Synchronization of Uncertain Time Delay Chaotic Systems using the Adaptive Fuzzy Method

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We consider the synchronization problem of a class of first-order differential-delay chaotic systems. We utilize time-delay fuzzy logic systems to approximate continuous nonlinear time-delay functions, so that the precise mathematical model need not be known. Adopting the adaptive fuzzy control method, we construct a class of state feedback controllers which can render the closed-loop error systems to be asymptotically stable. We carry out simulations of synchronizing Mackey–Glass and logistic chaotic systems, and the results are reasonable.

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Chaos has been extensively studied in the scientific, engineering and mathematical communities for three decades. Recently, the traditional trend of analysing and understanding chaos has evolved to a new situation of investigating, controlling and utilizing chaos. Many results have been obtained on chaos control[1,2,11] and chaos synchronization.[3–8,13]

The synchronization problem of chaotic systems was first investigated by Pecora *et al.*[3] For its successful applications to secure communications, synchronization of chaotic systems has been extensively investigated over the past few years. In Ref. [7] an adaptive control method has been adopted for synchronization. Based on nonlinear H_∞ theory, Suykens *et al.*[8] proposed the master–slave H_∞ synchronization for chaotic Lurie systems. Employing a backstepping method, Wang *et al.*[13] considered the synchronization of a class of nonlinear chaotic systems in the strict-feedback form. In most of the literature investigating the synchronization problem of chaotic systems, low-dimensional systems are often considered, such as Lorenz systems and Rössler systems. However, it is well known that low-dimensional chaotic signals can easily be detected, so they are not desirable for applications to secret communications. Time-delay chaotic systems are infinite-dimensional and have finite high-dimensional hyperchaotic attractors, which can present low detectability and high safety for applications to secure communication.

In this Letter, we investigate the synchronization problem of time-delay chaotic systems. Considering that the structures and parameters of systems are difficult to obtain, we need not know the precise mathematical models of chaotic systems. Time-delay fuzzy logic systems are used to approximate unknown time-delay nonlinear functions. Based on the adaptive fuzzy method, state feedback controllers are constructed to render the synchronization of driving systems and response systems successfully.

We consider the following class of time-delay chaotic systems as driving systems

$$\dot{x} = -ax + F(x(t-\tau)), \tag{1}$$

where $x \in R$, a is a scalar, and $F(x(t-\tau))$ is a nonlinear function containing the time-delay state. Tian *et al.*[11] pointed out that if system (1) satisfies eight conditions (C1–C8 in Ref. [11]), it may be chaotic, and many time-delay chaotic systems investigated are in the form of system (1).

Considering the Mackey–Glass time-delay system[12]

$$\dot{x} = -k_1 x + \frac{k_2 x(t-\tau)}{1 + x^{k_3}(t-\tau)}, \tag{2}$$

where k_1, k_2 and k_3 are the parameters. We know that this belongs to the form of system (1). When parameters k_1, k_2 and k_3 are properly selected, system (2) will be chaotic. If $k_1 = 20$, $k_2 = 40$, $k_3 = 10$ and $\tau = 0.2$, the delay function results in chaotic dynamics.[12] We consider the following logistic delay function

$$\dot{x} = -\lambda x + \gamma x(t-d)(1 - x(t-d)), \tag{3}$$

which also belongs to the form of system (1). When $\lambda = 26, \gamma = 104$ and $d = 0.5$, system (3) is chaotic.

System (1) represents a large class of time-delay chaotic systems. Here we investigate the synchronization problem of system (1).

The response systems are as follows

$$\dot{y} = G(y) + u, \tag{4}$$

where $G(y)$ is unknown.

For driving system (1) and response system (4), the synchronization problem is formulated as follows.

Problem: Design the controller u to render

$$y \to x, \quad \text{as} \quad t \to \infty,$$

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when $a, F(x(t-\tau))$ and $G(y)$ are unknown.

Before giving our main results, we present some details of fuzzy logic systems. Fuzzy logic systems, which are constructed from the fuzzy IF–THEN rules using specific inference, fuzzification and defuzzification strategies, are universal approximations from the point of view of human experts. They can uniformly approximate nonlinear continuous functions to arbitrary accuracy.[10] Thus, linguistic information from human experts can be directly incorporated into controllers.

A fuzzy system is a collection of fuzzy IF–THEN rules of the form:

$$\begin{aligned} R^{(j)}: &\text{ IF } x \text{ is } A^j \\ &\text{THEN } y \text{ is } B^j. \end{aligned} \tag{5}$$

By using the strategy of centre-average defuzzification, singleton fuzzification and product inference, the output of the fuzzy system is

$$y(x) = \frac{\sum_{j=1}^{l} y^j \mu_{A^j}(x)}{\sum_{j=1}^{l} \mu_{A^j}(x)}, \tag{6}$$

where $\mu_{A^j}(x)$ is the membership function of linguistic variable x, and y^j is the point in R at which μ_{B^j} achieves its maximum value.

