Tetraquarks Production in Quark–Gluon Plasma with Diquarks *

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We present a way to calculate tetraquarks ratios for quark–gluon plasma with diquarks. The ratios of tetraquarks over baryons produced from quark matter are high than hadronic gas model limits. It is a better way to search for four-quark states in relativistic heavy ion collisions. It may become a criterion to judge whether quark–gluon plasma has formed to search for four-quark states in relativistic heavy ion collisions.

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Multi-quark states means that the system can be decomposed into more than one colour singlet, such a simplest system consists of four quarks. Four-quark mesons were first considered in Refs. [1,2] in the MIT (Massachusetts Institute of Technology) version of the bag model. Theoretically, the existence of the four-quark states was investigated in various ways and some hadron nonet could be interpreted as four-quark states, such as $a_0(980)$, $f_0(980)$, $f_1(1420)$, $f_2(1525)$ and $\Psi(4040)$, etc.[3−5] In the experiments, the BaBar collaboration has discovered a narrow state, the D_s^+ with a mass $2317.6 \pm 1.3\,\mathrm{MeV}$ and a width $8.8 \pm 1.1\,\mathrm{MeV}$. The SELEX collaboration has observed a new narrow $D_s^+\pi$ resonance,[6] $D_s^+(2632)$ with total width $\Gamma < 17\,\mathrm{MeV}$. It is suggested that two resonances are scalar four-quark mesons.[7]

The research of ultra-relativistic heavy ion collisions is motivated by the search for quark matter. It is suggested that in a volume of nucleus size, the hadronic matter phase will transit into the phase of quark–gluon plasma under conditions of high energy density and high temperature.[8] The quark–gluon plasma phase can not be observed directly. The directly observable objects are leptons and photons emitted by quark–gluon plasma and hadrons emitted by the hadronization of the quark–gluon plasma phase, which happens as the phase transition from quark–gluon plasma phase to hadronic matter phase. Based on the theoretical analysis, several kinds of experimental phenomena are proposed as the possible signals of quark–gluon plasma formation.[9,10]

Due to large gluon density and low energy threshold for $q\bar{q}$ formation, strangeness production is enhanced in heavy ion collisions which has been suggested as one of possible signals of quark–gluon plasma. Low energy threshold for $q\bar{q}$ formation may similarly lead to the formation of tetraquarks. Four-quark states may be produced in relativistic heavy ion collision and may get larger yields, while quark–gluon plasma is formed in relativistic heavy ion collisions. The four-quark states could be a new candidate in relativistic heavy ion collisions.

For the scalar mesons, $J^{PC} = 0^{++}$ is very particularly important in the theoretical point of view. It needs at least one unit of angular momentum to obtain a positive parity state from a $q\bar{q}$ pair. Apparently, this costs an energy around 1 GeV since similar meson state lie about 1.2 GeV. However, four-quark states $q^2\bar{q}^2$ can couple to 0^{++} without orbital excitation and therefore could explain the structure of the scalar mesons.

Theoretically, the four-quark system has previously been studied in the frameworks of the bag model[1,11] and the nonrelativistic potential models,[12] from many other points of view. However, the conclusions for existence of four-quark states are quite model dependent and few investigations are based on quantum chromodynamics. Several mesons have been interpreted to the four-quark states in many papers, but none of them has been definitely conformed as the four-quark state by experiments so far. Whereas there exist numerous evidences in favour of the $q^2\bar{q}^2$ structure of these states. A look at the particle data group (PDG) review gives an idea of the four-quark structure of the light scalar meson nonet, $\sigma(600)$, $f_0(980)$ and $a_0(980)$. While the scalar nonet can not be treated as the p-wave $q\bar{q}$ in the naive quark model, it can be easily understood as the $q^2\bar{q}^2$ nonet, where $\sigma(600)$ has no strange quarks, $\kappa(700-900)$ has the s quark, $f_0(980)$ and $a_0(980)$ have a $s\bar{s}$ pair.

It is obvious that the four-quark scalar states can consist of diquark–antidiquark or $q\bar{q}$ pairs, which can be considered either as bound states of two ordinary pseudoscalar mesons or as bound states of two vector

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mesons. We concentrate our analysis on the former only.

