Scheme for Generation of Entanglement among Bimodal Cavities *

SONG Xin-Guo(宋新国)**, FENG Xun-Li(冯勋立)

Key Laboratory for High Intensity Optics, Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Shanghai 201800

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We present a scheme for generation of an entangled state in many spatially separated bimodal cavity modes via cavity quantum electrodynamics. A V-type three-level atom, initially prepared in a coherent superposition of its excited states, successively passes through both the bimodal cavities. If the atom is measured in its ground state after leaving the last cavity, an entangled state of many cavity modes can be generated. The conditions to generate the maximally entangled state with unity probability are worked out.

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Entanglement is recognized nowadays as a cornerstone in the fundamental of quantum physics and source of diverse applications in quantum information and computation devices.[1,2] In recent years, various quantum systems have been suggested as possible candidates for engineering of quantum entanglement; among them the cavity quantum-electrodynamics (QED) systems are always paid to with more attention. On the one hand, numerous proposals in cavity QED have been made for entangling atoms coupling to the cavity modes.[3−10] On the other hand, many proposals have been presented to generate entanglement of cavity modes. For example, several schemes were proposed to entangle single-mode cavities.[11−13] Gerry studied a model of single-photon nonlocality in which two separated cavities are prepared in an entangled state containing only one photon.[11] Plenio *et al.* presented a proposal for the robust experimental generation of a single-photon entangled Bell states in the modes of two spatially separated cavities mediated by short interaction with an two-level atom.[12] Guo *et al.* presented a scheme for preparation of the three-cavity W-state via cavity QED.[13] Furthermore, many proposals have been put forward to entangle two modes in a single cavity.[14−16] However, to the best of our knowledge, the scheme for the generation of entanglement among bimodal cavities has not been reported up to now. In this Letter, we solve such a problem.

Let us first examine the case of two cavities. The setup considered here is depicted in Fig. 1. The cavities A and B are spatially separated bimodal cavities. One mode is left-circularly polarized and the other is right-circularly polarized. A V-type three-level atom is successively sent into two bimodal cavities. After leaving the second cavity, the atom is measured by ionization detector D. Suppose the cavities A and B are the identical bimodal cavities with one mode being left-circularly polarized (L mode) and the other right-circularly polarized (R mode). A V-type three-level atom, with a ground state $|g\rangle$ and two excited states $|L\rangle$ and $|R\rangle$, is successively sent into the cavities A and B.

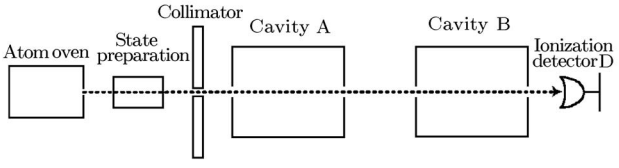


Fig. 1. Experimental setup.

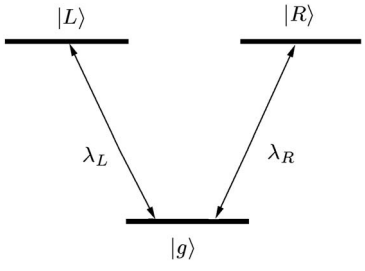


Fig. 2. Atomic level structure.

The atomic level structure is shown in Fig. 2, with a V-type three-level atom with a ground state $|g\rangle$ and two degenerate excited states $|L\rangle$ and $|R\rangle$. Here we assume the transition $|L\rangle \leftrightarrow |g\rangle$ is coupled to the L mode and the transition $|R\rangle \leftrightarrow |g\rangle$ to the R mode, respectively. For simplicity, we assume that the atomic transitions are resonant to cavity modes and the atomic spontaneous emissions to the other modes are neglectable. In this case, the interaction Hamiltonian takes the following form

$$H_I = \hbar \sum_{k=L,R} \lambda_k (a_k |k\rangle\langle g| + a_k^+ |g\rangle\langle k|), \qquad (1)$$

where the subscripts $k = L$ and R present the left- and

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** Email: xinguosong@mail.siom.ac.cn

