Smooth Transition from Shakura–Sunyaev Disc to Advection-Dominated Accretion Flow *

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(Received 21 April 2003)

We solve a set of basic equations describing black hole accretion flows using the standard Runge–Kutta method and a bridging formula for the radiative cooling, and show that a smooth transition from a Shakura–Sunyaev disc to an advection-dominated accretion flow is realizable for the high-viscosity case, without the need of involving any extra mechanism of energy transport.

PACS: 97. 60. Lf, 98. 62. Mw, 95. 30. Gv

The most famous model of accretion discs is the Shakura–Sunyaev disc (SSD).[1] Since the SSD was constructed in 1973, the most important breakthrough in the field of accretion disc theory has been the proposal of advection-dominated accretion flow (ADAF).[2,3] The SSD and the ADAF appear to be adequate to describe the outer and the inner region of black hole accretion flows, respectively, and a phenomenological SSD+ADAF model has been quite successfully applied to black hole x-ray binaries and galactic nuclei (see Ref. [4] for a review). From the physical point of view, however, the question remains whether (and how) an outer SSD can match an inner ADAF.

There have been basically three classes of answers to this question. In Refs. [5,6] the authors gave negative answers, arguing that a smooth transition from an SSD to an ADAF was not possible. Whereas their conclusions were under some certain conditions: Ref. [5] considered only the low-viscosity case (with the viscosity parameter $\alpha \sim 0.1$); and Ref. [6] referred only to the plain ADAF, i.e., that with zero cooling. The second class of answers, on the other hand, is positive. A number of authors showed that the SSD-ADAF transition was realizable if an extra heat flux caused by thermal conduction was invoked either in the radial direction,[7–9] or in the vertical direction.[10,11] The cost of this class of answers is, in our opinion, the involvement of an additional mechanism of energy transport, and in particular, the introduction of a new unknown parameter α_T to measure thermal conduction.[8] The third answer was in Refs. [12–14], the authors suggested that the thermal instability of a radiation pressure-supported SSD could trigger the flow to jump from the SSD state to the ADAF state. This answer is also a positive one, but without involving any extra mechanism of energy transport.

In this Letter, we present our answer to the question of SSD-ADAF transition. We demonstrate that such a transition in a smooth way is possible for flows with large values of α (different from the case of Ref. [5]) and non-zero radiative cooling (different from the case of Ref. [6]). We do not involve any extra energy transport mechanism such as conduction, and this is different from the above-mentioned second class of answers, and is similar to the third answer. We discuss the relation between our results here and those of Refs. [12–14] in detail.

We consider steady-state axisymmetric accretion flows and adopt the set of governing equations which is usually in the literature.

The mass conservation equation reads

$$\dot{M} = -4\pi R H \rho v, \tag{1}$$

where $\dot{M}$ is the constant mass accretion rate; R is the radius; H is the half-thickness of the flow; ρ is the density of the accreted gas; and v is the radial velocity.

The radial momentum equation

$$v\frac{\mathrm{d}v}{\mathrm{d}R} = \Omega^2 R - \Omega_K^2 R - \frac{1}{\rho}\frac{\mathrm{d}p}{\mathrm{d}R}, \tag{2}$$

where Ω is the angular velocity of the gas; Ω_K is the Keplerian angular velocity, and $\Omega_K^2 = GM/(R-R_g)^2 R$ in the well known Paczyński–Wiita potential,[15] with M being the mass of the central black hole, and R_g being the gravitational radius; and p is the pressure.

The vertical equilibrium equation

$$H = c_{\mathrm{s}}/\Omega_K, \tag{3}$$

where c_{s} is the isothermal sound speed of the gas, defined as $c_{\mathrm{s}}^2 = p/\rho$.

The angular momentum equation

$$\frac{\mathrm{d}\Omega}{\mathrm{d}R} = \frac{v\Omega_K(\Omega R^2 - j)}{\alpha R^2 c_{\mathrm{s}}^2}, \tag{4}$$

where j is an integration constant that represents the specific angular momentum accreted by the black hole.