Considering $N_F = 4$ quark flavours u, d, s and c, the diquarks have 16 basis states of the form of $|\, q_i q_j\rangle$, where $1 \le i,\ j \le 4$. Ten symmetric states form a 10-representation and six antisymmetric states form a $\bar{6}$-representation. The spin of the quarks in each diquark can couple to both singlets and triplets, while colour can couple to both triplets $\bar{3}$ and 6-representation. Diquarks may be in the following colour, spin and flavour:

$$3_c \otimes 3_c = 6_c \oplus \bar{3}_c,\ 2_s \otimes 2_s = 1_s \oplus 3_s,\ 4_f \otimes 4_f = 10_f \oplus \bar{6}_f. \tag{1}$$

Since the quark is fermions with a spin of $1/2$, the state function of any diquark must be symmetric under interchange of two quarks. Thus the basis for diquarks q^2 state

$$|\bar{3}_c 1_s 10_f\rangle,\ |\bar{3}_c 3_s \bar{6}_f\rangle,\ |6_c 1_s \bar{6}_f\rangle,\ |6_c 3_s 10_f\rangle. \tag{2}$$

Then from combination of a diquark and an antidiquark which are colour singlets and obey the Pauli principle we can construct the basis for $q^2\bar{q}^2$ states:

$$|q^2, \bar{q}^2\rangle = |(q_c^2, q_s^2, q_f^2), (\bar{q}_c^2, \bar{q}_s^2, \bar{q}_f^2)\rangle, \tag{3}$$

which includes the two possible colour singlet, $\{3\bar{3}\}$ and $\{6\bar{6}\}$.

All the possible four-quark mesons can be classified into four types, $q^2\bar{q}^2 = [qq][\bar{q}\bar{q}] + (qq)(\bar{q}\bar{q}) + \{[qq](\bar{q}\bar{q}) \pm (qq)[\bar{q}\bar{q}]\}$, $(q = u, d, s, c)$, where parentheses and square brackets denote symmetry and antisymmetry, respectively. As to the spin degrees of freedom, the ground state of the $q^2\bar{q}^2$ system can be in $J^P = 0^+,\ 1^+,\ 2^+$. The scalar $[qq][\bar{q}\bar{q}]$ can only have $J^{P(C)} = 0^{(++)}$ at the lowest level, the last two on the right-hand side can have spin parity $J^P = 1^+$ at their lowest level, while the $(qq)(\bar{q}\bar{q})$ with $J^P = 0^+, 1^+, 2^+$. Each of them can be again classified into two classes, since there exist two ways to obtain the colour singlet $q^2\bar{q}^2$ states. For the combination of four colour quarks into a colour singlet hadron, we can combine two colour triplets q into colour 6 or $\bar{3}$; similarly. Then we can combine two colour antitriplet $\bar{q}$'s into colour 3 or $\bar{6}$. Therefore, there are two ways to combine the four quarks $q^2\bar{q}^2$ into the final colour singlet, to take the $SU_c(3)\bar{3} \times 3$ or the $6 \times \bar{6}$ configuration.

For the quark–gluon plasma at the temperature T, the numbers density of diquarks in the quark matter is

$$n(\mu_i) = \frac{\omega_i}{(2\pi)^3} \int_0^\infty \frac{4\pi p^2 dp}{e^{(\sqrt{p^2+m_i^2}-\mu_i)/T} - 1}, \tag{4}$$

where ω_i is the number of degrees of freedom of diquarks, and μ_i is chemical potential. If $\mu_i < T$, one can obtain

$$n(\mu_i) = \frac{\omega_i}{2\pi^2} T^3 \sum_{k=0}^{\infty} \int_0^\infty x^2 dx \cdot e^{-(k+1)(\sqrt{x^2+(m_i/T)^2}-\mu_i/T)}. \tag{5}$$

From the relation[13]

$$\int_0^\infty x^2 dx e^{-(k+1)(\sqrt{x^2+(m_i/T)^2}} = \frac{1}{k+1}\Big(\frac{m_i}{T}\Big)^2 K_2\Big(\frac{(k+1)m_i}{T}\Big), \tag{6}$$

where K_2 is Bessel function, one has

$$n(\mu_i) = \frac{\omega_i}{2\pi^2} T m_i^2 \sum_{k=1}^{\infty} \frac{1}{k} K_2\Big(\frac{k m_i}{T}\Big) e^{k\mu_i/T}. \tag{7}$$

