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Dynamic Localization of a One-Dimensional Quantum Dot Array in an ac Electric Field *

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We investigate the dynamics of two interaction electrons confined to one-dimensional quantum dot array in an ac electric field. We find that initially localized electrons will remain localized in the absence of Coulomb interaction if the ratio of the ac field magnitude to the frequency is a root of the ordinary zero-order Bessel function. In contrast to the case without Coulomb interaction, no matter what the value is, the electrons are delocalized and the delocalization effect depends on the ratio U/ω and eaE/ω, where U is the strength of Coulomb interaction, a is the lattice constant, and E and ω are the ac field amplitude and frequency, respectively.

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Recent progress in fabrication technology make it possible to fabricate a wide variety of semiconductor microstructures, such as quantum wells, superlattices (SLs), quantum dots, and quantum dot arrays. There has been increasing interest in the problem of propagation of carriers in novel quantum systems because they provide a convenient system for studying electron localization on quantum systems.[1−20] With the help of this convenient system, there has been intensive investigation of the dynamics of charged particles (or carriers) on such novel quantum structures under the action of static and time-dependent electric fields.[1−13]

For example, Bloch oscillation and Wannier–Stark localization can be experimentally verified in semiconductor superlattices.[1−5] With the tight-binding approximation, it is found that an initially localized particle will remain localized in an ac field if the ratio of the field magnitude to the field frequency is a root of zero-order Bessel function.[6−8] This phenomenon has been called the dynamic localization and was later found to be related to the effect of band collapse of the quasi-energy miniband,[9] which has been verified experimentally in optical lattices.[10] An array of coupled quantum dots, in which electrons are confined by a heterostructure of compound semiconductors and electrons can move between adjacent quantum dots by tunnelling, is of interest as a new artificial material. Unlike the bulk material, the physical properties of the quantum dot arrays can be easily varied, which gives theorists and experimentalists the opportunity to study quantum effects in a well-controlled system and has many potential applications, such as implementing novel logical computing elements and even quantum computing.[21−23]

In this Letter, we study the dynamical behaviour of one-dimensional quantum dot arrays containing an arbitrary number of quantum dots connected in a series by tunnelling under an ac electric field. In the quantum dot array, each quantum dot has a strong electron–electron interaction,[24,25] so intradot Coulomb interaction cannot be neglected. Therefore, the model of a single electron employed in a SL is not completely suitable to a quantum dot array, and the electron correlation effect should be taken into account when one studies the dynamical behaviour of an array of the coupled quantum dot system driven by external fields. The analysis is based on exact calculations of discrete lattices by the numerical method.

Our model consists of a one-dimensional quantum dot chain. The dominant effect of interdot coupling in this system is introduced by hopping matrix element V between the corresponding single-particle states in the nearest-neighbour dots. We neglect the hopping matrix element between nonequivalent states. When each quantum dot is sufficiently small, the excited states of each quantum dot may be ignored. Then the system can be mapped to a single-band Hubbard model.[18] Thus the Hamiltonian in this linear quantum dot chain for electrons of sites m $(-\infty < m < \infty)$ with spin σ under an ac electric field can be written as

$$H = V\sum_{m,\sigma}(a^{+}_{m,\sigma}a_{m+1,\sigma} + a^{+}_{m+1,\sigma}a_{m,\sigma}) - eaE(t)\sum_{m,\sigma} m a^{+}_{m,\sigma}a_{m,\sigma} + U\sum_{m} N_{m,\uparrow}N_{m,\downarrow}, \quad (1)$$

where $a^{+}_{m,\sigma}$ $(a_{m,\sigma})$ creates (annihilates) an electron with spin σ on the mth site, V is the hopping between the nearest neighbours, and a is the lattice constant.

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The ac electric field $E(t)$ can be written explicitly as $E(t) = E\cos(\omega t)$ with the amplitude E and frequency ω. Here U is the Coulomb interaction in one dot.

