Calculation of Chiral Lagrangian Coefficients

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We present a systematic way to combine the global colour model and the instanton liquid model to calculate the chiral Lagrangian coefficients. Our numerical results are in agreement well with the experimental values.

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Quantum chromodynamics (QCD) is successful to describe strong interaction in terms of basic force at high energy. However, the large coupling constant at low energy makes the perturbation theory break down. At low-energy scale, the freedoms of quarks and gluons realize themselves in hadronic level as mesons and baryons, which is characterized by chiral perturbation theory (χPT).[1,2] The χPT can give a lot of satisfactory explanations for experimental data while the coefficients of the chiral expansion are input parameters and their values are determined from experimental observables. Since the present technique cannot derivate the chiral Lagrangian from QCD directly, a class of chiral invariant QCD based models are used in deriving the chiral Lagrangian which is characterized by the effective quark–quark interaction.

The global colour model (GCM), which has provided a lot of successful descriptions of the long distance properties of strong interaction and the QCD vacuum as well as hadronic phenomena at low energy is based on a truncation of QCD Lagrangian preserves global colour gauge transformation and permits a large N_c expansion.[3−6] The instanton liquid model (ILM) of the QCD vacuum is based on a semiclassical approximation, in which all gauge configurations are replaced by an ensemble of topological nontrivial fields, instantons and anti-instantons.[7,9] Although it does not give rise to a long range confining force between quarks, the instanton vacuum has been shown to provide a good phenomenological description of many hadronic properties. Therefore, it is interesting to calculate the chiral coefficients with the two coupling nonperturbative models. In fact, the method of the two coupling nonperturbative models to calculate physical quantities already exists in the literature and is successful in describing some hadronic phenomena.[17] In this Letter, we approximate the effective quark–quark interaction by a dilute instanton liquid ensemble and calculate the chiral coefficients from the GCM following Ref. [4], while the pioneer work for obtaining chiral hadronic phenomena through GCM was given by Roberts *et al.*[10]

Here we describe briefly the main skeleton of the GCM. The global colour model, based upon an effective quark–quark interaction which is approximated by the dilute instanton liquid configurations, is defined by a truncation of QCD Lagrangian while maintaining all global symmetries.

The generation function for QCD in Euclidean space can be written as

$$Z[\bar\eta,\eta]=\int \mathcal{D}\bar q\mathcal{D}q\mathcal{D}A\exp\Big\{-S+\int \mathrm{d}^4x(\bar\eta q+\bar q\eta)\Big\}, \tag{1}$$

where

$$S=\int \mathrm{d}^4x\Big\{\bar q\Big[\gamma_\mu\Big(\partial_\mu-\mathrm{i}g\frac{\lambda^a}{2}A^a_\mu\Big)+\psi\Big]q+\frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu}\Big\},$$

and $G^a_{\mu\nu}=\partial_\mu A^a_\nu-\partial_\nu A^a_\mu+gf^{abc}A^b_\mu A^c_\nu$. We leave the gauge fixing term, the ghost field term and its integration measure to be understood. Here A^a_μ are approximated by the fields of the dilute instanton liquid. In order to preserve the chiral invariance of the above action S (not the full action), the quarks have to be coupled to an external field $\psi(x)$ instead of current quark mass m, the field $\psi(x)$ transforms in a certain way under chiral transformations[1,2,4] (For detailed discussion one can see Ref. [4]).

Here we introduce

$$\mathrm{e}^{W[J]}=\int \mathcal{D}A\mathrm{e}^{\int \mathrm{d}^4x(-\frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu}+J^a_\mu A^a_\mu)}, \tag{2}$$

with $J^a_\mu(x)=\bar q(x)\gamma_\mu\frac{\lambda^a}{2}q(x)$ and rewrite the generation function as

$$Z[\psi,\bar\eta,\eta]=\int \mathcal{D}q\mathcal{D}\bar q\mathrm{e}^{-\int \mathrm{d}^4x[\bar q(\gamma\cdot\partial+\psi)q-\bar\eta q-\bar q\eta]}\mathrm{e}^{W[J]}, \tag{3}$$

then expand $W[J]$ in the current J^a_μ:

$$W[J]=\frac{1}{2}\int \mathrm{d}^4x\mathrm{d}^4yJ^a_\mu(x)D^{ab}_{\mu\nu}(x,y)J^b_\nu(y)+\frac{1}{3!}\int J^a_\mu J^b_\nu J^c_\rho D^{abc}_{\mu\nu\rho}+\cdots. \tag{4}$$