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right-circularly polarized cavity modes respectively; a_k^+ and a_k are the creation and the annihilation operator of the photon of the mode k, and λ_k is the coupling constant between the atom and the mode k. If the initial atomic state is prepared in $\frac{1}{\sqrt{2}}(|L\rangle+|R\rangle)$ and the cavities are in vacuum state $|0_L0_R\rangle = |0\rangle_L \otimes |0\rangle_R$, we can solve the Schrödinger equation and calculate exactly the state of the subsystem (e.g. atom + cavity A) after the interaction has taken place. We suppose that the time interval of the atom crossing the cavity A is τ_1 and that of the atom crossing the cavity B is τ_2. After the atom leaving the cavity A, the state of the subsystem atom plus cavity A evolves to

$$\begin{aligned}|\Phi(\tau_1)\rangle = \frac{1}{\sqrt{2}}\big[&\cos(\lambda_L\tau_1)|L\rangle|0_L0_R\rangle_A \\ &+ \cos(\lambda_R\tau_1)|R\rangle|0_L0_R\rangle_A \\ &- i\sin(\lambda_L\tau_1)|g\rangle|1_L0_R\rangle_A \\ &- i\sin(\lambda_R\tau_1)|g\rangle|0_L1_R\rangle_A\big]. \end{aligned} \tag{2}$$

Now we consider the interaction between the atom and the modes of cavity B. In this process the initial state is $|\Phi(\tau_1)\rangle \otimes |0_L0_R\rangle_B$. Note that the last two terms on the right-hand side of Eq. (2) for the atom in its ground state would remain unchanged after the atom crosses the cavity B. In this case, only the portion

$$\frac{1}{\sqrt{2}}\big[\cos(\lambda_L\tau_1)|L\rangle+\cos(\lambda_R\tau_1)|R\rangle\big]\otimes|0_L0_R\rangle_A\otimes|0_L0_R\rangle_B$$

can contribute to the evolution of the whole system. In a similar way, the state of the subsystem atom plus cavity B can be worked out. The state of the whole system can thus be obtained, which is given in the following form

$$\begin{aligned}|\Psi(\tau_1+\tau_2)\rangle = &C_1|L\rangle|0_L0_R\rangle_A|0_L0_R\rangle_B \\ &+ C_2|R\rangle|0_L0_R\rangle_A|0_L0_R\rangle_B \\ &+ C_3|g\rangle|0_L0_R\rangle_A|1_L0_R\rangle_B \\ &+ C_4|g\rangle|0_L0_R\rangle_A|0_L1_R\rangle_B \\ &+ C_5|g\rangle|1_L0_R\rangle_A|0_L0_R\rangle_B \\ &+ C_6|g\rangle|0_L1_R\rangle_A|0_L0_R\rangle_B, \end{aligned} \tag{3}$$

where

$$\begin{aligned} C_1 &= \cos(\lambda_L\tau_1)\cos(\lambda_L\tau_2)/\sqrt{2}, \\ C_2 &= \cos(\lambda_R\tau_1)\cos(\lambda_R\tau_2)/\sqrt{2}, \\ C_3 &= -i\cos(\lambda_L\tau_1)\sin(\lambda_L\tau_2)/\sqrt{2}, \\ C_4 &= -i\cos(\lambda_R\tau_1)\sin(\lambda_R\tau_2)/\sqrt{2}, \\ C_5 &= -i\sin(\lambda_L\tau_1)/\sqrt{2}, \\ C_6 &= -i\sin(\lambda_R\tau_1)/\sqrt{2}. \end{aligned} \tag{4}$$

