Resolution Ability and Surface Bright Spots of a Lossy Left-handed Material Slab*

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The imaging quality of a lossy left-handed material (LHM) slab is studied. The exact solution is obtained for the interaction of a monochromatic source with a lossy LHM slab by Fourier integrals in wave number domain. It is found that a small amount of loss may have a strong suppression of the super-resolution ability. However, the resolution of a lossy slab can still be better than a conventional lens because the evanescent wave can still be partly restored if the loss is small enough. The localized field with large amplitude (surface bright spots) near the interfaces of the slab is related to the resonance of the coupling between surface waves at the interfaces. The evanescent wave restoration and the surface bright spots are two relevant phenomena. The distance of the bright spots is close to the half of the resolution length.

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Left-handed materials with negative permittivity and permeability gained significant interest since their practical realizations in microwave frequency band were demonstrated.[1] An intriguing property of a left-handed material (LHM) slab was predicted by Pendry.[2] Unlike conventional optical components, an LHM slab can provide a phase correction to restore both the propagating and evanescent components, thus achieving the super-resolution. This opened an exciting gateway to create an unprecedented optical and imaging system. However, Pendry's theory is restricted to the rigorous conditions that the relative permittivity ε and permeability μ are simultaneously -1. This aroused special interest in the theoretical study of the possibility of developing a "superlens", such as the sensitivity of the image quality to the small deviations from the perfect lens conditions,[3] the finite-difference time-domain (FDTD) study on the sub-wavelength focusing by a left-handed material slab,[4] the study on the finite-size effects on the image quality,[5] and the localization of the electromagnetic waves using an LHM slab,[6] which is related to the slab focusing properties. In addition, some experiments[7,8] have also been performed to verify the focusing and resolution ability of LHMs made of different geometries.

In this Letter, the Fourier integrals in wave-number domain were developed to obtain the exact solution of the interaction of a monochromatic source with a lossy LHM slab. Based on the solutions of the Fourier integrals, the influence of the loss on the image quality and the surface bright spots (localized field with large amplitude near the interfaces) was clearly observed. A criterion function was introduced to evaluate the evanescent wave restoration ability of a lossy slab and the results show that the restoration of the evanescent waves is partly achievable if the loss is small enough. Therefore the resolution of a lossy LHM slab can still be better than the conventional lens to some extent. The spatial spectrum of the field along the interfaces was analysed and the results show that the surface bright spots are related to the resonance of the coupling of surface waves between the interfaces of the slab. Our results also show that the restoration of the evanescent waves and the resonance of the surface waves are two relevant phenomena. The distance of the surface bright spots is close to the half of the resolution length for an LHM slab with little loss.

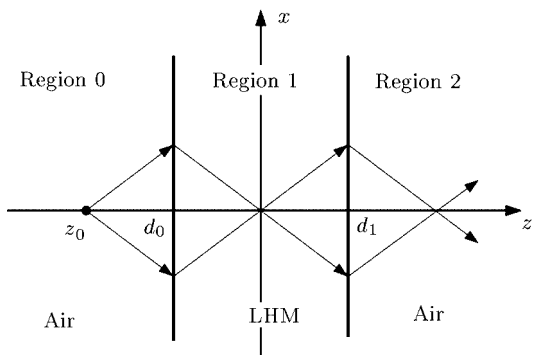


Fig. 1. A line source in front of an LHM slab.

Now let us consider an imaging system formed by an LHM slab with permittivity ε and permeability μ as shown in Fig. 1. The slab is positioned between $z = d_0$ and $z = d_1$. The thickness of the slab is $d = d_1 - d_0$. A monochromatic line source parallel to

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to the y axis is positioned at $x = x_0$, $z = z_0$ in front of the slab. The field excited by a source can be represented by a series of plane wave components with different wave vectors. The fundamental function of the slab is to apply a phase and amplitude correction to each of these components. With a direct solution of Maxwell equations under the corresponding boundary conditions, the transmission of individual wave components by the slab can be readily obtained. Then, the total electric fields upon the interaction of a line source with the slab can be expressed in the form of Fourier integrals in wave number domain as in Eq. (1):

$$E(x,z) = \int_{-\infty}^{\infty} \tilde{E}(k_x,z) e^{ik_x(x-x_0)} dk_x. \tag{1}$$