The GCM is defined by a truncation of the functional $W[J]$ in which the higher order $n(\geq 3)$-point

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functions are neglected, and only the gluon 2-point function $D^{ab}_{\mu\nu}(x,y)$ is retained. This model maintains all global symmetries of QCD and permits a $1/N_c$ expansion, however, local $SU(3)$ gauge invariance is lost by the truncation. For simplicity, we use a Feynman-like gauge $D^{ab}_{\mu\nu}(x-y)=\delta_{\mu\nu}\delta^{ab}D(x-y)$. Using the functional integral approach, we introduce bilocal field integration variables $\mathcal{B}^\theta(x,y)$ and integrate out the quarks and gluons fields to change dynamical freedoms of fundamental QCD to hadronic variables. This method has an obvious advantage that the effective hadron field interactions retain knowledge of their subhadronic origin. The resulting function $Z_{\rm GCM}$ takes the following form:

$$Z_{\rm GCM}=\int\mathcal{D}\mathcal{B}\mathrm{e}^{-S[\mathcal{B}]}, \tag{5}$$

where the action is given by

$$S[\bar\eta,\eta,\mathcal{B}]=-\,\mathrm{Trln}[G^{-1}]+\int\mathrm{d}^4x\mathrm{d}^4y\frac{\mathcal{B}^\theta(x,y)\mathcal{B}^\theta(y,x)}{2g^2D(x-y)} +\int\mathrm{d}^4x\mathrm{d}^4y\bar\eta(x)G(x,y;[\mathcal{B}^\theta])\eta(y), \tag{6}$$

and G^{-1} is defined as

$$G^{-1}(x,y)=(\gamma\cdot\partial+\psi(x))\delta(x-y)+\Lambda^\theta\mathcal{B}^\theta(x,y). \tag{7}$$

Here the quantity Λ^θ arises from the Fierz reordering of the current–current interaction term in Eq. (4) and represents the direct product of Dirac, flavour $SU(3)$ and colour matrices.

The vacuum configurations are defined by minimizing the bilocal action:

$$\frac{\delta S[\mathcal{B}]}{\delta\mathcal{B}}\Big|_{\bar\eta,\eta=0}=0,$$

following this mean field approximation, we obtain

$$\mathcal{B}^\theta_0(x-y)=g^2D(x-y)\mathrm{Tr}[\Lambda^\theta G_0(x-y)]. \tag{8}$$

These configurations provide self-energy dressing of the quarks through the definition $\Sigma(p)\equiv\Lambda^\theta\mathcal{B}^\theta_0(p)=\mathrm{i}\gamma\cdot p[A(p^2)-1]+B(p^2)$. In fact, Eq. (8) is the rainbow Schwinger–Dyson equation. In this Letter, the Schwinger–Dyson functions $A(p^2)$ and $B(p^2)$ are approximated by the results of ILM through equating the two point Green functions of the corresponding quark in the two nonperturbative methods.[17]

With the GCM generation function Eq. (5), it is straightforward to calculate the vacuum expectation value of any quark operator of the form $O_n\equiv(\bar q_{i_1}\Lambda^{(1)}_{i_1j_1}q_{j_1})(\bar q_{i_2}\Lambda^{(2)}_{i_2j_2}q_{j_2})\cdots(\bar q_{i_n}\Lambda^{(n)}_{i_nj_n}q_{j_n})$ in the mean field vacuum. Here $\Lambda^{(i)}$ represents the operator in Dirac, flavour and colour space.

