Finite-Temperature Equation of State with Three-Body Force Correlations *

LI Zeng-Hua(李增花)**, ZUO Wei(左维), LU Guang-Cheng(陆广成), YONG Gao-Chan(雍高产)
Institute of Modern Physics, Chinese Academy of Sciences, PO Box 31, Lanzhou 730000

(Received 16 September 2003)

The equation of state of asymmetric nuclear matter at finite temperature is studied in the framework of the Brueckner–Hartree–Fock approximation which is extended to include three-body force correlations. Due to the additional repulsion produced by three-body force contribution, the proton or neutron potential energy and effective mass are correspondingly enhanced. The symmetry energy gradually becomes more sensitive to the temperature with the increasing density compared with the result that only includes the two hole-line contribution. The asymmetry parameter dependence of symmetry energy is also indicated. It is shown that there is the same linear relation between symmetry energy and β^2 as the zero temperature case with β being the asymmetry parameter.

PACS: 05. 70. Ce, 05. 70. − a

With the development of heavy ion collision physics, researchers have intensely investigated the properties of strongly interacting hot and dense nuclear matter, and it is expected that new information of the nuclear equation of state (EOS) at finite temperature can be obtained at different densities. In addition, a comparable interest in EOS also exists in astrophysics, in which the dynamics of supernovae explosions and the structure of neutron stars are very sensitive to the properties of nuclear matter far from the saturation point. Under astrophysical conditions, mainly in the later stage of the supernovae collapse, the temperature dependence of the nuclear EOS is of great relevance anyhow, even if the temperatures reached in the process are probably below the critical temperature. Therefore, the properties of hot nuclear matter has been studied with a variety of models. A semi-empirical method introduced many years ago [1] to derive the finite-temperature EOS (FTEOS) is based on the application of the so-called Skyrme force.[2] The Skyrme forces (SF) are phenomenological effective nucleon–nucleon (NN) interactions, widely employed in nuclear structure calculations. In addition, there have been microscopic calculations reported.[3−5]

In this Letter, based on the finite-temperature Brueckner–Hartree–Fock (FTBHF) theory, we introduce the microscopic temperature-dependent three-body force (TBF). As far as the present understanding of the microscopic structure of nuclei is concerned, the NN potential originates from the exchange of mesons. In the NN interaction by absorbing and emitting the mesons, it should follow the correction to the nuclear structure (such as the virtual excitation of intermediate nuclei). In a nuclear medium, due to Pauli blocking, the nuclear virtual excitation is inevitably connected with the third degree of freedom. Thus, the three-body force is introduced by considering this freedom degree. Many authors [6] have used three-body forces to implement the saturation curve calculation of nuclear matter at zero temperature and obtained a satisfactory result, which improves the saturation point properties to the great extent. Thus, it is necessary to introduce TBF into the

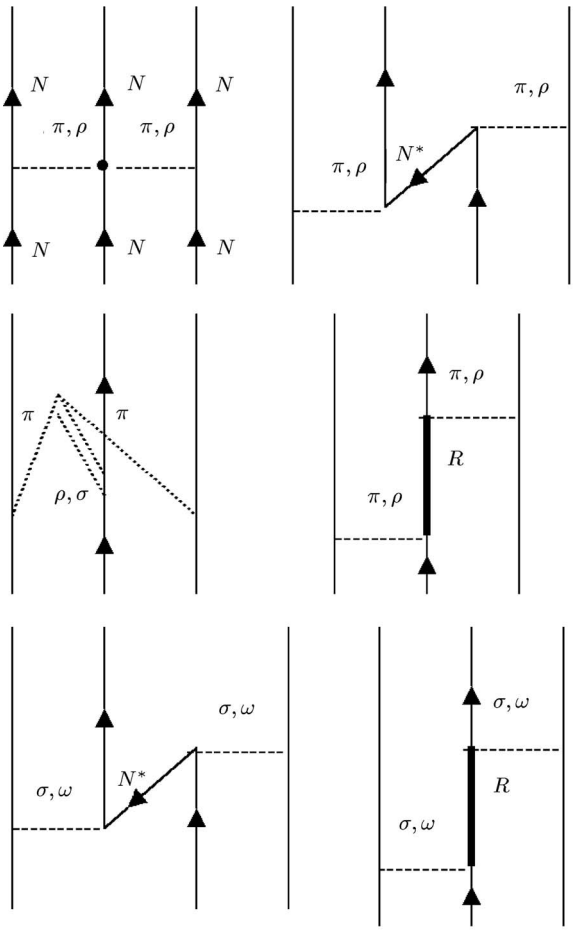


Fig. 1. Some processes contributing to the three-body force.