The detailed expressions of $\tilde{E}(k_x,z)$ are derived to be

$$\tilde{E}(k_x,z) = \begin{cases} \Big[\dfrac{1}{k_z^2 \tilde{E}_{fs}(k_x,d_0)} + R(k_x)\tilde{E}_{fs}(k_x,d_0)\Big] \\ \quad \cdot e^{-ik_z(z-d_0)}, & z < z_0, \\ \tilde{E}_{fs}(k_x,d_0)[e^{ik_z(z-d_0)} \\ \quad + R(k_x)e^{-ik_z(z-d_0)}], & z_0 \le z < d_0, \\ \tilde{E}_{fs}(k_x,d_0)[A(k_x)e^{ik_z(z-d_1)} \\ \quad + B(k_x)e^{-ik_z(z-d_1)}], & d_0 \le z < d_1, \\ \tilde{E}_{fs}(k_x,d_0)T(k_x)e^{ik_z(z-d_1)}, & z \ge d_1, \end{cases} \tag{2}$$

where the corresponding solution of the field excited by the same source in free space reads

$$\tilde{E}_{fs}(k_x,z) = \frac{e^{ik_z|z-z_0|}}{k_z}. \tag{3}$$

The coefficients $R(k_x)$, $A(k_x)$, $B(k_x)$, and $T(k_x)$ can be obtained by matching the boundary conditions at $z = d_0$ and $z = d_1$. The detailed expressions read

$$R(k_x) = \frac{(\mu k_z - k_z')(\mu k_z + k_z')[1 - e^{2ik_z'(d_1-d_0)}]}{(\mu k_z + k_z')^2 - (\mu k_z - k_z')^2 e^{2ik_z'(d_1-d_0)}}, \tag{4}$$

$$T(k_x) = \frac{4\mu k_z k_z' e^{ik_z'(d_1-d_0)}}{(\mu k_z + k_z')^2 - (\mu k_z - k_z')^2 e^{2ik_z'(d_1-d_0)}}, \tag{5}$$

$$A^{(1)} = \frac{\mu k_z + k_z'}{2k_z'} T(k_x), \tag{6}$$

and

$$B^{(1)} = \frac{k_z' - \mu k_z}{2k_z'} T(k_x). \tag{7}$$

The transmission of an individual component with wave numbers k_x from source plane $z = z_0$ to image plane $z = z_{im}$ can be expressed as

$$\begin{aligned} t(k_x) &= \frac{\tilde{E}(k_x,z_{im})}{\tilde{E}_{fs}(k_x,z_0)} \\ &= \frac{4\mu k_z k_z' e^{ik_z'd} e^{ik_z(z_{im}-z_0-d)}}{(\mu k_z + k_z')^2 - (\mu k_z - k_z')^2 e^{2ik_z'd}}. \end{aligned} \tag{8}$$