Taking the appropriate number of derivatives with respect to the external sources η_i and $\bar\eta_j$ of Eq. (5) and putting $\eta_i=\bar\eta_j=0$, we obtain

$$\langle 0|:O_n:|0\rangle=(-)^n\sum_p[(-)^p\Lambda^{(1)}_{i_1j_1}\cdots\Lambda^{(n)}_{i_nj_n}\cdot(G_0)_{j_1i_{p(1)}}\cdots(G_0)_{j_ni_{p(n)}}], \tag{9}$$

where p represents the permutation of the n indices. In fact, the above equation can only be correct with the assumption of vacuum saturate or vacuum dominance.[22] Furthermore, in the calculation we should subtract the perturbative part of the two point Green function the quark,[18] which is here neglected due to the small current quark mass.

The quark condensate $\langle 0|:\bar q(0)q(0):|0\rangle$ can be inferred from Eq. (9),

$$\langle 0|:\bar q(0)q(0):|0\rangle=-\frac{12}{16\pi^2}\int_0^\mu\mathrm{d}s\frac{B(s)}{sA^2(s)+B^2(s)}. \tag{10}$$

In the instanton liquid model, $A(q)=1$. Here μ is taken to be $1\,\mathrm{GeV}^2$, the scale dependence of the quark condensate is thoroughly discussed in the previous paper.[21] The quark condensate is calculated for later use in the expressions for the Chiral coefficients.

We expand the bilocal fields about the saddle point,

$$\mathcal{B}^\theta(x,y)=\mathcal{B}^\theta_0(x-y)+\hat{\mathcal{B}}^\theta(x,y), \tag{11}$$

and then examine the effective interactions of the fluctuations, $\hat{\mathcal{B}}$. The bilocal fields contain information about internal excitions of the $q\bar q$ in addition to centre-of-mass motion which is associated with the usual local field variables. Thus, they can be projected onto a complete set of internal excitations Γ^θ_n with the remaining centre-of-mass degree freedom represented by the coefficients $\phi^\theta_n(P)$. In fact, the coefficients $\phi^\theta_n(P)$ can be identified with the meson fields. The bilocal fluctuations can be written as

$$\hat{\mathcal{B}}^\theta(P,q)=\sum_n\phi^\theta_n(P)\Gamma^\theta_n(P,q). \tag{12}$$

The functions Γ^θ_n are in general eigenfunctions of the free kinetic operator of the bilocal fields. At the mass shell point, $P^2=-M_n^2$, they satisfy the homogeneous Bethe–Salpeter equation in the ladder approximation for the given quantum number θ and modes n.[4]

To obtain the chiral coefficients, we take a digression to discuss Goldstone theorem. Dynamical chiral symmetry breaking is associated with the occurrence of a massless Goldstone mode which can be related to the dynamical generated scalar self-energy of the quark propagator in the limit of vanishing current quark mass. By analysing the corresponding zero momentum behaviour of the axial-vector Ward identity in the chiral limit, we can obtain the Goldberger–Treiman relation for the quark-pseudoscalar vertex and then reduce the bound state function $\Gamma^\theta_n(P,q)$

from ladder Bethe–Salpeter equation for the pseudoscalar Goldstone mode with $P^2 = 0$ to the quark self-energy function.[4]

Our main goal here is to obtain the effective action for the Goldstone mode, the higher mass fluctuations present in the bilocal fields are neglected. In fact, the higher mass fluctuations should have some effects if they have been integrated out. Based on the above discussion, we can write the full bilocal field of quark fields in the following form[4]:

$$\Lambda^\theta \mathcal{B}^\theta = \Sigma(x-y) + B(x-y)\Big[U_5\Big(\frac{x+y}{2}\Big) - 1\Big], \quad (13)$$

where $U_5(x) = P_R U(x) + P_L U^\dagger(x)$ with $P_{R,L}$ the standard right-left projection operators. For the $SU(3)$ flavour case under consideration here the chiral field U is defined as $U \equiv \mathrm{e}^{\mathrm{i}\lambda^a\phi^a/f_\pi}$.