After leaving the cavity B, the atom is measured by an ionization detector D.[17] If it is found in the ground state, then state (3) will collapse to

$$\begin{aligned}|W'\rangle = &C_3|0_L0_R\rangle_A|1_L0_R\rangle_B + C_4|0_L0_R\rangle_A|0_L1_R\rangle_B \\ &+ C_5|1_L0_R\rangle_A|0_L0_R\rangle_B + C_6|0_L1_R\rangle_A|0_L0_R\rangle_B \end{aligned} \tag{5}$$

with the probability

$$P = 1 - (|C_1|^2 + |C_2|^2). \tag{6}$$

It is obvious that the probability P is determined by the coupling constants λ_L and λ_R and the interaction times τ_1 and τ_2.

In order to obtain a maximally entangled state in the following form

$$\begin{aligned}|W\rangle = \frac{1}{2}[&|0_L0_R\rangle_A|1_L0_R\rangle_B + |0_L0_R\rangle_A|0_L1_R\rangle_B \\ &+ |1_L0_R\rangle_A|0_L0_R\rangle_B + |0_L1_R\rangle_A|0_L0_R\rangle_B], \end{aligned} \tag{7}$$

the following conditions are required

$$C_3 = C_4 = C_5 = C_6. \tag{8}$$

Supposing that the coupling constants are equal, that is, $\lambda_L = \lambda_R \equiv \lambda$, we can obtain the following relation for τ_1 and τ_2,

$$\mathrm{tg}(\lambda\tau_1) = \sin(\lambda\tau_2). \tag{9}$$

As a result, the successful probability P can be simplified as

$$P = 2\sin^2(\lambda\tau_1) = 1 - \cos(2\lambda\tau_1).$$

When the interaction times τ_1 and τ_2 are chosen to fulfill the conditions $\lambda\tau_1 = \pi/4$ and $\lambda\tau_2 = \pi/2$, the successful probability P is unity. In fact, the maximally entangled state obtained here is a W state of two bimodal cavities. Compared to the Greenberger-Horne-Zeilinger (GHZ) state, the W state is much less fragile relative to particle loss. When one traces over one of the particles, entanglement is completely lost in the GHZ state. In the case of the W state, the remaining two parties still share significant entanglement. Recently, many proposals have been put forward for the generation of a W state in various systems.[18–23]

The scheme proposed here can be easily generalized to produce entangled states of more than two bimodal cavities. In the case of three bimodal cavities, starting with the system initial state $\frac{1}{\sqrt{2}}(|L\rangle+|R\rangle)\otimes|0_L0_R\rangle_A\otimes|0_L0_R\rangle_B\otimes|0_L0_R\rangle_C$, we can calculate exactly the whole system state by solving the Schrödinger equation in a similar way. Under the assumption of the interaction time in the last cavity being τ_3, the state evolves to

$$|\Psi(\tau_1+\tau_2+\tau_3)\rangle$$

$$
\begin{aligned}
&= D_1|L\rangle|00\rangle_A|00\rangle_B|00\rangle_C + D_2|R\rangle|00\rangle_A|00\rangle_B|00\rangle_C \\
&\quad + [D_3|00\rangle_A|00\rangle_B|10\rangle_C + D_4|00\rangle_A|00\rangle_B|01\rangle_C \\
&\quad + D_5|00\rangle_A|10\rangle_B|00\rangle_C + D_6|00\rangle_A|01\rangle_B|00\rangle_C \\
&\quad + D_7|10\rangle_A|00\rangle_B|00\rangle_C + D_8|01\rangle_A|00\rangle_B|00\rangle_C]|g\rangle, \qquad (10)
\end{aligned}
$$

