Phase Leading of Temperature Variations in Cavity Caused by Heat Conduction Between Air and Rock *

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It is well known that the terrestrial temperature varies with the period of 24 hours mainly due to the rotation of the earth in the field of the solar radiation. However, we observed the phenomenon that the semidiurnal (12 hour) temperature oscillation is dominant in our underground laboratory. By the spectrum analysis, the close correlation between the variations of temperature and atmospheric pressure was discovered, and the result of phase analysis showed that the phase of semidiurnal pressure lags behind that of the semidiurnal temperature. A model of heat conduction between air and rock is presented to explain the semidiurnal temperature oscillation observed in the underground laboratory.

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The ambient temperature observation is important for the high precise experiments, such as the determination of the gravitational constant G[1−3] and observation of earth tides.[4,5] In determination of the gravitational constant G, the torsion pendulum is one of the most precise instruments, and the time-of-swing method has commonly been used. The accuracy of those experiments depends on the constancy of the torsion spring constant of the torsion fibre. While the research works about the physical properties of the fibre indicate that the torsion spring constant is dependent not only on the oscillation frequency and amplitude due to the anelasticity and the nonlinearly,[6−9] but also on temperature.[10−12] The experimental result shows that the ambient temperature fluctuation with 1°C will introduce a considerable uncertainty about 165 ppm in the torsion spring constant of the fibre. Therefore, the thermoelastical effect of torsion fibre should be corrected by monitoring the ambient temperature. In the earth tides observations, the signal is very weak. For the instruments designed to measure the subtle earth tides, the thermal expansion derived from temperature variation will not be neglected. To reduce the ambient temperature disturbance, most such experiments as determination of the gravitational constant G and the earth tides observations were performed in underground laboratories, and the ambient temperature should be monitored in the whole process of experiment.

Since 1999, the temperature variation in the underground laboratory has been continuously measured. The spectrum was analysed and was shown in Fig. 1. The fact that the semidiurnal (12 hour) temperature oscillation with an uncertainty of 1 min is dominant in our underground laboratory was discovered.

As we know, the terrestrial temperature varies with the period of 24 hours mainly due to the rotation of the earth in the field of the solar radiation.[13] Obviously, the semidiurnal temperature has a different activating mechanism. According to our analysis, this phenomenon can not be considered as the result of the outer atmospheric temperature variation or the tidal forces.

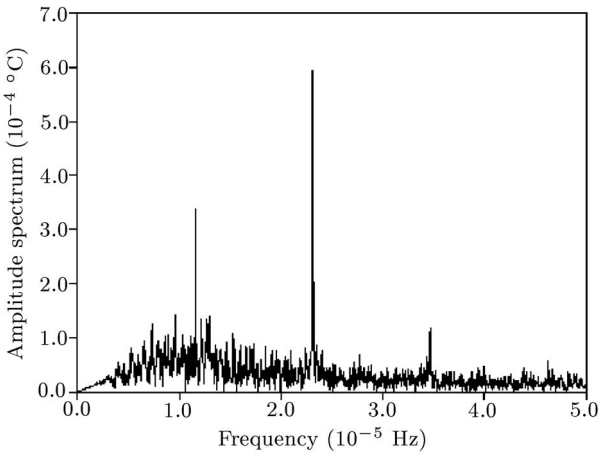


Fig. 1. Amplitude spectrum of the temperature variation observed in the underground laboratory. The main peaks are 1.157×10^{-5} Hz (24 h), 2.314×10^{-5} Hz (12 h) and 3.471×10^{-5} Hz (8 h), respectively.

The external atmospheric temperature has periodic oscillation, but it cannot be used to explain the semidiurnal temperature oscillation occurred in our underground laboratory. Firstly, it will be inevitable to confront the difficulty to explain the disparity of frequencies. On the earth ground, it is not difficult to understand that the dominant component is diurnal oscillation by considering the earth rotation in the solar radiant field. Supposing that the external temperature contributes the temperature oscillation in the underground laboratory, we could draw the conclusion that the period of temperature should be 24 hours. However, this is in contradiction to the experimental result. Secondly, the effect contributed by the heat conduction through the rock cover can be approximated. According to the heat conducting

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equation and its boundary condition, the amplitude of the temperature variation decreases with the increase of the thickness of cover, and it abides by the exponent law. In our underground laboratory, the least thickness of the cover is about 40 m. With the employment of the typical parameters of the rock and the amplitude of the external temperature oscillation in our local area: 10°C per day, the effect is approximated on the order of 10^{-96}°C. Apparently, the effect of heat conduction by the rock cover can be neglected completely.

The earth tides also cannot be used to explain the semidiurnal temperature oscillation phenomenon because the dominant period of the tidal effect is half a lunar day (12 h and 25 min) instead of half a solar day (12 h).