* Supported in part by the 100 Person Project and the Knowledge Innovation Project of Chinese Academy of Sciences (KJCX2-SW-N02), the Major State Basic Research Development Program of China under Grant No G2000077400, and the Important Preresearch Project of the Ministry of Science and Technology of China under Grant No 2002CCB00200.

** Email: zhli@ns.lzb.ac.cn

©2004 Chinese Physical Society and IOP Publishing Ltd

FTBHF theory at finite temperature. The microscopic three-body force adopted in the present calculations is constructed from the meson-exchange current approach as described in Ref. [7]. In the one-boson exchange (OBE) approximation, four important mesons π, ρ, σ and ω are considered.[8] They are schematically depicted in Fig. 1, as in Ref. [7].

On the basis of the difference between the Fermi distributions at zero temperature and at finite temperature, we extend the TBF at zero temperature to the one at finite temperature and introduce it into the FTBHF theory. At finite temperature the Fermi distribution can be expressed by

$$f_\tau(k,T,k_F^\tau(\rho,\beta)) = \left[1+\exp\left(\frac{e_\tau(k)-\mu_\tau}{T}\right)\right]^{-1}. \quad (1)$$

The formalism of the Brueckner–Bethe–Goldstone (BBG) theory is described for the case of asymmetric nuclear matter.[9,10] The proton and neutron Fermi momenta are $k_F^\tau = [3\pi^2\rho_\tau]^{1/3}$ with $\tau = p$ or $\tau = n$, where $\rho_n = \frac{1}{2}(1+\beta)\rho$, $\rho_p = \frac{1}{2}(1-\beta)\rho$, and the isospin asymmetry parameter is $\beta = \frac{\rho_n-\rho_p}{\rho}$ by the definition. The basic ingredient of the BBG approach is the reaction matrix G that satisfies the generalized Bethe–Goldstone equation

$$G_{\tau,\tau'}(\rho,T,\omega,\beta) = v + v\sum_{k_a k_b}\frac{|k_a k_b\rangle Q_{\tau,\tau'}\langle k_a k_b|}{\omega - e(k_a) - e(k_b)} \cdot G_{\tau,\tau'}(\rho,T,\omega,\beta), \quad (2)$$

where v is the bare NN interaction, ω is the starting energy, and $Q_{\tau,\tau'}$ is the finite-temperature Pauli operator,

$$\begin{aligned} Q_{\tau,\tau'} &= Q_{\tau,\tau'}(k_a,k_b,T,\rho,\beta) \\ &= [1-f_\tau(k_a,T,k_F^\tau(\rho,\beta))] \\ &\quad\cdot[1-f_{\tau'}(k_b,T,k_F^{\tau'}(\rho,\beta))]. \end{aligned} \quad (3)$$

The single-particle energy is expressed by

$$e_\tau(k,T,\rho,\beta) = \frac{\hbar^2}{2m}k^2 + U_\tau(k,T,\rho,\beta) + V_3^\tau(k,T,\rho,\beta), \quad (4)$$

where $U_\tau(k,T,\rho,\beta)$ is the two hole-line contribution to the BBG expansion using continuous choice [11] for the single-particle potential, namely

$$U_\tau(k,T,\rho,\beta) = \frac{1}{2}\sum_{\tau'}\sum_{k'} f_{\tau'}(k',T,k_F^{\tau'})f_\tau(k,T,k_F^\tau) \cdot \langle kk'|G_{\tau,\tau'}(T,\rho,e(k)+e(k'),\beta)|kk'\rangle_A, \quad (5)$$