Next, we follow Ref. [4] to calculate the chiral coefficients and we consider only the real contribution to the effective action. The pioneer work for obtaining the chiral hadronic phenomenon through global colour model was given by Roberts and co-workers[10] The restriction of the fluctuations to the Goldstone modes with $UU^\dagger = 1$ entails that the second term of the action in Eq. (6) is independent of the field U and can be neglected. Thus, the real contribution to the action can be written as

$$\mathcal{S} \equiv \mathrm{Re}[S] = -\frac{1}{2}\mathrm{Trln}(G^{-1}[G^{-1}]^\dagger), \quad (14)$$

where

$$\begin{aligned} G^{-1} =& \gamma\cdot\partial_x A(x-y) + \psi\Big(\frac{x+y}{2}\Big)\delta(x-y) \\ &+ B(x-y)U_5\Big(\frac{x+y}{2}\Big). \end{aligned} \quad (15)$$

Here $A(x-y) = \delta(x-y)$ in the ILM.

By expanding the logarithm and dropping the irrelevant constant terms, we obtain the chiral Lagrangian to the fourth order (in Euclidean space):

$$\begin{aligned} \mathcal{S} = \int \mathrm{d}^4x\Big\{ & \frac{f_\pi^2}{4}\mathrm{Tr}[\partial_\mu U\partial_\mu U^\dagger] - \frac{f_\pi^2}{4}[U\chi^\dagger + \chi U^\dagger] \\ & - L_1(\mathrm{Tr}[\partial_\mu U\partial_\mu U^\dagger])^2 \\ & - L_2\mathrm{Tr}[\partial_\mu U\partial_\nu U^\dagger]\mathrm{Tr}\partial_\mu U\partial_\nu U^\dagger] \\ & - L_3\mathrm{Tr}[\partial_\mu U\partial_\mu U^\dagger\partial_\nu U\partial_\nu U^\dagger] \\ & + L_5\mathrm{Tr}[\partial_\mu U\partial_\mu U^\dagger(U\chi^\dagger + \chi U^\dagger)] \\ & - L_8\mathrm{Tr}[\chi U^\dagger\chi U^\dagger + U\chi^\dagger U\chi^\dagger]\}, \end{aligned} \quad (16)$$

where $\chi(x) = -2\langle\bar{q}q\rangle\psi(x)/f_\pi^2$ and the remaining trace is over flavour. The coefficients L_1, L_2 ($L_2 = 2L_1$), L_3, L_5, L_8 and π decay constant f_π are functions of the scalar part of quark self-energy and its derives, as the expressions are lengthy, one can see Ref. [4] for details.

In performing the gradient expansion it is important to keep the x dependence of the external field $\psi(x)$. After carrying out the gradient expansion to the fourth order, we employ the equation of motion which is obtained at the second order and depends on the external field $\psi(x)$. Finally, we can identify $\psi(x)$ with the current quark mass matrix. Failure to keep the x dependence of ψ to the very end violates chiral invariance and will render some unphysical results for the low-energy coefficients.

From the above expressions, we can see that the coefficients L_4, L_6 and L_7 vanish, which is consistent with the large N_c QCD result. Some modifications may be obtained if we have integrated out higher mass states.

Instantons, as the solutions of the classical Yang–Mills equation of motion, give many successful descriptions of strong interaction phenomena, such as $U(1)_A$ problem, dynamical chiral symmetry breaking, tunnelling the θ vacuum and so on. After the great achievement of 't Hooft's computing the functional determinant in the one-instanton background, attempt was made to understand quantitatively the nonperturbative QCD vacuum configurations in terms of instanton ensembles.[11,12] However, the notorious infrared problem due to large size instanton made many researchers no longer believe in the dilute instanton gas model. To circumvent the disaster, one may think the overcome that the vacuum obtains more filled with instantons of increasing size for larger distances, at some scale there might be some repulsive interactions to stabilize the medium while the semiclassical treatment is still possible and the instantons are not much deformed through their interactions, thus form a dilute instanton liquid ensemble. In Ref. [13,14], the authors used variational techniques and mean field approximation to deal with the statistical mechanics of the instanton liquid while stabilizing it by phenomenological hard core and repulsive interactions derived from ansatz of gauge field. Subsequently, the propagation of light quarks in the dilute instanton liquid ensemble was formulated quantitatively and refined due to the treatment of the nonzero mode propagation.[15,16] Although it does not give rise to a long range confining force between quarks, the instanton vacuum has been shown to provide a good phenomenological description of many hadronic properties.[7,9] Furthermore, lattice calculations support that the vacuum gluon configurations can be approximated by the dilute instanton liquid.[20] Its shortcoming will be that the vector self-energy function of quark propagator in the instanton ensembles $A(q^2) = 1$ which is not consistent with the corresponding one in lattice QCD and other potential models.[8,19] In addition, the $q^2 \gg 1\,\mathrm{GeV}^2$ behaviour of the scalar self-energy function $B(q^2)$ of quark propagator in the instanton medium falls as $1/p^6$ which