For the tetraquarks production process $D_i + \bar{D}_j \to T_l$, it can be described with the effective Lagrangian $L = igD_i\bar{D}_j T_l$, where g is the effective coupling constant. It is assumed that g is the same in these process. D is diquark with spin 0 or 1, and T is tetraquarks with spin 0, 1, or 2. We suppressed the Lorentz indices for mesons. For the processes $D\bar{D} \to T$, tetraquarks produced in a time-step dt are

$$dN_T = |\nu_{D_i} - \nu_{\bar{D}_j}| \rho_{D_i} \rho_{\bar{D}_j} \sigma(D_i + \bar{D}_j \to T_l) dt, \tag{8}$$

where ρ_D is the number density of diquarks, and cross-section $\sigma(D_i + \bar{D}_j \to T_l)$ is

$$\sigma(D\bar{D} \to T) = (2\pi)^4 \iint \frac{|M|^2}{4E_{D_i}E_{D_j}|\nu_{D_i} - \nu_{D_j}|} \cdot \delta({p_{T_l}}^2 - {m_{T_l}}^2) \cdot \delta^4(p_{D_i} + p_{D_j} - p_{T_l}) \theta(E_{T_l}) \frac{d^4 p_{T_l}}{(2\pi)^3}, \tag{9}$$

where p_T and p_D are momentums of tetraquarks and diquarks, respectively. The number of tetraquarks produced in a time-step dt is[16]

$$dN_T = \frac{3|M|^2}{32\pi^2} T^2 F(m_{D_i}, m_{D_j}, m_{T_l}, T) dt, \tag{10}$$

where

$$F(m_{D_i}, m_{D_j}, m_{T_l}, T) = \iint \frac{dE_{D_i} dE_{D_j}}{[e^{(E_{D_i}-\mu_{D_i})/T} + 1][e^{(E_{D_j}-\mu_{D_j})/T} + 1]}, \tag{11}$$

and the integrating ranges is

$$m_D \le E_D \le \infty. \tag{12}$$

Then one can obtain that the tetraquarks produced in the process $D_i + \bar{D}_j \to T_l$ read

$$\frac{dN}{dt} = \frac{3}{32\pi^2} |M|^2 T^2 F(m_{D_i}, m_{D_j}, m_{T_l}, T). \tag{13}$$

For a given tetraquark state T_l, several production process can probably produce it. Thus one needs to sum over all possible reactions that can produce it. The relative yields of two possible tetraquark states are

$$\frac{N_1}{N_2} = \frac{\Sigma|M_1|^2 F_1(m_{D_i}, m_{D_j}, m_{T_l}, T)}{\Sigma|M_2|^2 F_2(m_{D_i'}, m_{D_j'}, m_{T_l'}, T)}, \tag{14}$$

where 1 and 2 stand for the possible tetraquark states.

The study of hadronic particle production in relativistic heavy ion collisions offers an opportunity to explore the physical properties of the deconfined quark matter phase. In principle, production and emission of particles of particles can occur throughout the evolution of the dense matter fireball. However, it has been generally assumed that the bulk of particles is produced in the final moments of evolution when the temperature of dense matter sinks below the deconfinement condition.[16] In the deconfined region, diquarks and anti-diquarks can combine to produce a tetraquarks, baryons in the hadronic gas come from the fusion reaction between quarks and diquarks.