and $V_3^\tau(k,T,\rho,\beta)$ is the three-body contribution to the potential energy at finite temperature at some momentum value. We can sum up all contributions depicted in Fig. 1 in the momentum space and obtain

$$\begin{aligned} V_3^\tau(T,\rho,\beta) =& V_3^{\tau,2\sigma}(T,\rho,\beta) + V_3^{\tau,2\omega}(T,\rho,\beta) \\ &+ V_3^{\tau,(\sigma,\omega)}(T,\rho,\beta) + V_3^{\tau,(\pi,\rho)}(T,\rho,\beta), \end{aligned} \quad (6)$$

where $V_3^{\tau,2\sigma}(T,\rho,\beta)$, $V_3^{\tau,2\omega}(T,\rho,\beta),\cdots$ represent the 2σ, $2\omega,\cdots$ meson exchange TBF contributions. In the FTBHF approach with TBF, Eqs. (2) and (4)–(6) can be solved self-consistently for each density. The bare potential adopted as input in Eq. (2) is the Argonne V_{18}.[12] The equation needs five iterations to reach a satisfactory convergence. Thus, the single-particle energy $e(T,\rho,\beta)$ is obtained.

Using the extended FTBHF theory including TBF we have obtained the critical temperature of the liquid–gas phase transition for symmetric nuclear matter, i.e. $T_C \approx 14\,\mathrm{MeV}$,[13], which is analogous with some results.[1,3] Therefore, here we mainly discuss the thermal properties of some physical quantities and their variations after the TBF is introduced, which is important for understanding of this phase transition phenomenon and nuclear interaction.

Figure 2 shows the proton or neutron potential energy of the symmetric nuclear matter at density $\rho = 0.16\,\mathrm{fm}^{-3}$ for three values of temperature $T = 0\,\mathrm{MeV}$, $10\,\mathrm{MeV}$, $20\,\mathrm{MeV}$. As T increases, the proton or neutron potential energy becomes higher; the increment is about 4 MeV. The behaviour leads to the enhancement of the binding energy per nucleon, and then heightens the pressure. The solid line shows a slight difference in shape compared with the dashed and dotted lines. We believe that this fact can be attributed to the numerical treatment of the proton or neutron potential energy in the self-consistent procedure, and in this way Eq. (2) can be numerically solved, and the potential energy can be obtained fully self-consistently up to $5\,\mathrm{fm}^{-1}$.

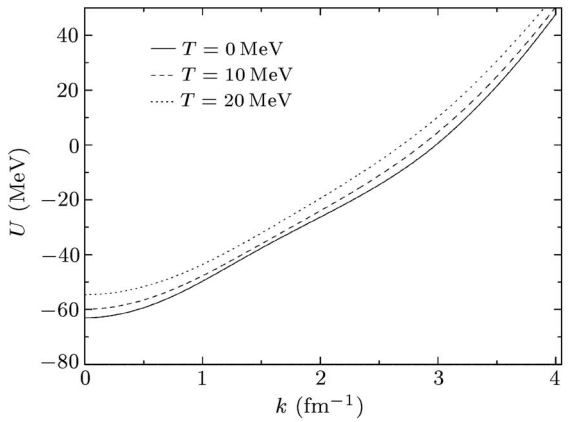


Fig. 2. Proton or neutron potential energy as a function of momentum with different temperatures at density $\rho = 0.16\,\mathrm{fm}^{-3}$ for symmetric nuclear matter ($\beta = 0$).

In order to indicate the effect brought about by the TBF on the proton and neutron potential energy, Fig. 3 displays the result as the function of asymmetry parameter β at the density $\rho = 0.16\,\mathrm{fm}^{-3}$ and the temperature $T = 10\,\mathrm{MeV}$. Briefly, the introduction of the TBF leads to an additional repulsion for the potential energy. The additional repulsion is slightly larger than that at zero temperature.[16] To the neutron potential, the additional variable is 3 MeV to 2 MeV from

symmetric nuclear matter to the pure neutron matter, and it is 3 MeV to 6 MeV for the proton potential. The inequilibrium in the increasing value is mainly due to the reduction of the protons and the addition of the neutrons in the quantity with the increasing β. Therefore, it can be further seen that the additional repulsion produced by the TBF becomes slightly stronger when one moves from symmetric nuclear matter to the pure neutron matter.