By introducing the concept of the fuzzy basic function vector $\xi(x)$, Eq. (6) can be rewritten as

$$y(x) = \theta^T \xi(x), \tag{7}$$

where $\theta = (y^1, \cdots, y^l)^T, \xi(x) = (\xi^1(x), \cdots, \xi^l(x))^T$ and $\xi^j(x)$ is defined as

$$\xi^j(x) = \frac{\mu_{A^j}(x)}{\sum_{j=1}^{l} \mu_{A^j}(x)}. \tag{8}$$

Here we take the two universal fuzzy logic systems $F_1(x(t-\tau), \Theta_1)$ and $F_2(y, \Theta_2)$ with $x(t-\tau) \in U_x$ and $y \in U_y$ for some compact sets U_x and U_y to approximate the uncertain terms $F(x(t-\tau))$ and $G(y)$, where Θ_1 and Θ_2 contain the tunable parameters. We construct the fuzzy rule bases of $F_1(x(t-\tau), \Theta_1)$ and $F_2(y, \Theta_2)$ consisting of rules:

$$\begin{aligned} R^j_{F_1}: &\text{ IF } x \text{ is } A^j \\ &\text{THEN } \widehat{F}_1(x, \Theta_1) \text{ is } F_1^j. \\ R^j_{F_2}: &\text{ IF } x\ (t-d) \text{ is } A^j \\ &\text{THEN } \widehat{F}_2(x(t-d), \Theta_2) \text{ is } F_2^j. \end{aligned}$$

We construct the following fuzzy systems

$$\begin{aligned} F_1(x(t-d), \Theta_1) &= \Theta_1^T \xi_1(x(t-d)), \\ F_2(y, \Theta_2) &= \Theta_2^T \xi_2(y), \end{aligned}$$

where $\xi_1(x(t-d))$ and $\xi_2(y)$ are the fuzzy basis functions. According to the universal approximation theorem, there are optimal approximation parameters Θ_1^* and Θ_2^*, so that $\xi_1(x(t-d))^T\Theta_1^*$ and $\xi_2(y)^T\Theta_2^*$ can approximate $F(x(t-d))$ and $G(y)$ as well as possible. Θ_1^* and Θ_2^* are defined as follows

$$\begin{aligned} \theta_1^* =& \arg \min_{\theta_1 \in \Omega_{F_1}} \Big[\sup_{x(t-\tau)\in\Omega_x} |F_1(x(t-\tau), \Theta_1) \\ &- F(x(t-\tau))| \Big], \\ \theta_2^* =& \arg \min_{\theta_2 \in \Omega_{F_2}} \Big[\sup_{y\in\Omega_y} |F_2((y), \Theta_2) - G(y)| \Big], \end{aligned}$$

where Ω_{F_1}, Ω_{F_2}, Ω_x and Ω_y denote the sets of suitable bounds on θ_1, θ_2, x and y, respectively. We assume that θ_1, θ_2, x and y never reach the boundaries of Ω_{F_1}, Ω_{F_2}, Ω_x and Ω_y. The minimum approximation error satisfies the following assumption.

Assumption 1: The approximation error between $F(x(t-\tau))$, $G(y)$ and $\xi_1(x(t-\tau))\Theta_1^*, \xi_2(y)^T\Theta_2^*$ can satisfy the following inequality

$$\begin{aligned} &\big|F(x(t-\tau)) - \xi_1(x(t-\tau))^T\Theta_1^* + G(y) - \xi_2(y)^T\Theta_2^*\big| \\ &\leqslant \eta_0^*, \end{aligned} \tag{9}$$

where η_0^* is an unknown scalar. The two parameters Θ_1^* and Θ_2^* can be learned by using the adaptive algorithms.

We can obtain the error system from Eqs. (1) and (4) as follows

$$\dot{e} = -ax + F(x(t-\tau)) - G(y) - u, \tag{10}$$

where $e = x - y$. The problem of designing controller u to render $y \to x$ as $t \to \infty$ becomes that of designing a controller u such that $e \to 0$ as $t \to \infty$.

If $a, F(x(t-\tau))$ and $G(y)$ are known, the controller can be chosen as follows

$$u = -ax + F(x(t-\tau)) - G(y) + ke, \tag{11}$$

where $k > 0$. Substituting Eq. (11) into Eq. (10), we can obtain

$$\dot{e} = -ke.$$

It is easy to see $e \to 0$ for $t \to \infty$.