Under the special case $\varepsilon = \mu = -1$, it is explicitly verified that the transmission coefficient $t(k_x)$ of each components equals to 1 at the image plane $z_{im} = z_0 + 2d$. Hence all the propagating wave components (with $k_x < k_0$) and evanescent wave components (with $k_x > k_0$) are perfectly restored and contribute to the resolution of the image, which results in Pendry's well-known perfect lens.[2] The calculated contour plot of the magnitude of electric field (in logarithmic scale) based on Eq. (1) is illustrated in Fig. 2. The line source is at $x_0 = 2\lambda$, $z_0 = 0.5\lambda$, where λ is the wavelength in free space. The LHM slab with $\varepsilon = \mu = -1$ and $d = \lambda$ is located at $\lambda < z < 2\lambda$. Clearly, two super-resolution focuses are observed at $x = 2\lambda$ and $z = 2.5\lambda$ behind the slab and at $x = 2\lambda$ and $z = 1.5\lambda$ in the slab. Near the back interfaces, the field intensity diverges due to the exponential increase of the evanescent waves in the LHM slab. Note that the unbounded field intensity cannot be represented accurately in the source and image point. If loss exists, an imaginary part should be added to the permittivity and permeability as $\varepsilon = -1 + \varepsilon''$, $\mu = -1 + \mu''$. Here, $\varepsilon'' = \mu'' = \sigma$ is assumed for simplicity and this does not lose the generality. The calculated contour plot of the electric field intensity (in linear scale) for $\varepsilon = \mu = -1 + 10^{-5}i$ is illustrated in Fig. 3(a), and the field intensity near the source and the image was enlarged and plotted in Figs. 3(b) and 3(c).

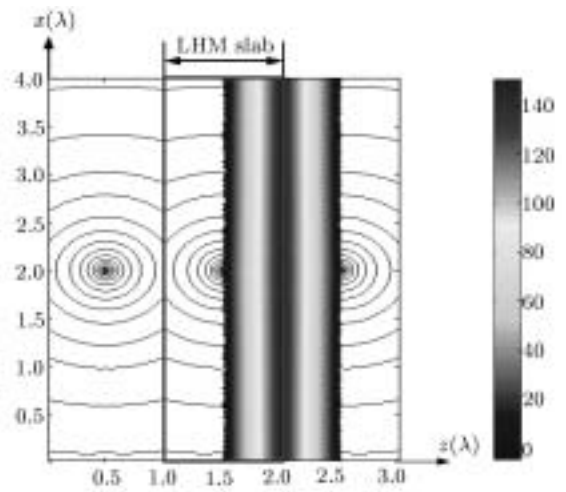


Fig. 2. Contour plot of the magnitude of field (logarithmic scale). The line source is at $x = 2\lambda$ and $z = 0.5\lambda$. The slab with $\varepsilon = \mu = -1$ is located at $\lambda < z < 2\lambda$ and indicated by the red rectangle.

Unlike in the lossless case, it is clearly observed that the centre of the focusing behind the LHM slab is blurred. This means that the slab super-resolution ability is strongly suppressed by the small loss and the quality of the image is heavily degraded compared to the lossless case. This may result from the

suppression of the evanescent wave restoration ability by the small loss. To analyse the evanescent wave restoration ability, we can study the deviation of the transmission coefficient $t(k_x)$ with and without losses. Since $t(k_x)$ is a complex value in general, both the magnitude and phase deviation should be considered. These can be successfully accomplished by introducing a criterion function $C(k_x,\sigma) = \left|\dfrac{t(k_x,\sigma)-t(k_x,0)}{t(k_x,0)}\right|$, where $t(k_x,\sigma)$ is the transmission of individual component with wave number k_x when $\varepsilon'' = \mu'' = \sigma$. If $C(k_x,\sigma) \ll 1$, the component with wave number k_x has been effectively restored and contributed to the resolution of the lossy slab. For $\sigma = 10^{-5}$ and $d = \lambda$, $C(k_x,\sigma)$ is computed and illustrated in Fig. 4 (solid line). It is shown that just for such a small loss, only the evanescent waves with $|k_x| < k_{x\,\max} = 2k_0$ have been successfully restored and contribute to the resolution of the image. For $|k_x| > k_{x\,\max}$, $C(k_x,\sigma)$ increases rapidly, which means that the corresponding evanescent waves are absent in the image plane. Hence, the corresponding resolution length is $L_R = 2\pi/k_{x\,\max}$.

