[11,26,27] and using simplifying approximations.[28,29] It is remarkable that all calculations yield qualitatively similar results for the region of existence of $^1S_0$ pairing. However, the size of maximum gap at the Fermi surface as well as the condensation energy is controversial due to the uncertainty of many complex factors. In fact, besides the interaction between nucleons, the parameter sets such as the effective mass, density of nucleons and the composition of neutron star etc. can also influence the superfluidity of nucleons in dense neutron star matter. In the past, the superfluidity of nucleons was usually studied in the neutron star matter composed of only nucleons, electrons and muons. As pointed out in the second paragraph, however, hyperons may appear in neutron star. These hyperons influence the superfluidity of nucleons by changing the effective mass and density of nucleons. The primary aim of this study is to compare the $^1S_0$ pairing gap of nucleons in neutron star matter with and without hyperons, and to investigate the influences of hyperons on the superfluidity of nucleons. The model Lagrangian and the gap equation for the $^1S_0$ nucleon superfluidity are illustrated. The results of the pairing properties in neutron star matter are presented.

The particles we considered here are neutrons, protons, electrons, muons, Λ, Σ, Ξ and Δ, and the Lagrangian density in the relativistic $\sigma-\omega$ model is given by[30]

$$\begin{aligned} L = & \sum_B \bar{\psi}_B \Big(i\gamma_\mu \partial^\mu + m_B - g_{\sigma B}\sigma \\ & - g_{\omega B}\gamma_\mu \omega^\mu - \frac{1}{2} g_{\rho B}\gamma_\mu \boldsymbol{\tau}\cdot\boldsymbol{\rho}^\mu \Big)\psi_B + \frac{1}{2}(\partial\sigma)^2 \\ & - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu \\ & - U(\sigma) - \frac{1}{4}\rho_{\mu\nu}\rho^{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\boldsymbol{\rho}^\mu \\ & + \sum_l \bar{\psi}_l(i\gamma_\mu\partial^\mu - m_l)\psi_l, \end{aligned} \tag{1}$$

where

$$U(\sigma) = a\sigma + \frac{1}{3!}c\sigma^3 + \frac{1}{4!}d\sigma^4, \tag{2}$$

$$F_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu, \tag{3}$$

$$\rho_{\mu\nu} = \partial_\mu\boldsymbol{\rho}_\nu - \partial_\nu\boldsymbol{\rho}_\mu, \tag{4}$$

ψ_B, ψ_l, σ, ω^μ and $\boldsymbol{\rho}^\mu$ are the field operators of baryon B (B =n, p, Λ, Σ, Ξ, Δ), lepton l ($l = e, \mu$), σ, ω and ρ meson, respectively; g_{iB} ($i = \sigma$, ω, ρ) the coupling constants between meson and baryon B, respectively. In general, $g_{in} = g_{ip}$ ($i = \sigma$, ω, ρ) are assumed and decided by saturated property of nuclear matter or the symmetry energy of nuclear matter. Due to the poor knowledge of coupling between mesons and hyperons, their coupling constants are assumed in accordance with a little experimental data and nucleon coupling constants.[31,32] As for leptons, we assume they are free Fermi gas. Here m_B, m_l and m_i ($i = \sigma, \omega, \rho$) are the mass of baryon, lepton and three species of meson, respectively; $\boldsymbol{\tau}$ is the isospin operator. Besides the relativistic $\sigma-\omega$ model, we also employ the Walecka model for comparison.

When the attraction in the $^1S_0$ partial wave is dominant, the BCS theory predicts to a transition to a superfluid (neutron)/superconducting (protons) state if we can obtain a non-zero solution $\Delta_{n(p)}(k)$ of the gap equation

$$\Delta_{n(p)}(k) = -\frac{1}{8\pi^2}\int_0^\infty \frac{\Delta_{n(p)}(k')}{\sqrt{(E_{k'} - E_{k'_F})^2 + \Delta^2_{n(p)}(k')}} \cdot \tilde{v}(k,k')k'^2 dk', \tag{5}$$

where $\Delta_{n(p)}(k)$ is the so-called gap function of neutrons (protons). Here k, k' are single-particle momenta. $E_{k'} = \sqrt{k'^2 + m_B^{*2}} + g_{\omega B}\langle\omega_0\rangle + g_{\rho B}\langle\rho_0^{(3)}\rangle$, $B = n, p$. $\tilde{v}(k,k')$ is the S-wave component of the particle-particle ($p-p$) channel interaction between protons or neutrons.