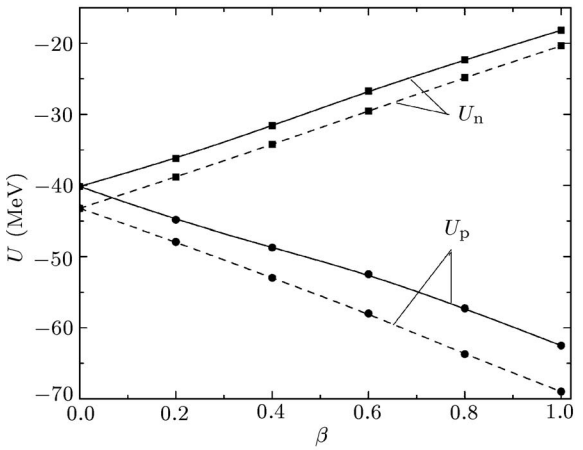


Fig. 3. Proton and neutron potential energy including (solid) and excluding (dashed) TBF.

According to the extended FTBHF theory with TBF, it is essential to discuss the isospin effect of the symmetry energy. In heavy-ion collision with exotic beams, regions of large charge asymmetry could be formed and their properties are expected to be very sensitive to the EOS of asymmetric nuclear matter. It is hoped to decide whether nuclear matter is asy-stiff or asy-soft for the density dependence of the symmetry energy. This point has been of importance for a long time. In the prompt explosion model of a type-II supernova, the β-capture process drives the star to the last stage of the collapse where the proton fraction is about 0.33. Thus, the cooling mechanism of a protoneutron star via direct URCA processes requires knowledge of the symmetry energy in hot nuclear matter with high accuracy.[15,16] At zero temperature, studies based on microscopic many-body theories have shown that the energy per nucleon in asymmetric matter can be approximated by a parabolic function,[9,17,18] so the symmetric energy E_{sym} is normally extracted from the energy difference between pure neutron matter and symmetric matter. The results for the temperature $T = 20\,MeV$ are depicted in Fig. 4. It is indicated that there is the same linear relation between symmetry energy and β^2 as the one at the zero temperature after the TBF is included based on the FTBHF theory.

Density dependence of the symmetry energy is described in Fig. 5(a). It is obvious that the symmetry energy is almost insensitive to the variation of T. However, compared to the result given by Bombaci in Ref. [19] that only included the two hole-line contribution, the symmetry energy gradually becomes more sensitive to the temperature with the increasing density, which is the same as the result including the TBF, as shown in Fig. 5(b). At high density the effect of the TBF is more evident.

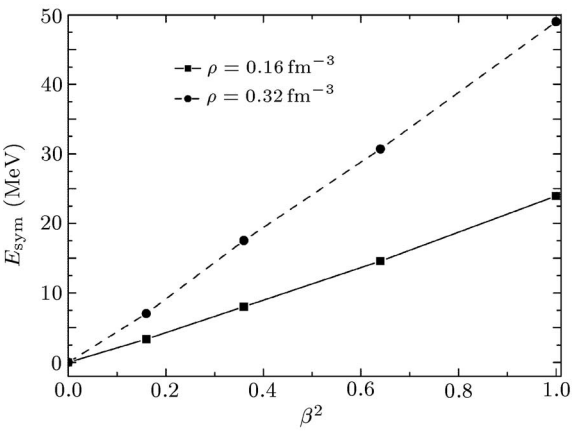


Fig. 4. Symmetry energy versus β^2 in the range $0 \le \beta \le 1$ at two densities.