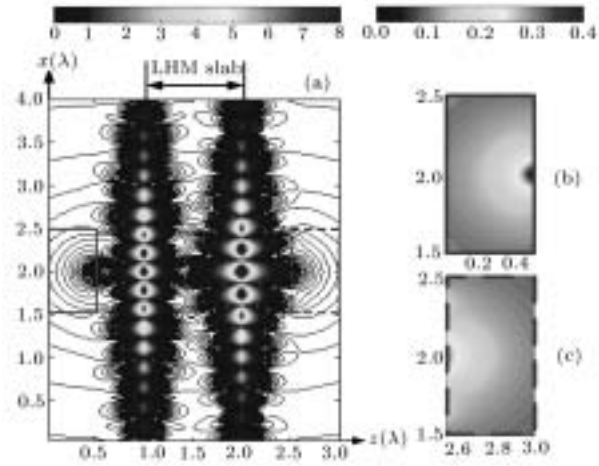


Fig. 3. (a) Contour plot of the field intensity (linear scale). The parameters are similar to that of Fig. 2, but $\varepsilon = \mu = -1 + 10^{-5}i$. (b) The field intensity of the region indicated by the solid red rectangle. (c) The field intensity of the region indicated by the dotted red rectangle.

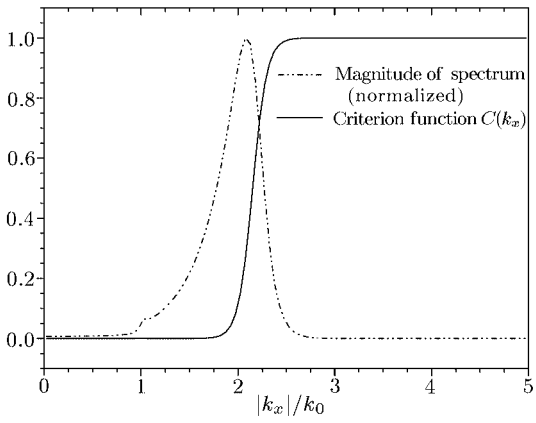


Fig. 4. Dependence of criterion function and magnitude of spectrum on $|k_x|/k_0$ in the case of $\sigma = 10^{-5}$ and the thickness of the slab λ.

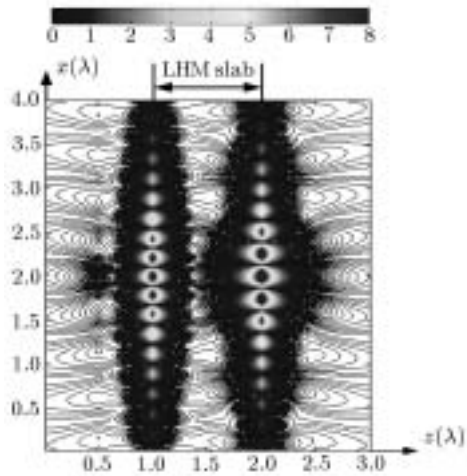


Fig. 5. Contour of field only composed of resonance wave components in the case of $\sigma = 10^{-5}$ and the thickness of the slab λ.

Unlike the lossless case, it is also clearly shown (Fig. 3) that a series of bright spots distribute in almost equal distance along the interfaces of the slab. This localized field with large amplitude distributes near the interfaces and decreases rapidly out of the slab. The spatial spectrum of the field along the interfaces of the slab can be easily obtained by letting $z = d_0$ for the front interface and by letting $z = d_1$ for the back interface in Eq. (3). Here, the magnitude of the spectrum at the back interface is computed with respect to k_x and plotted in Fig. 4 (dot-dashed line) for $\sigma = 10^{-5}$, $d = \lambda$. It is seen that there is a resonance-like peak approximately at $k_{x\,\mathrm{peak}} = 2k_0$. This means that the resonance of the coupling between the surface waves at the two interfaces takes place and the corresponding wave components experience a strong amplification in the slab. Note that the meaning of the resonance here differs from that of surface plasmon polarization, which is usually related to the eigenmodes of the slab. To extract the wave components of the resonance, a window function is applied to the spectra and the modified spectra are transformed back to the physical space through the inverse Fourier transform. Here, a bandpass window function with the lower truncation $k_x = k_{x\,\mathrm{peak}} - 0.1k_0$ and upper truncation $k_x = k_{x\,\mathrm{peak}} + 0.1k_0$ is applied and the corresponding field only composed of resonance wave components is computed and plotted (linear scale) in Fig. 5. The localized bright spots near the interfaces can still be clearly observed, which are almost identi-