Let us generally denote quarks by q, diquarks by D and baryons by B. The possible fusion reactions are

$$q(\boldsymbol{q}) + D(\boldsymbol{k}) \longrightarrow B(\boldsymbol{p}). \tag{15}$$

For the process $qD \to B$, baryons produced in a time-step dt read

$$dN_B = |\nu_q - \nu_D|\rho_q\rho_D\sigma(q + D \to B)dt, \tag{16}$$

where ρ_q is the number density of quarks

$$\rho_q = \frac{1}{\pi^2}\int_{m_q}^{\infty} \frac{p_q E_q dE_q}{e^{(E_q-\mu_q)/T}+1},$$

and cross-section $\sigma(q + D \to B)$ is

$$\sigma(qD \to B) = (2\pi)^4 \iint \frac{|M|^2}{4E_q E_D|\nu_q - \nu_D|} \cdot \delta({p_B}^2 - {m_B}^2)\delta^4(p_q + p_D - p_B)\theta(E_B)\frac{d^4 p_B}{(2\pi)^3}. \tag{17}$$

Baryons have spin 1/2 or 2/3, therefore the effective Lagrangian density of the reactions has three cases:

$$\begin{aligned} \mathcal{L}_1 &= ig\bar{u}_B(p)\gamma_\mu\gamma_5 u_q(q)e_\mu^\lambda(k), \\ \mathcal{L}_2 &= ig\bar{u}_B(p)u_q(q)e(k), \\ \mathcal{L}_3 &= ig\bar{u}_{B\mu}(p)u_q(q)e_\mu^\lambda(k), \end{aligned} \tag{18}$$

where it is assumed that g is the same in these processes. For particles with spin 1/2 we have

$$\sum_{s_z s_z'} u^{s_z}(p)\bar{u}^{s_z'}(p) = \frac{m - i\gamma\cdot p}{2m}, \tag{19}$$

where s_z and s_z' are the spin z-components. For particles with spin 3/2 we have

$$\sum_{s_z s_z'} u_\mu^{s_z}(p)\bar{u}_\nu^{s_z'}(p) = \frac{m - i\gamma\cdot p}{2m}\Big[\delta_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu + \frac{i}{3m}(\gamma_\mu p_\nu - \gamma_\nu p_\mu) + \frac{2}{3m^2}p_\mu p_\nu\Big]. \tag{20}$$

Then

$$\begin{aligned} |M_1|^2 &= \frac{1}{4}g^2[(m_B + m_q)^2 - m_D^2], \\ |M_2|^2 &= \frac{1}{12}g^2\Big[\frac{(m_B^2 - m_q^2)^2}{m_D^2} - m_B^2 - m_q^2 + 6m_B m_q\Big], \\ |M_3|^2 &= \frac{1}{6}g^2\Big[16m_q m_B - 8(m_B^2 + m_q^2 - m_D^2) - \frac{(m_B^2 + m_q^2 - m_D^2)^2}{m_B^2 m_D^2} \cdot (2m_q m_B - m_B^2 + m_q^2 - m_D^2)\Big]. \end{aligned} \tag{21}$$

Diquarks $|qq\rangle$ can fuse into baryons with three colour quarks. For these fusion reactions, it is obvious that the number of baryons is equal to the number of diquarks fusing with quarks in the same fusion reaction. Thus we have

$$\frac{dN_i}{dt} = \frac{3}{32\pi^2}|M_i|^2 T^2 F(m_{q_i}, m_{d_i}, m_{B_i}, T), \tag{22}$$

where i represents the possible reaction. From the rates mentioned above, it can be figured out how many percent in diquarks $|ud\rangle$ will fuse into baryon with u, d or s quark respectively, which is also the number of baryons. Therefore, in the same way, we can obtain the numbers of all kinks of baryons which can be expressed by the number of diquarks.

However, the rate for baryon production is complicated. In the quark fusion processes, fusing with corresponding quarks, different diquarks can form baryons which contain equal favour quarks. For example, the reaction $|uu\rangle + |u\rangle \longrightarrow |uuu\rangle$ makes only one $[uuu]$ baryon (Δ^{++}) but not three. On the other hand, $[uds]$ can be constructed from three different quark–diquark fusion processes. Therefore for the number of baryons $|uds\rangle$, produced in the system hadronization, must include contribution from these three kinds of diquarks. For the number of all these three quarks states, we must sum over all the possible cases.

After calculating the evolution of the number of a given type of three quarks state (e.g. uud), at the final time, the number is distributed between the corresponding multiplets, i.e. the lowest lying octet and decuplet for baryons (anti-baryons), according to the spin degeneracy. In order to determine how this number distributes between octet and decuplet for baryons (anti-baryons), it is necessary to write out all $SU(6)$

wave functions of octet and decuplet for baryons (anti-baryons). From all these wave functions, it is obvious that one should distribute the number of a given type of quark between the corresponding multiplets according to the spin degeneracy. e.g. the number of uds quark group is distributed between the Λ, Σ^0 and Σ^{*0} in the ratios of 1, 2 to 4 while uuu quark group populates only the Δ^{++} resonance, and so on.