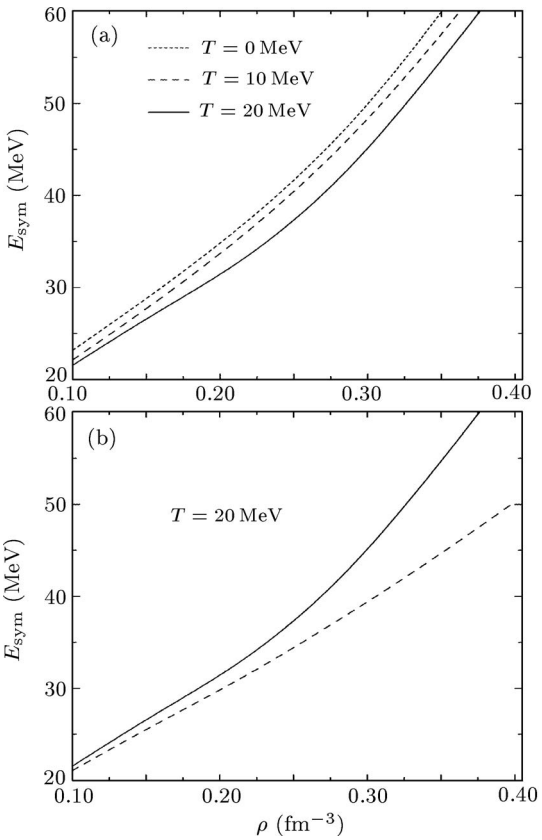


Fig. 5. (a) Symmetry energy as a function of density at three temperatures and (b) density dependence of symmetry energy including (solid) and excluding (dashed) the TBF at temperature $T = 20$ MeV.

In addition, we also study the effect of the TBF on the effective mass and the temperature dependence of the effective mass. The effective mass m^* is expressed

by

$$\frac{m^*_\tau(k)}{m} = \frac{k}{m}\Big(\frac{dE^\tau(k)}{dk}\Big)^{-1},$$

where τ represents proton or neutron. This formula incorporates the nonlocal part of the mean field that makes the local part less attractive for a nucleon travelling with momentum $k > 0$. Figure 6 shows the momentum dependence of m^* with three temperatures $T = 0$, 10, 20 MeV at density $\rho = 0.16\,\mathrm{fm}^{-3}$ for symmetric nuclear matter. The results are in good agreement with the ones reported in Refs. [20,21]. As in the zero-temperature regime, the peak of $m^*(k)$ stays around the Fermi momentum k_F. However, the values of the peak become lower as the temperature increases. When the momentum $k > 2\,\mathrm{fm}^{-1}$, the value of m^* becomes higher than the result of Ref. [4] that excluded the modification of the TBF in the BHF theory, which is due to the fact that the TBF can produce a stronger repulsion at densities above the saturation point. This repulsion is illustrated in Fig. 7, from which we can see that the addition of the three body repulsive potential results in enhancement of the proton and the neutron effective mass. However, in the whole range of asymmetry β, this increment induced by the introduction of TBF hardly changes. The reason is that the repulsion cannot lead to too large an enhancement of potential energy along with β (see Fig. 3), so that it can seldom cause the change of the proton and neutron effective masses to any great extent as the asymmetry parameter increases.

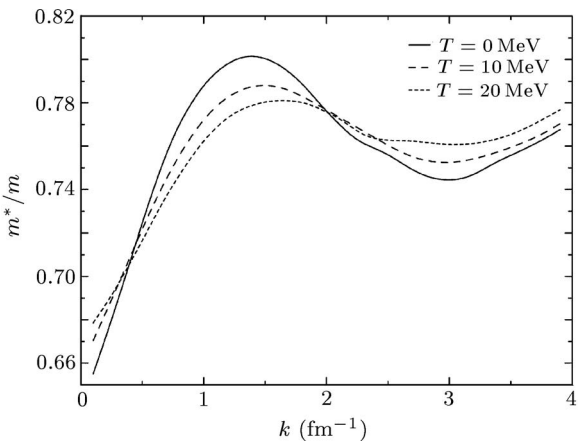


Fig. 6. Momentum dependence of effective mass at three temperatures for symmetric nuclear matter.