In order to solve the time-dependent Schrödinger equation (henceforth $\hbar = 1$)

$$\mathrm{i}\frac{\partial\psi(t)}{\partial t} = H\psi(t), \tag{2}$$

we expand the wavefunction $|\psi(t)\rangle$ with a complete set of two-electron determinants $|k,l\rangle$. Since Eq. (1) contains no spin-flip terms, there is no mixture of singlet and triplet sub-spaces. In particular, if the initial state has a definite parity, its symmetry will be preserved in the process of evolution, and consequently only the basis functions of the same symmetry as the initial state need expansion. Considering two electrons with opposite spins in an initial state, we can write the wavefunction as

$$\psi = \sum_{k,l} C_{k,l}(t)|k,l\rangle, \tag{3}$$

where $C_{k,l}(t)$ are the expansion coefficients and $|k,l\rangle$ refers to the situation in which one electron occupies dot k and the other one resides in dot l with the opposite spin. Substituting Eq. (3) into Eq. (2) and using Eq. (1), we obtain the time-dependent equation for $C_{k,l}(t)$

$$\begin{aligned}\mathrm{i}\frac{\partial C_{k,l}(t)}{\partial t} &= V[C_{k+1,l}(t) + C_{k-1,l}(t) \\ &\quad + C_{k,l+1}(t) + C_{k,l-1}(t)] \\ &\quad - eaE\cos(\omega t)(k+l)C_{k,l}(t) \\ &\quad + \delta_{k,l}UC_{k,l}(t). \end{aligned} \tag{4}$$

There is no essential difference between the large number $N = a$ and $N \longrightarrow \infty$ if the system (N) is large enough. For example, the solution of Eq. (4) with $k,l = 24$ is the same as that of Eq. (4) with $k,l = 25$. Thus we assume that the pair of electrons is initially located at site 0, i.e.

$$C_{k,l}(0) = \delta_{k,0;l,0}. \tag{5}$$

Our analysis here is based on the observable probability: propagator $\psi_{k,l}(t) = |C_{k,l}(t)|^2$, which denotes the probability that the particle is at site k and l at time t. Here we assumed that they are at site 0 initially. We choose $k,l = 25$ and solve Eq. (4) by a Runge–Kutta integrator. Once we have solved for the amplitude $C_{k,l}(t)$, we can obtain the probability propagator $\psi_{k,l}(t)$. If the self-propagator $\psi_{0,0}(t)$ (the probability that two electrons simultaneously occupy the site 0 at time t) decays in time, we can conclude that an initially localized particle becomes delocalized, otherwise having a tendency of localization.

In order to clearly explain our result and study the dynamics property of the system, we would like to discuss the feature of our solution in the following discussion: (1) eaE/ω fixed at 2.405 while varying the strength of the Coulomb interaction U—the special case $eaE/\omega = 2.405$, the first root of the zero-order Bessel function, is the condition of the dynamical localization in the absence of the Coulomb interaction; (2) the reduced Coulomb interaction U/ω fixed at 1 and 1.7 while considering different values of eaE/ω. Both cases are depicted in Figs. 1 and 2. In these figures, the self-propagator $\psi_{0,0}$ (the probability of the initially occupied site 0) is plotted as a function of dimensionless time ωt for $V/\omega = 0.05$.

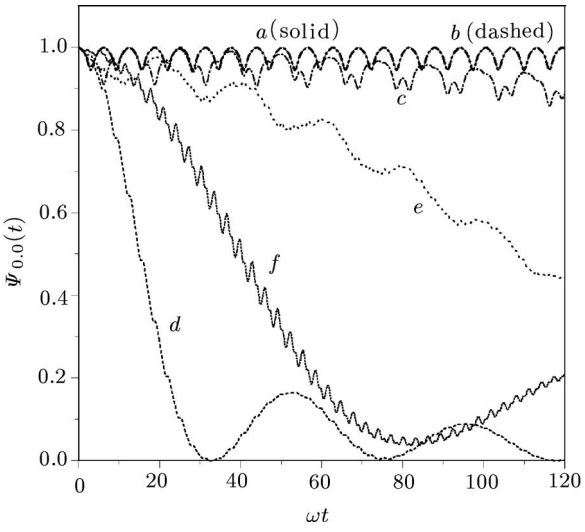


Fig. 1. Self-propagator $\psi_{0,0}(t)$ versus time ωt with $eaE/\omega = 2.405$,$V/\omega = 0.05$: (*a*) $U/\omega = 0.0$; (*b*) $U/\omega = 0.001$; (*c*) $U/\omega = 0.5$; (*d*) $U/\omega = 1$; (*e*) $U/\omega = 1.7$; (*f*) $U/\omega = 3$.