We consider baryons (anti-baryons) including the lowest lying octet and decuplet between the corresponding multiplets according to the spin degeneracy. However, in experiment, hadrons can be observed to be $p, n, \Lambda, \Sigma, \Xi$, Ω and their anti-particles emitted by the hadronization of the quark–gluon plasma phase, which happens to be the phase transition from quark gluon plasma phase to hadronic matter phase. Finally Δ, Σ^* and Ξ^* hadronic decays are taken into account with the dominant branching ratios obtained from the particle data table.[14] It is assume that secondary hadron–hadron interactions have a negligible effect on the finally observed hadronic composition.

Therefore the total number of a baryons produced in the hadronization process of quark matter is

$$N_B = \sum \frac{3}{32\pi^2} |M_B|^2 T^2 F(m_{q_i}, m_{d_i}, m_{B_i}, T), \quad (23)$$

where we sum over all possible reactions that can produce baryon B. Then the relative yields of two baryons read

$$\frac{N_{B_1}}{N_{B_2}} = \frac{\sum |M_1|^2 F(m_{q_1}, m_{d_1}, m_{B_1}, T)}{\sum |M_2|^2 F(m_{q_2}, m_{d_2}, m_{B_2}, T)}. \quad (24)$$

The ratio of tetraquarks to the baryons is

$$\frac{N_T}{N_B} = \frac{\sum |M_T|^2 F(m_{d_i}, m_{d_i'}, m_{T_i}, T)}{\sum |M_B|^2 F(m_{q_j}, m_{D_j}, m_{B_j}, T)}. \quad (25)$$

In Ref. [15], the light four-quark states are calculated using the traditional sum rules while four-quark states containing one heavy quark are computed with the heavy quark effective theory (HQET). It is found that the spin–colour combination has little effect on the mass of four-quark states. Therefore, we can constituent quark masses: $m_u = m_d = 310\,\mathrm{MeV}$, $m_s = 450\,\mathrm{MeV}$, and $m_c = 1430\,\mathrm{MeV}$.

The minimum of the coupling constants ratio of g_T to g_B can be fixed by the hadronic gas model.[16,17] For the hadronic gas of thermal chemical equilibrium, the relative yields of hadrons can be calculated, based on the free gas model. The temperature of hadronic matter should be lower than the critical temperature T_c of phase transition, so the ratios of tetraquarks over the other hadrons from hadronic matter can give the upper limits of the relative yields of tetraquarks. The minimum of the coupling constant ratio $a_{\min} = g_T/g_B$ can be fixed by upper bounds of relative yields of tetraquarks to Ω in the hadronic gas.

$D_{sJ}^+(2632)$ is interpreted as an iso-singlet four-quark meson and is supposed to be dominated by the $c\bar{s}s\bar{s}$ structure.[3,18] Therefore $D_{sJ}^+(2632)$ mass is $M_{D_{sJ}^+(2632)} = m_u + 2m_s + m_c = 2640\,\mathrm{Mev}$. Relative yields of tetraquarks to Ω are obtained to be 0.0656 when $g_T/g_B = 2a_{\min}$, where chemical potential of u, d and s quark are 13.3, 13.3, and 5.1 MeV.[17,19]

In summary, we have presented a way to calculate hadrons ratios produced in relativistic heavy ion collisions. In contrast with the normal hadrons, existence of the multiquark state is neither forbidden nor required by colour confinement. Even though there exists no multiquark state, it is helpful to study multiquark configurations to understand final state interactions. Four-quark states is probably produced largely in relativistic heavy ion collision, so it is a better way to search for four-quark states in relativistic heavy ion collisions. The ratio of tetraquarks to baryons produced from quark matter with diquarks is higher than the hadronic gas model limits. It may become a criterion to judge whether quark–gluon plasma has been formed to search for four-quark states in relativistic heavy ion collisions.

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