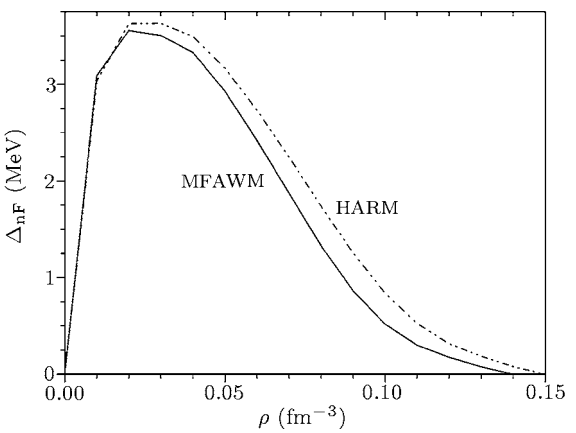

Fig. 1. The $^1S_0$ pairing gaps of neutrons at Fermi surface as a function of baryon density.

We pick out the parameter sets of cases I and Ⅲ from the table in Ref. [30] for the mean field approximation of the Walecka model (MFAWM) and the Hartree approximation of the relativistic $\sigma-\omega$ model (HARM), respectively. The effective nucleon mass in symmetric nuclear matter at saturation density is $\sim 0.5m_n$ in the former and $\sim 0.8m_n$ in the latter. The hyperon–meson coupling constants are from Ref. [31] as listed in Table 1. We use the parameter sets 1 in Table 1 when we include hyperons in the first stage of our numerical simulation.

In neutron star matter composed by the models, we solve the gap equation for the $^1S_0$ proton and neutron pairing correlation at zero temperature using Bonn-B potential[33] as $p-p$ channel interaction. To

Table 1. Ratios of hyperon–meson to nucleon–meson coupling constants, $x_{iH} = g_{iH}/g_{iN}$, where $i = \sigma, \omega, \rho$ and H represent hyperon species. The cases 1, 2 and 3 are GM2,3, B91 in Table II and the second case in Table LX in Ref. [31]

Case	$x_{\sigma\Lambda}$	$x_{\omega\Lambda}$	$x_{\rho\Lambda}$	$x_{\sigma\Sigma}$	$x_{\omega\Sigma}$	$x_{\rho\Sigma}$	$x_{\sigma\Xi(\Delta)}$	$x_{\omega\Xi(\Delta)}$	$x_{\rho\Xi(\Delta)}$
1	0.6	0.659	0.6	0.6	0.659	0.6	0.6	0.659	0.6
2	0.6	0.649	0.6	0.6	0.649	0.6	0.6	0.649	0.6
3	0.6	0.65	0.77	1.00	0.67	0.6	0.65	0.65	0.6

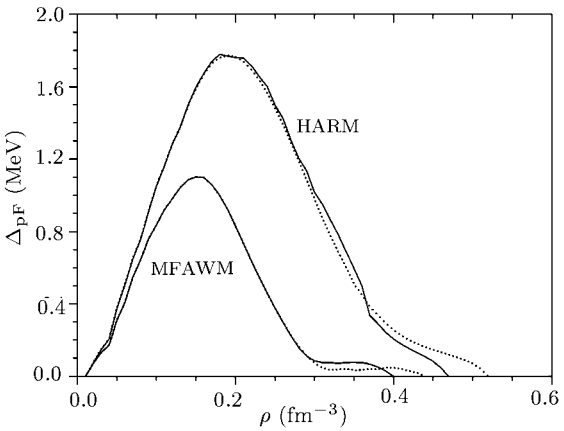

Fig. 2. The $^1S_0$ pairing gaps of protons at Fermi surface as a function of baryon density. The pairing gaps with hyperons corresponding to case 1 in Table 1.