The research in meteorology shows that the amplitude of the semidiurnal atmospheric pressure is larger than the diurnal variation because of the resonance of the free air with period of 12 h.[14,15] It is reasonable to think that the temperature oscillation in the underground laboratory is caused by the pressure variation.

In our underground laboratory, a barometer (GK-03, Chinese Seismological Bureau) was used to observe the atmospheric pressure. The amplitude spectrum of pressure data in 2000 is shown in Fig. 2. From this figure, we can see that the semidiurnal variation of the atmospheric pressure is larger than the diurnal variation. This result is reasonable according to the resonant and nonresonant theory.[14,15]

Compared Fig. 1 with Fig. 2, the remarkable similarity between the two amplitude spectra suggests that the temperature variation in the underground laboratory closely correlates with the atmospheric pressure variation. With the cross-correlation equilibrium method,[16] the phases of the semidiurnal pressure oscillation and temperature oscillation were determined to be 5.15 ± 0.15 rad and 6.10 ± 0.18 rad, respectively. Now that the semidiurnal pressure variation is the cause of the semidiurnal temperature variation in the underground laboratory, it is quite strange that the phase of semidiurnal pressure variation lags behind that of temperature by 0.95 rad.

Generally, we can assume that the atmosphere in the underground laboratory obeys the Poisson equation for polytropic processes, i.e.

$$PV^n = C, \tag{1}$$

where n is the polytropic parameter, specifically $n = 0$ represents the constant pressure process, $n = 1$ represents the isothermal process, $n = 1.4$ represents the adiabatic process, and $n = \infty$ represents the constant volume process.

The temperature of air in the cavity also changes when the atmospheric pressure changes. According to the first law of thermodynamics, the internal energy of air in the cavity is contributed by the work carried out by atmospheric pressure in air and the heat quantity conducted from rock, i.e.

$$\Delta U = \Delta W_p + \Delta Q, \tag{2}$$

where ΔU is the variation of internal energy of air in the cavity, ΔW_p is the work carried out by atmospheric pressure in air, and ΔQ is the heat extracted by the air from ambient rock.

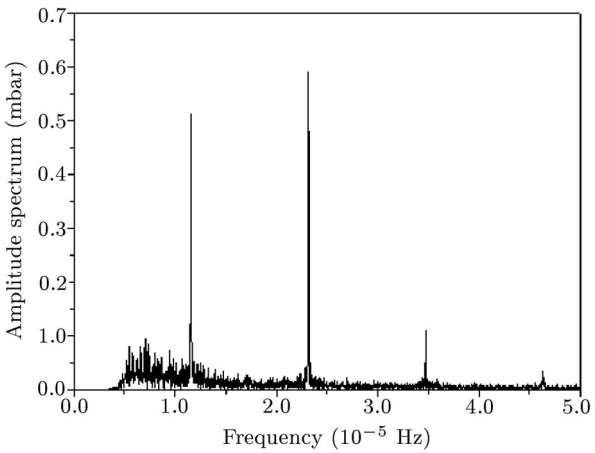


Fig. 2. Amplitude spectrum of atmospheric pressure. The main peaks are 1.157×10^{-5} Hz (24 h), 2.314×10^{-5} Hz (12 h) and 3.471×10^{-5} Hz (8 h), respectively.

When a temperature gradient exists in a solid medium, heat will be transferred from the higher- to the lower-temperature region. According to Fourier's law of heat conduction, the rate at which heat is transferred by conduction is proportional to the temperature gradient $\partial T/\partial r$ and is proportional to the area through which heat will be transferred. Therefore, in the time interval of $\Delta\tau$, the heat quantity extracted by air through the area S reads as

$$\Delta Q = kS\frac{\partial T}{\partial r}\Delta\tau, \tag{3}$$

where k is the thermal conductivity of materials.

In order to compute the heat quantity by conduction, the temperature gradient in rock should be determined.

It can be supposed that the temperature in the cavity is expressed by

$$T = T_0 + A\cos(\omega\tau + \varphi_t), \tag{4}$$

where $T_0, A,\ \omega$ and φ_t are the average temperature, the amplitude of temperature variation, the temperature oscillation frequency and the phase of temperature variation, respectively.

According to the equation of heat conduction, the temperature of rock which is r away from the contacting surface $T(r,\tau)$ reads as

$$T(\tau,r) = T_0 + A\exp\Big(-r\sqrt{\frac{\omega\rho c}{2k}}\Big) \cdot\cos\Big(\omega\tau - r\sqrt{\frac{\omega\rho c}{2k}} + \varphi_t\Big), \tag{5}$$

where ρ and c are the density and the specific heat of the rock, respectively. Therefore, according to Eq. (3), the heat extracted from rock by air in unit time interval can be expressed as

$$\dot{Q} = \frac{\Delta Q}{\Delta \tau} = kS\sqrt{\frac{\omega\rho c}{2k}}A[\sin(\omega\tau+\varphi_t) - \cos(\omega\tau+\varphi_t)]. \quad (6)$$

The pressure variation can be expressed as

$$p = p_0 + \Delta p\cos(\omega\tau+\varphi_p), \quad (7)$$

where p_0, Δp and φ_p represent the average pressure, the amplitude of the pressure variation and the phase of the pressure oscillation, respectively.