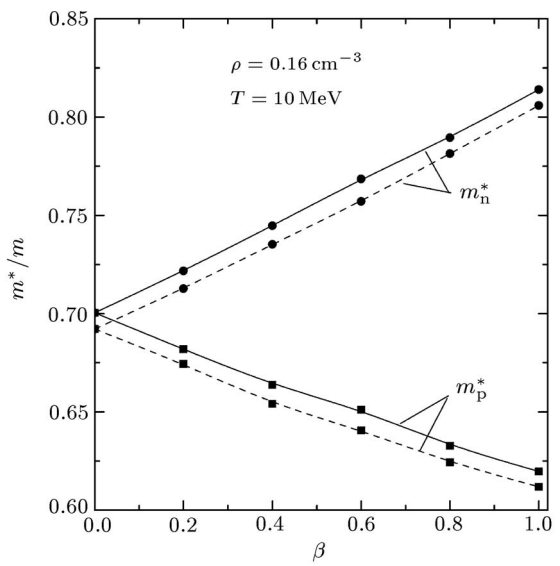


Fig. 7. Proton and neutron effective masses as a function of asymmetry parameter including (solid) and excluding (dashed) three-body force.

In summary, we have discussed the equation of state at finite temperature based on the FTBHF theory by introducing the temperature-dependent TBF contributions. We have also shown the temperature dependence and the isospin dependence of the proton and neutron potential energy, symmetry energy and the effective mass for nuclear matter. In virtue of the additional repulsion produced by the TBF contribution, the proton or neutron potential energy and effective mass are correspondingly enhanced, and the symmetry energy gradually becomes more sensitive to the temperature with the increasing density compared with the result that only includes the two hole-line contribution. The linear relation between the symmetry energy and β^2 proves the availability of the empirical parabolic law of binding energy per particle for the finite-temperature case. Moreover, the TBF contribution manifests stronger repulsion at densities above the saturation, which is very important for studying the dynamics of supernovae explosions and the structure of neutron stars. Therefore, the extended FTBHF theory including TBF is more adapted to investigate the properties of compact objects with high densities in astrophysical conditions.

References

[1] Sauer G et al 1976 *Nucl. Phys.* A **264** 221
[2] Ge L X et al 1986 *Nucl. Phys.* A **459** 77
[3] Friedman B et al 1981 *Nucl. Phys.* A **361** 502
[4] Baldo M et al 1988 *Phys. Lett.* B **215** 19
[5] Baldo M and Ferreira L S 1999 *Phys. Rev.* C **59** 682
[6] Zuo W et al 2002 *Nucl. Phys.* A **706** 418
[7] Grangé P et al 1989 *Phys. Rev.* C **40** 1040
[8] Machleidt R 1989 *Adv. Nucl. Phys.* **16** 189
[9] Bombaci I and Lombardo U 1991 *Phys. Rev.* C **44** 1892
[10] Bombaci I et al 1994 *Phys. Rep.* **242** 165
[11] Jeukenne J P et al 1976 *Phys. Rep. Phys. Lett.* C **25** 83
[12] Wiringa R B et al 1995 *Phys. Rev.* C **51** 38
[13] Li Z H, Zuo W and Lu G C 2004 *Chin. Phys. Lett.* **21** 29
[14] Brown G E et al 1987 *Commun. Nucl. Part. Phys.* **17** 39
[15] Pethick C J 1992 *Rev. Mod. Phys.* **64** 1133
[16] Lattimer J M et al 1991 *Phys. Rev. Lett.* **66** 2701
[17] Huber H et al 1993 *Phys. Lett.* B **317** 485
Huber H et al 1994 *Phys. Rev.* C **50** 1287
[18] Bordbar G H and Modarres M 1998 *Phys. Rev.* C **57** 714
[19] Li B A and Schröder U W 2001 *Isospin Physics in Heavy-Ion Collisions at Intermediate Energies* (New York: Nova Science) p 24
[20] Grange P et al 1987 *Nucl. Phys.* A **473** 365
[21] Hasse R W and Schuck P 1986 *Phys. Lett.* B **179** 313