The system satisfies the dynamic localization condition ($eaE/\omega = 2.405$), which can be derived from the single-electron system. Our goal is to investigate the effects of the Coulomb interaction U on the dynamic localization of the system with the help of the increasing strength of the Coulomb interaction. Firstly, we wish to investigate the case without the Coulomb interaction, i.e. $U = 0.0$ in Eq. (1). The numerical result is shown by curve *a* in Fig. 1 for $V/\omega = 0.05$. This result is the same as that in Refs. [6] and [7]. In other words, the two electrons undergo the dynamical localization, and return at the initial site. The behaviour of an electron in this case is just like that of a single electron. Since we are interested in the influence of the Coulomb interaction on the dynamic localization, we gradually increase the strength of the Coulomb interaction to observe the behaviour of the electrons of the system.

Curve *b* in Fig. 1 shows the time evolution of the self-propagator $\psi_{0,0}(t)$ for the case of $U/\omega = 0.001$. We can see that the self-propagator oscillations are the same as those of curve *a*. The motion of the

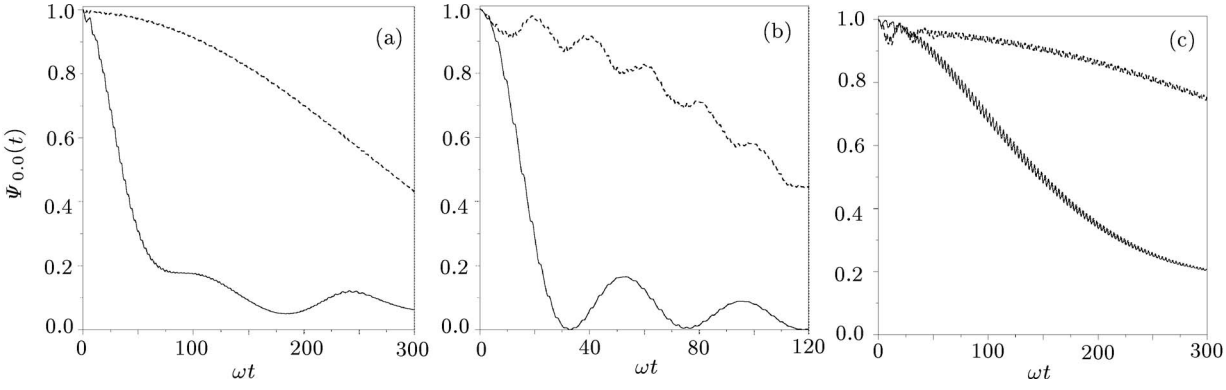


Fig. 2. Self-propagator $\psi_{0,0}(t)$ versus time ωt with $V/\omega = 0.05$, $U/\omega = 1$ (solid lines) and $U/\omega = 1.7$ (dashed lines): (a) $eaE/\omega = 0.5$; (b) $eaE/\omega = 2.405$; (c) $eaE/\omega = 4.0$.

two electrons has a tendency of dynamical localization, which indicates that the Coulomb repulsion interaction in this case is so weak that it cannot affect the dynamical localization of the system. With the increasing Coulomb interaction, i.e. $U/\omega = 0.5$ as shown by curve c in Fig. 1, the stronger the Coulomb interaction, the more effective the Coulomb interaction U on the electron dynamic behaviour. Furthermore, the amplitude and period of the oscillation become large and $\psi_{0,0}(t)$ never returns to its original value 1. This means that the electrons are delocalized at the initial site and the dynamical localization of the system is destroyed by the Coulomb interaction. If the Coulomb interaction increases to $U/\omega = 1.0$, as shown by curve d in Fig. 1, we can see clearly that the recurrences are incomplete and the magnitude of successive peaks decreases. This is the same as the case without an electric field in Refs. [6] and [7], which signifies the delocalization phenomena, i.e. the two electrons do not return site 0 at one time and finally move away from initial site 0.