Therefore, the power carried out by atmospheric pressure is

$$\dot{W}_p = -\frac{V}{n}\omega\Delta p\sin(\omega\tau+\varphi_p). \quad (8)$$

The internal energy variation of air can be expressed by

$$\Delta U = \rho_a V c_v \Delta T, \quad (9)$$

where ρ_a and c_v are the density and the specific heat of air, respectively. According to the temperature expression (4), we can obtain the variation rate of temperature, and the internal energy variation of air in the unit time can be expressed by

$$\dot{U} = -\rho_a V c_v \omega A\sin(\omega\tau+\varphi_t). \quad (10)$$

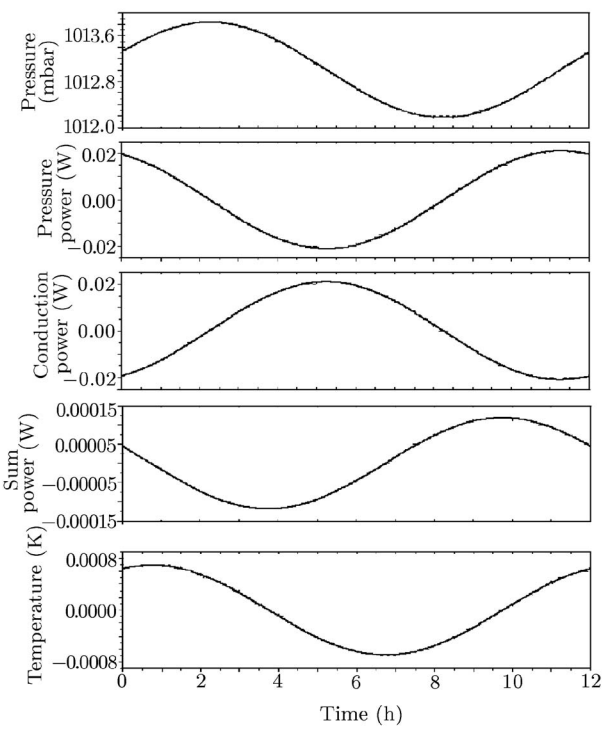


Fig. 3. Variation of parameters in the polytropic process as the heat conduction between air and rock is considered. (a) Atmospheric pressure variation, (b) power carried out by atmospheric pressure, (c) power of heat conduction, (d) sum power of heat conduction and work carried out by atmospheric pressure, (e) air temperature observed in the cavity.

According to the first law of thermodynamics, we can obtain

$$\dot{U} = \dot{Q} + \dot{W}_p. \quad (11)$$

Substituting Eqs. (6), (8), and (10) into Eq. (11) yields the phase lag of pressure behind that of temperature, i.e.

$$\Delta\varphi = \mathrm{arctg}\frac{\frac{S}{V}\sqrt{k\omega c\rho/2}}{\rho_a c_v\omega + \frac{S}{V}\sqrt{k\omega c\rho/2}}, \quad (12)$$

where we take the parameters $\rho_a = 1.29\,\mathrm{kg\cdot m^{-3}}$, $c_v = 716\,\mathrm{J\cdot kg^{-1}\cdot K^{-1}}$, $\rho = 2.54\times10^3\mathrm{kg\cdot m^{-3}}$, $c = 888\,\mathrm{J\cdot kg^{-1}\cdot K^{-1}}$, $k = 2.7\,\mathrm{Wm^{-1}K^{-1}}$, $S/V = 1.5\,\mathrm{m^{-1}}$, we can know that the phase lag of the semidiurnal pressure oscillation behind that of temperature is 0.78 rad, which is close to the observed result.

Figure 3 shows the variation of parameters in the polytropic process. The variation of atmospheric pressure was shown in Fig. 3(a). According to the model of heat conduction between air and the rock in the cavity, the power carried out by atmospheric pressure and the conduction power can be obtained as shown in Figs. 3(b) and 3(c), respectively. Thus, the sum power can be obtained as shown in Fig. 3(d). The power carried out by the atmospheric pressure is proportional to the slope of the pressure variation. Obviously, when the pressure is a maximum, the power carried out by the atmospheric pressure is zero. However, at this time, the air temperature which is shown in Fig. 3(e) is not a maximum because the sum of pressure power and the conduction power is not zero. Therefore, it is not difficult to understand the fact that the temperature and the pressure oscillation are not in phase, and the phase of pressure lags behind that of temperature.

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