cal with that in Fig. 3, although the field patterns in the other regions in these two figures differ greatly. At this point, the conclusion can clearly be drawn that the surface bright spots are related to the resonance peak of the coupling between the surface waves at the two interfaces. Hence, the spatial spectrum of the field along the interfaces has a very narrow bandwidth with its band centre at $k_x = k_{x\,\mathrm{peak}}$. Therefore the magnitude of the physical field must have periodical maxima along the interfaces with its period approximately equal to $L_D = \pi/k_{x\,\mathrm{peak}}$. Bright spots appear at the positions where the field magnitude reaches one of its maxima. Hence the bright spots will distribute nearly in the equal distance $L_D = \pi/k_{x\,\mathrm{peak}}$ along the interfaces of the slab. This is in good agreement with the field pattern shown in Fig. 3.

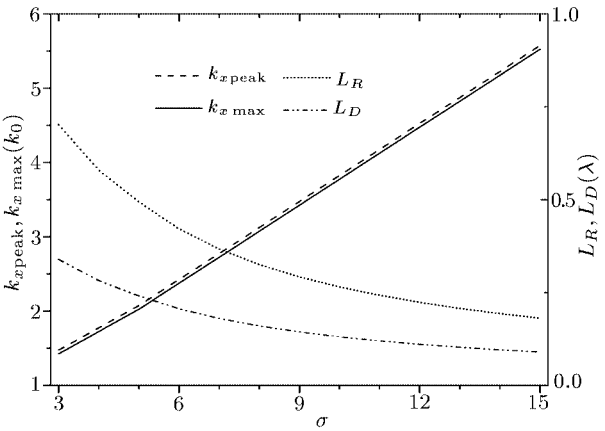


Fig. 6. Characteristics of $k_{x\,\mathrm{max}}$, $k_{x\,\mathrm{peak}}$, L_R, and L_D versus loss σ. The LHM slab thickness $d = \lambda$.

For different losses of σ, the values of $k_{x\,\mathrm{max}}$, $k_{x\,\mathrm{peak}}$, resolution length L_R and bright spots distances L_D are determined similarly with the above procedure and the results are plotted in Fig. 6. It can be seen that for little loss, $k_{x\,\mathrm{max}}$ is greater than k_0, which means that the evanescent waves can be partly restored and the resolution can be better than that of the conventional lens. Here $k_{x\,\mathrm{max}}$ is almost linear with respect to $\log\sigma$. Hence as the loss σ goes to zero (lossless limit), $\log\sigma$ goes to infinity, then $k_{x\,\mathrm{max}}$ and $k_{x\,\mathrm{peak}}$ goes to infinity, which gives rise to the super-resolution ability of the lossless LHM slab. The values of $k_{x\,\mathrm{max}}$ and $k_{x\,\mathrm{peak}}$ are very close to each other, which means that the restoration of the evanescent waves and the resonance of the coupling between the surface waves are two relevant phenomena. Accordingly, the bright-spot distance is close to half of the resolution length.

In summary, the exact solution has been obtained by Fourier integrals in wave-number domain for the interaction of a monochromatic source with a loss LHM slab. Combining the numerical solutions with the analysis on the criterion function and the spatial spectrum of the physical field along the interfaces, we have found that: (1) A small amount of loss has a strong suppression of the super-resolution ability of an LHM slab. However, the resolution of a lossy slab can still be better than a conventional lens because the evanescent wave can still be partly restored if the loss is little enough. (2) The surface bright spots are related to the resonance of the coupling of surface waves between the interfaces of the lossy slab. (3) The evanescent wave restoration and the surface bright spots are two relevant phenomena. The distance between the bright spots is close to half the resolution length.

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