begin with, we calculate the $^1S_0$ pairing gaps of neutrons in neutron star matter, Δ_{nF}, by the parameter sets for the HARM. The result is shown by the dashed curve in Fig. 1. Δ_{nF} is non-zero in the extent of baryon density $\rho = 0.0 \sim 0.15\,\mathrm{fm}^{-3}$ corresponding to the Fermi momentum of neutron $k_{nF} = 0 \sim 1.6\,\mathrm{fm}^{-1}$ which agrees with other works,[4,28] and its maximum value is 3.7 MeV which is larger than the results in these works. Δ_{nF} in neutron star matter with hyperons are equal to that in the neutron star matter without hyperons, so they are both depicted by the dashed curve in Fig. 1. Because hyperons appear after Δ_{nF} vanishes as in Fig. 3, hyperons have no influence on the $^1S_0$ superfluidity of neutron. It is needed to elucidate that the fractions of particles in case 1 are almost the same as that in case 2 in Fig. 3. Similar results can be obtained by using the parameter sets for the MFAWM as the solid curve in Fig. 1 except that Δ_{nF} become smaller in this sets in the region of $\rho = 0.05 \sim 0.15\,\mathrm{fm}^{-3}$. Figure 2 shows the $^1S_0$ pairing gaps of protons at Fermi surface in neutron star matter, Δ_{pF}. Δ_{pF} in neutron star matter with hyperons are larger than that in neutron star matter without hyperons in the density range of $0.30\,\mathrm{fm}^{-3} \sim 0.35\,\mathrm{fm}^{-3}$ when the parameter sets for HARM are used, but this relation is reversed in the range of $0.35\,\mathrm{fm}^{-3} \sim 0.52\,\mathrm{fm}^{-3}$, and they are equal in other range of baryon density.

The first hyperon Σ^- exists as shown in Fig. 3 in the range of $\rho \geq 0.3\,\mathrm{fm}^{-3}$, and until $\rho \sim 0.5\,\mathrm{fm}^{-3}$ the second hyperon Λ appear, so that the influence it imposes on $^1S_0$ pairing gaps of protons can be neglected. Thus we can see that only the hyperon Σ^- imposes influence on Δ_{pF}. Similar relations of Δ_{pF} with baryon density can be found when the parameter sets for the MFAWM are used. The extent of existence and the maximum value of Δ_{pF} agree with Ref. [11].

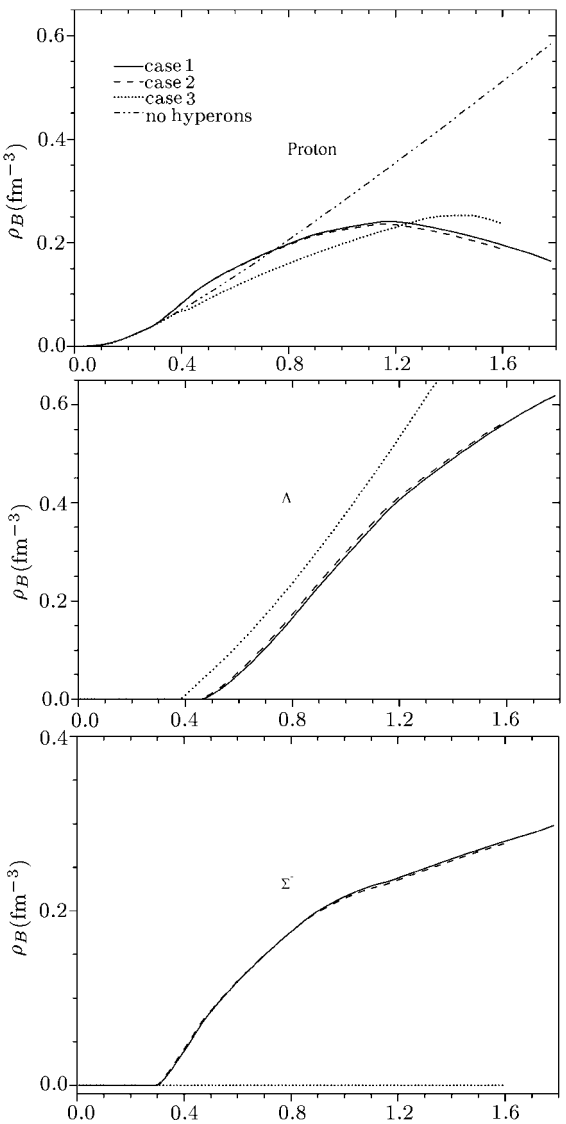

Fig. 3. The density of proton, Λ and Σ^- as a function of total baryon density in the relativistic $\sigma - \omega$ model.