* Supported by the National Natural Science Foundation of China under Grant No 10233030.
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The energy equation

$$Q_{\rm vis} = Q_{\rm adv} + Q_{\rm rad} \tag{5}$$

needs some more explanations. In this equation $Q_{\rm vis}$ is the viscous heating rate,

$$Q_{\rm vis} = 2\alpha c_{\rm s}\rho H^2 R^2 \Big(\frac{{\rm d}\Omega}{{\rm d}R}\Big)^2;$$

$Q_{\rm adv}$ is the advective cooling rate,

$$Q_{\rm adv} = 2Hv\Big(\frac{\rho}{\gamma-1}\frac{{\rm d}c_{\rm s}^2}{{\rm d}R} - c_{\rm s}^2\frac{{\rm d}\rho}{{\rm d}R}\Big),$$

where γ is a constant adiabatic index; the radiative cooling rate $Q_{\rm rad}$ is calculated using a bridging formula which is valid in both optically thick and optically thin regimes,[5]

$$Q_{\rm rad} = 8\sigma T^4\Big(\frac{3\tau}{2} + \sqrt{3} + \frac{8\sigma T^4}{Q_{\rm br}}\Big)^{-1},$$

where T is the temperature, τ is the total (election scattering plus absorption) optical depth, $\tau = \tau_{\rm es} + \tau_{\rm abs} = (0.34\,{\rm cm^2 g^{-1}})\rho H + Q_{\rm br}/8\sigma T^4$, and the bremsstrahlung cooling $Q_{\rm br}$ is expressed as $Q_{\rm br} = 1.24\times 10^{21} H\rho^2 T^{1/2}$ ergs$\cdot$s$^{-1}\cdot$cm^{-2}.[3]

Finally, the equation of state is needed to close the system of equations,

$$p = p_g + p_r = 2\Re\rho T + Q_{\rm rad}(\tau + 2/\sqrt{3})/4c. \tag{6}$$

There are eight equations [Eqs. (1)–(6) plus the definition $p = \rho c_{\rm s}^2$ and the expression of τ] for eight unknown variables $v, \Omega, c_{\rm s}, H, \rho, p, T$ and τ as functions of R, with M, $\dot{M}$, α, j, and γ being the given parameters. We use the standard Runge–Kutta method to solve a set of three differential Eqs. (2), (4), and (5) for three unknown v, Ω, and $c_{\rm s}$, and then obtain other variables from the remaining algebraic equations. We integrate the three differential equations from a given sonic point $R_{\rm s}$ (where the radial velocity is equal to the sound speed) both inward and outward. The derivatives ${\rm d}v/{\rm d}R$, ${\rm d}\Omega/{\rm d}R$, and ${\rm d}c_{\rm s}/{\rm d}R$ at $R_{\rm s}$, which are needed to start the integration, are evaluated by applying the l'Hôpital rule. The inward, supersonic part of the solution extends to the inner boundary of the flow, i.e., to a radius $R_{\rm in}$ where the no-torque condition ${\rm d}\Omega/{\rm d}R = 0$ (i.e. $\Omega R^2 = j$) is satisfied. More important for our purpose here is the outward, subsonic part of the solution. It should be stressed that we do not specify any ad hoc outer boundary conditions. We just observe if and where the outward solution can experience a smooth transition from the ADAF state to the SSD state.

We now present our numerical results. Figure 1 shows an example of global solution of accretion flow, i.e., the flow quantities as functions of R, with the given parameters $\gamma = 1.5, \alpha = 0.7$, $\dot{m} = 0.1$ ($\dot{m} \equiv \dot{M}/\dot{M}_{\rm Edd}$, with $\dot{M}_{\rm Edd}$ being the Eddington accretion rate), $j = 0.717$ (cR_g), and $R_{\rm s} = 3.86\,R_g$. In Fig. 1(a) the solid line and the dashed line denote the radial velocity v and the sound speed $c_{\rm s}$, respectively; and the crossing point of the two lines is the sonic point. It is seen that the radial motion is transonic for the inner part of the flow, and is highly subsonic for the outer part. Figure 1(b) shows the angular momentum l ($= \Omega R^2$, the solid line) in comparison with the Keplerian angular momentum l_K ($= \Omega_K R^2$, the dashed line), implying that the rotating motion of the flow is sub-Keplerian for the inner part, and is essentially Keplerian for the outer part. Figure 1(c) draws the relative flow thickness H/R, which changes from ~ 0.4 (relatively thick) to ~ 0.1 (relatively thin) as R increases. Figure 1(d) is for the optical depth τ, again with increasing R, the flow becomes from being optically thin ($\tau < 1$) to being optically thick ($\tau > 1$). In Fig. 1(e), which is for the advection factor $Q_{\rm adv}/Q_{\rm vis}$, one sees that the value of this factor is ~ 1 (advection-dominated) for the inner part of the flow, and decreases dramatically with increasing R, and reaches zero finally (radiative cooling-dominated). Noticing that SSD has the following properties: subsonic radial motion, Keplerian rotating, being geometrically thin and optically thick, and with radiation as the dominant cooling mechanism, while ADAF is characterized by transonic radial motion, sub-Keplerian rotating, being geometrically thick, optically thin, and of course, advection-dominated, we conclude that the global solution of Fig. 1 does contain a smooth SSD-ADAF transition. We also note from Fig. 1 that the transition occurs gradually in a radial region, rather than suddenly at a single radius. It is seen that it is difficult to determine a unique transition radius R_{tr} as suggested in the phenomenological SSD+ADAF model (e.g. Ref. [4]): if R_{tr} is defined as the radius where $l = l_K$, then it is nearly $25\,R_g$ (Fig. 1b); if it is defined so that $\tau = 1$ there, then it is about $12\,R_g$ [Fig. 1(d)]. In the solution of Fig. 1 the transition region ranges from ~ 10 to $\sim 25\,R_g$.