For $U/\omega = 1.7$, as shown by curve e in Fig. 1, the oscillation amplitude decreases strikingly, and is comparable to that of $U/\omega = 0.5$, but the successive peaks decline more slowly than that of $U/\omega = 1.0$. When the Coulomb interaction U continually increases to $U/\omega = 3$, as shown by curve f in Fig. 1, we can see again that the recurrences become very incomplete, and the peaks of the self-propagator decline. This behaviour is comparable to that shown by curve d in Fig. 1. The change (1 to 0) of $\psi_{00}(t)$ means that both electrons leave from site 0 rapidly during the evolution of the system.

From curves a–f in Fig. 1, we can see that the Coulomb interaction U competes with the electric field. (i) In the absence of the Coulomb interaction, the dynamic localization condition of the system is the same as that of the single-electron system. (ii) When the Coulomb interaction is very weak, it can slightly change the condition of the original dynamic localization, which usually depends on the external electric field, as shown by curve b. (iii) With the increase of the Coulomb interaction, the dynamic localization of the system is gradually destroyed although $eaE/\omega = 2.405$ is still satisfied. If $U/\omega = n$ (n is integer), curves d and f show that the self-propagator varies with time in the same way. However, when $U/\omega \neq n$ the self-propagator varies slowly with time and there are explicit oscillations due to the electric field and the Coulomb interaction. The system still keeps the character of dynamics localization, as revealed by curves c and e. This means that the influence of the Coulomb interaction on the system is stronger for $U/\omega = n$ than for $U/\omega \neq n$.

Next, we study the effects of the external electric field on the dynamic localization of the two-electron system by fixing the value of the Coulomb interaction. Due to the fact that the effect of the Coulomb interaction U for $U/\omega = n$ on the dynamical behaviour is different from that for $U/\omega \neq n$, we fix the values of the Coulomb interaction at $U/\omega = 1$ and $U/\omega = 1.7$. We vary the value of eaE/ω and investigate the dynamical properties of a quantum dot array. Figure 2 shows the cases for $U/\omega = 1$ (solid line) and $U/\omega = 1.7$ (dashed line) but different values of eaE/ω. The self-propagator $\psi_{00}(t)$ is shown in Fig. 2(a)–2(c) for $eaE/\omega = 0.5$, 2.405 and 4.0, respectively. From Fig. 2, we clearly see the incomplete recurrences and successive decrease in magnitude, signifying the delocalization phenomena. Despite $eaE/\omega = 2.405$, which is the condition of dynamic localization in the absence of the Coulomb interaction, the evolution of electrons is still delocalized. We thus conclude that no matter what the value eaE/ω is, the dynamic localization cannot occur. The delocalization properties of the system depend on both eaE/ω and U/ω. Figure 2 shows that the time evolution of the self-propagator $\psi_{00}(t)$ changes rapidly when eaE/ω is a root of the

ordinary zero-order Bessel function. Comparing the case of $U/\omega = 1$ with that of $U/\omega = 1.7$ in Fig. 2, we notice that the evolution of $\psi_{00}(t)$ is more quick for $U/\omega = 1$ than that for $U/\omega = 1.7$, which indicates that the delocalization phenomenon is very strong if U/ω is an integer and it has nothing with the value of eaE/ω.

Generally speaking, the Coulomb repulsion tends to destroy the dynamic localization of the system. In the absence of the Coulomb interaction, the dynamic localization of the two-electron system is the same as that in a single-electron system. When the Coulomb interaction is very weak, the condition of the dynamic localization in a two-electron system is basically realized. The two electrons initially located at site 0 may absorb electromagnetic energy from the external field and transit between different levels. The stronger the Coulomb interaction, the worse the dynamic localization behaviour of the system. When the Coulomb interaction is strong enough, the dynamic localization of the system is completely destroyed. At the same time, the system displays the delocalization phenomenon and the delocalization effect depends on the values of U/ω and eaE/ω.

In summary, we have studied the dynamic localization of two electrons in a quantum dot array driven by an ac electric field. Comparing our results of a two-electron system with those of a single-electron system, the Coulomb interaction between electrons can destroy the dynamic localization derived from a single-electron system, satisfying $J_0(eaE/\omega) = 0$. The evolution behaviour of Bloch electrons is quantitatively affected by the magnitude of the electric field and the Coulomb repulsion of the system. The interplay of the ac field and the Coulomb interaction leads to the occurrence of electron hopping, and consequently the dynamic localization property.

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