where we neglect the subscripts $k = L$ and R for the sake of simplicity,

$$
\begin{aligned}
D_1 &= \cos(\lambda_L\tau_1)\cos(\lambda_L\tau_2)\cos(\lambda_L\tau_3)/\sqrt{2}, \\
D_2 &= \cos(\lambda_R\tau_1)\cos(\lambda_R\tau_2)\cos(\lambda_R\tau_3)/\sqrt{2}, \\
D_3 &= -i\cos(\lambda_L\tau_1)\cos(\lambda_L\tau_2)\sin(\lambda_L\tau_3)/\sqrt{2}, \\
D_4 &= -i\cos(\lambda_R\tau_1)\cos(\lambda_R\tau_2)\sin(\lambda_R\tau_3)/\sqrt{2}, \\
D_5 &= -i\cos(\lambda_L\tau_1)\sin(\lambda_L\tau_2)/\sqrt{2}, \\
D_6 &= -i\cos(\lambda_R\tau_1)\sin(\lambda_R\tau_2)/\sqrt{2}, \\
D_7 &= -i\sin(\lambda_L\tau_1)/\sqrt{2}, \\
D_8 &= -i\sin(\lambda_R\tau_1)/\sqrt{2}. \qquad (11)
\end{aligned}
$$

Then we measure the atomic state. If it is in ground state, the entangled state of three bimodal cavities can be obtained in the following form

$$
\begin{aligned}
|W'\rangle &= D_3|00\rangle_A|00\rangle_B|10\rangle_C + D_4|00\rangle_A|00\rangle_B|01\rangle_C \\
&\quad + D_5|00\rangle_A|10\rangle_B|00\rangle_C + D_6|00\rangle_A|01\rangle_B|00\rangle_C \\
&\quad + D_7|10\rangle_A|00\rangle_B|00\rangle_C + D_8|01\rangle_A|00\rangle_B|00\rangle_C \qquad (12)
\end{aligned}
$$

with the successful probability

$$
P = 1 - (|D_1|^2 + |D_2|^2). \qquad (13)
$$

Supposing the coupling constants are equal, when the interacting times are chosen to fulfill the conditions $\lambda\tau_1 = \arcsin(1/\sqrt{3})$, $\mathrm{tg}(\lambda\tau_1) = \sin(\lambda\tau_2)$, and $\mathrm{tg}(\lambda\tau_2) = \sin(\lambda\tau_3)$, the successful probability P is unity.

Similarly, in the case of n identical bimodal cavities, we can obtain a maximally entangled state of $2n$ modes

$$
\begin{aligned}
|W\rangle &= \frac{1}{\sqrt{2n}}\sum_{j=1}^{n}(|00\rangle_1|00\rangle_2\cdots|01\rangle_j\cdots|00\rangle_n \\
&\quad + |00\rangle_1|00\rangle_2\cdots|10\rangle_j\cdots|00\rangle_n) \qquad (14)
\end{aligned}
$$

with the successful probability $n\sin^2(\lambda\tau_1)$. When the interaction times are chosen to fulfill the conditions $\lambda\tau_1 = \arcsin(1/\sqrt{n})$ and $\mathrm{tg}(\lambda\tau_{n-1}) = \sin(\lambda\tau_n)$, the probability is unity.

In the above discussion, we have neglected the influence of the atomic spontaneous emissions and that of the decay of each cavity. In fact, these influences decrease more or less the fidelity of the resultant entangled states. In order to overcome the influence of the atomic spontaneous emissions, a Rydberg atom can be used in the experiment. It has been reported that the radiative lifetimes of excited states have reached the order of 30 ms. In free space, the atom can propagate a few maters at thermal velocity before decaying.[8] As for the influence of the decay of each cavity, we can adopt high-Q cavities. In a recent paper, for example, the Q-value of the cavity has been reached as high as 4×10^{10}, corresponding to a photon lifetime of 0.3 s.[24] In this way, these decoherence influences on the entanglement fidelity can be controlled to a low extent.

In summary, we have presented a scheme for preparation of the maximally entangled state of modes in spatially separated n identical bimodal cavities via cavity QED. This scheme is based on the passage of a V-type three-level atom through resonant bimodal cavities. We find the conditions for generation of a maximally entangled state.

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