We have shown that a smooth SSD-ADAF transition is realizable for black hole accretion flows with high viscosity ($\alpha = 0.7$ in Fig. 1) and non-zero radiative cooling. Our argument is simple and naive. The equations we solve and the numerical method we use are usual. We do not introduce any extra energy transport mechanism such as thermal conduction. Perhaps the only tool somewhat special here is the bridging formula expressing the radiative cooling $Q_{\rm rad}$, which we need to join the optically thick regime to the optically thin regime of the flow.

What is really of importance in obtaining our results is use of the Runge–Kutta method. In the literature, three numerical methods are often used to solve

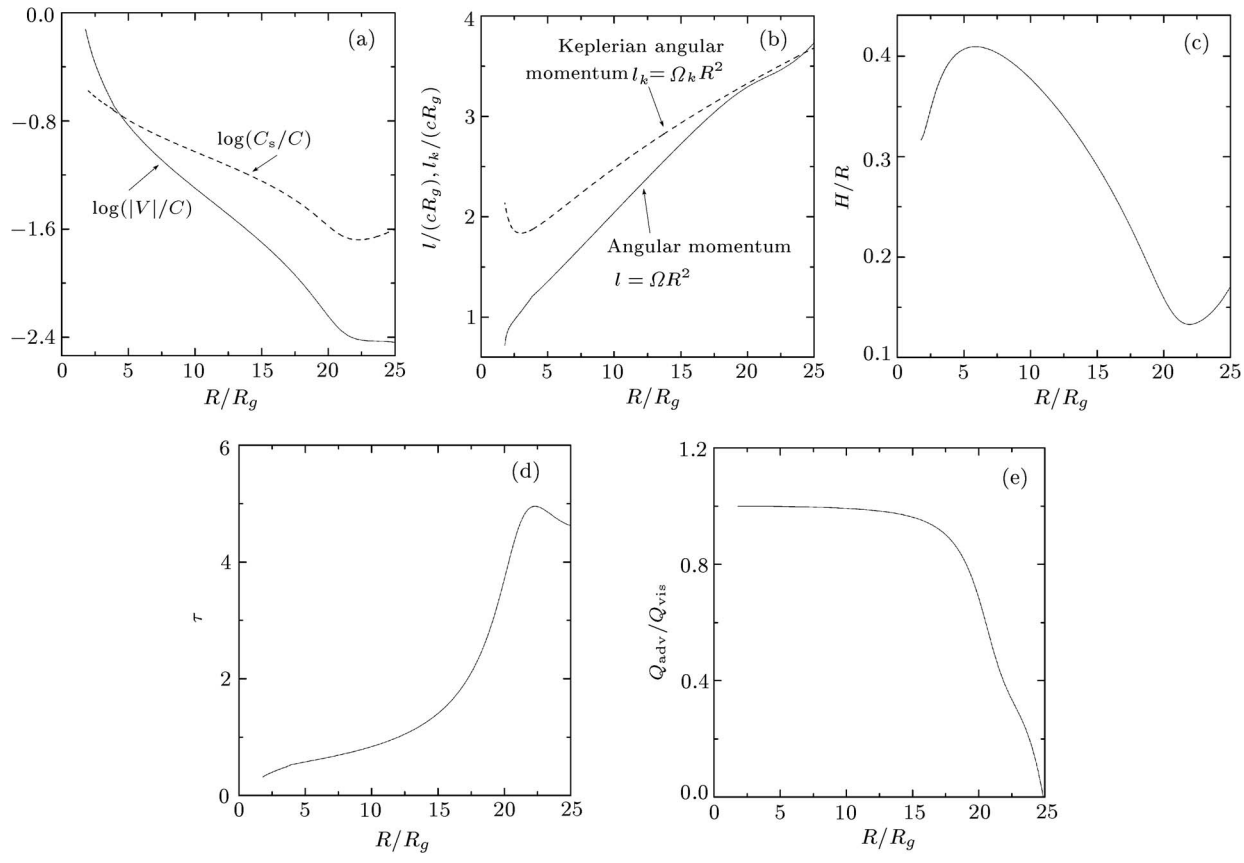


Fig. 1. A global solution with a smooth SSD-ADAF transition for (a) the radial variations of the radial velocity v and the sound speed c_s, (b) the angular momentum l, (c) the relative thickness H/R, (d) the optical depth τ, and (e) the advection factor Q_{adv}/Q_{vis}.

ordinary differential equations, namely the relaxation method, the shooting method, and the Runge–Kutta method. In principle, the solution obtained should not be related to the numerical method, but different methods suit solving different problems. An ADAF on to a black hole has three boundaries: the inner boundary, the sonic point, and the outer boundary. The supersonic motion of the flow between the inner boundary and the sonic point is not much related to the problem of SSD-ADAF transition. In order to give a solution for the subsonic flow between the sonic point and the outer boundary, the relaxation and the shooting method each require both the sonic point condition and the outer boundary condition, and the authors using these methods imposed the SSD properties (Keplerian rotating, radiation-dominated, etc.) as the outer boundary condition of ADAF (e.g. Ref. [8]). This might take some risk, in the sense that the solution containing the SSD-ADAF transition obtained this way could be a mathematically forced one, because the outer boundary of ADAF had been fixed to be in a state corresponding to an SSD. The Runge–Kutta method, on the other hand, requires only one boundary condition, and is adequate for the problem we address here. We do not know a priori what the outer boundary condition of ADAF ought to be, so we do not specify any; but we know for sure that black hole accretion must be transonic, so we use the Runge–Kutta method to integrate the equations starting from the sonic point, and observe how the