disagrees with QCD. Thus, it is interesting to substitute the quark propagator which is dressed through propagating in the dilute instanton medium into the expressions for the π decay constant and chiral coefficients. Our numerical results fit experimental values well. The quark propagator in the ILM can be written as

$$G(p) = [\mathrm{i}\gamma \cdot p + M(p)]^{-1} + O(N/VN_c), \qquad (17)$$

where

$$M(p) = \sqrt{\frac{N}{2VN_c}} \frac{p^2\varphi^2(p)}{\sqrt{\int \frac{\mathrm{d}^4k}{(2\pi)^4} k^2\varphi^4(k)}},$$
$$\varphi(k) = \pi\rho^2 \frac{\partial}{\partial z}(I_0(z)K_0(z) - I_1(z)K_1(z))|_{z=k\rho/2}, \qquad (18)$$

where I and K are the modified Bessel functions, $N/V \approx (200\,\mathrm{MeV})^{-4}$ and $\rho \approx (600\,\mathrm{MeV})^{-1}$.

In conclusion, we have provided a systematic way to combine the GCM with the ILM.[17] Then based on the nonperturbative quark propagator derived in the ILM, we calculate chiral coefficients which are expressed in terms of the scalar part of quark self-energy and its derives.

Our results are compatible with the experimental data. Our results are $f_\pi = 86\,\mathrm{MeV}$, $\langle\bar{q}q\rangle = -(217\,\mathrm{MeV})^3$, $L_1 = 0.8 \times 10^{-3}$, $L_2 = 1.6 \times 10^{-3}$, $L_3 = -3.9 \times 10^{-3}$, $L_5 = 2.0 \times 10^{-3}$, $L_8 = 1.0 \times 10^{-3}$. The experimental values are $L_1 = 0.9 \pm 0.3 \times 10^{-3}$, $L_2 = 1.7 \pm 0.7 \times 10^{-3}$, $L_3 = -4.4 \pm 2.5 \times 10^{-3}$, $L_5 = 2.2 \pm 0.5 \times 10^{-3}$, $L_8 = 1.1 \pm 0.3 \times 10^{-3}$.

Following the GCM, we truncate the QCD Lagrangian to include only two point gluon Green function which is approximated by the dilute instanton liquid ensemble and describe the mesons as fluctuations about the saddle point of the effective action. The chiral coefficients obtained in this way are consistent with the large N_c QCD, and embody their underlying theory through the dependence on the quark self-energy and its derives. Some modification will be made if we have integrated out the high mass fluctuations instead of neglecting them.

To obtain the nonperturbative quark propagator, we have to resolve the rain-bow Schwinger–Dyson equation. However, the form of the gluon propagator $g^2G(p)$ in the infrared region is unknown, one often uses model forms as input in the previous studies of the rainbow Schwinger–Dyson equation.[3,5,6] Obviously, this dressing comprises the notation of constituent quarks by providing a mass $m(p^2) = B(p^2)/A(p^2)$ with $(m(0) = 418\,\mathrm{MeV})$ in the ILM, which corresponds to dynamical symmetry breaking. Here we use the dilute instanton liquid to approximate the QCD vacuum configurations, the quark propagator is dressed through propagating in the dilute instanton liquid ensemble and contains valuable nonperturbative information which manifest itself in the coefficients. Although ILM has shortcoming as mentioned above, it does give satisfactory chiral coefficients and can be extended to other systems. Furthermore, the lattice calculations support that the dilute instanton liquid ensemble is a good approximation of vacuum configurations.

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