We also study the $^1S_0$ superfluidity of neutrons and protons in the HARM as in Fig. 4 by using the second and third parameter sets in Table 1, respectively. The $^1S_0$ pairing gaps of neutrons show the same behaviour with baryon density increasing in the three cases listed in Table 1. This is due to the $^1S_0$ pairing gaps of neutrons having vanished when hyper-

ons appear as we have pointed out. The $^1S_0$ pairing gaps of protons in case 1 are almost the same as that in case 2, however, the Δ_{pF} in the third case appears larger than that in the former two cases in the range of baryon density $\rho_B > 0.3\,\mathrm{fm}^{-3}$.

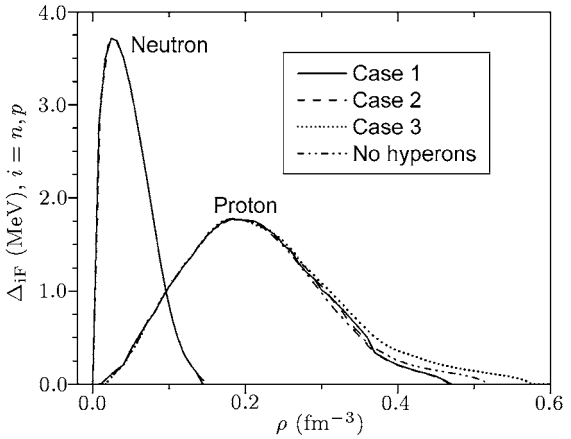

Fig. 4. The $^1S_0$ pairing gaps of neutrons and protons at Fermi surface as a function of baryon density in relativistic $\sigma-\omega$ model by using the three parameter sets in Table 1.

This phenomenon results from the different order of appearing of hyperons in different parameter sets. Σ^- do not appear in the third case as shown in Fig. 3, and Λ possesses larger density in this case than that in cases 1 and 2 in which Σ^- appears earlier than Λ. Σ^- can enhances the fraction of proton due to the neutrality of neutron star matter, while Λ reduces the fraction of proton due to its neutrality and the conservation of baryon number, so that the densities of protons in case 3 are smaller than that in cases 1 and 2 when the baryon density is in the range of $\sim 0.3 - 1.0\,\mathrm{fm}^{-3}$ as shown in Fig. 3. In this range of baryon density, the moderate density of protons in case 3 keeps the $^1S_0$ pairing gaps of protons larger than that in cases 1 and 2. Ξ^- have very small influence on the $^1S_0$ pairing gaps of protons, and other hyperons such as Δ, etc., which appear at very high density of baryon impose no influence on the superfluidity of neutron and proton, so we do not present them in Fig. 3.

In summary, we have investigated the property of $^1S_0$ superfluidity of neutrons and protons in the neutron star matter. On the basis of the result, we study the effects of hyperons on the $^1S_0$ pairing gaps of the two species of particles. The predicted domain of superfluidity is very close to other works, whereas differences appear in the predicted value of the maximum gap. A point of interest here is that hyperons have no influence on Δ_{nF}, but they may change Δ_{pF} when the density of baryon $\rho_B > 0.30\,\mathrm{fm}^{-3}$. The main cause of this phenomenon is that hyperons have changed the fraction and effective mass of proton before Δ_{pF} vanishing. Another impressive point is that the influences of hyperons on Δ_{pF} depend on hyperon-meson coupling constants which decide the critical density where the hyperons appear. Therefore, we expect more exact data about hyperon–meson coupling constants. These works are helpful for us to study the property of superfluidity of neutron star matter in which hyperons is usually anticipated to exist.

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