solution behaves as R increases. For wrong choices of the sonic point condition and the given constant parameters, the outward ADAF solution does not match an SSD solution, then we try again until a correct choice is made and the ADAF has an asymptotic approach to an SSD. The payment for using the Runge–Kutta method is that not only the variables themselves at the sonic point, but also their derivatives there must be supplied in order to start the integration, the calculations are troublesome, though still straightforward.

Finally, let us comment on the relation between our results here and the third answer to the question of SSD-ADAF transition mentioned in introduction. In Ref. [12] authors suggested firstly that for large $\alpha \sim 1$ the thermal instability of radiation pressure-supported SSD could trigger the SSD-ADAF transition. They made time evolutionary calculations and obtained very interesting results: because of the dominance of radiation pressure, the SSD becomes unstable at a radius ($\sim 3.5\,R_g$ in their example solution,

see their Figs. 2 and 3), and the outer stable parts of the flow are disturbed and evolve towards the ADAF state; this outward propagating disturbance damps and stops at a large radius ($\sim 160\,R_g$), then a transition backward to the SSD state starts from that radius, and propagates inward until $\sim 5\,R_g$ (it is larger than the original instability radius $\sim 3.5\,R_g$); finally, a two-phased flow structure really becomes persistent, i.e. the flow stays in the ADAF state inside the transition radius $R_{tr} \sim 5\,R_g$, and in the SSD state outside it. Later, Refs. [13,14] made a somewhat more extensive study on the mechanism of thermal instability-triggered SSD-ADAF transition, giving an α–$\dot{m}$ parameter diagram in which the region allowing the SSD-ADAF transition is clearly seen. Both Refs. [13,14] and the present paper are for stationary flows, so if the work of Refs. [13,14] corresponds to the first step of time evolutionary calculations of Ref. [12], i.e., it proves the cause of SSD-ADAF transition and shows how to determine the original instability radius where the transition starts to occur, then our work here corresponds to the final stage of evolutionary sequence of Ref. [12], i.e., it demonstrates that a stable two-phased flow can form and exist. Nevertheless, according to our results, the SSD-ADAF transition occurs in a radial region, rather than at a single radius, and this point was not addressed by Ref. [12].

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