If $a, F(x(t-\tau))$ and $F(y)$ are unknown, there is no existing method to solve this synchronization problem. Here we use the adaptive fuzzy method to solve this kind of problem.

For system (10), we consider the following controller

$$u = u_1 + u_2 + u_3, \tag{12}$$

where u_1, u_2 and u_3 are as follows

$$u_1 = -\widehat{a}x + \theta^T\xi_1(x(t-d)) - \vartheta^T\xi_2(y), \tag{13}$$

$$u_2 = \eta_0 \text{sign}(e), \tag{14}$$

$$u_3 = k\text{sign}(e). \tag{15}$$

Here, $\widehat{a}$, η_0, θ and ϑ are the adaptive parameters with corresponding adaptive laws

$$\dot{\widehat{a}} = -k_1 ex, \quad \dot{\eta}_0 = k_2|e|,$$
$$\dot{\theta} = \Gamma_1 \xi_1(x(t-d)), \quad \dot{\vartheta} = -\Gamma_2 \xi_2(y), \tag{16}$$

where k_1 and k_2 are the positive scalars, and Γ_1 and Γ_2 are the positive matrices.

In the above controller, u_1 contains two fuzzy logic systems used to approximate the unknown nonlinear functions, and u_2 is used to compensate for the approximation error. Equation (15) further guarantees that system (10) is asymptotically stable. Thus, we have the following theorem.

Theorem 1: For system (10), the controllers (12)–(16) render the error system asymptotically stable, i.e. as $t \to \infty$, $y \to x$.

Proof: For system (1), we define the following Lyapunov function

$$V = \frac{1}{2}(e^2 + k_1\widetilde{a}^2 + k_2\widetilde{\eta}_0^2 + \widetilde{\theta}^T\Gamma_1^{-1}\widetilde{\theta} + \widetilde{\vartheta}^T\Gamma_2^{-1}\widetilde{\vartheta}), \tag{17}$$

where $\widetilde{a} = \widehat{a} - a$, $\widetilde{\eta}_0 = \eta_0^* - \eta_0$, $\widetilde{\theta} = \theta^* - \theta$, $\widetilde{\vartheta} = \vartheta - \vartheta^*$.

The time derivative along the state trajectory of system (10) is

$$\dot{V} = e\dot{e} + k_1\widetilde{a}\dot{\widetilde{a}} + k_2\widetilde{\eta}_0\dot{\widetilde{\eta}}_0 + \widetilde{\theta}^T\Gamma_1^{-1}\dot{\widetilde{\theta}} + \widetilde{\vartheta}^T\Gamma_2^{-1}\dot{\widetilde{\vartheta}}, \tag{18}$$

since we know that

$$e\dot{e} = e(-ax + F(x(t-\tau)) - G(y) - u). \tag{19}$$

Substituting Eqs. (12) and (13) into Eq. (19) yields

$$\begin{aligned} e\dot{e} =& e(\widehat{a} - a)x + (\vartheta - \vartheta^*)^T\xi_2(y)e \\ &+ (\theta^* - \theta)^T\xi_1(x(t-d))e \\ &+ (F(x(t-\tau)) - \theta^{*T}\xi_1(x(t-d)) \\ &+ \vartheta^{*T}\xi_2(y) - G(y))e + (u_2 + u_3)e. \end{aligned} \tag{20}$$

From assumption 1, we further obtain

$$\begin{aligned} e\dot{e} \leqslant& e\widetilde{a}x + \widetilde{\theta}^T\xi_1(x(t-d))e + \widetilde{\vartheta}^T\xi_2(y)e \\ &+ \eta_0^*|e| + (u_2 + u_3)e. \end{aligned} \tag{21}$$

Substituting Eqs. (14)–(16) and (21) into Eq. (18), we obtain

$$\dot{V} \leqslant -k|e|. \tag{22}$$

From Eq. (22), we can obtain that the controllers (12)–(16) can render the closed-loop error system asymptotically stable.

In particular, if the functions $a, F(x(t-d))$ and $G(y)$ in systems (1) and (4) are partially known, the controller should be changed by using the known parts. For example, if $G(y)$ are known, then the controller becomes

$$u = -\widehat{a}x + \theta^T\xi_1(x(t-d)) - G(y) + \eta_0\text{sign}(e) + ke \tag{23}$$

with the corresponding adaptive law

$$\dot{\widehat{a}} = -k_1 ex, \qquad \dot{\eta}_0 = k_2|e|,$$
$$\dot{\theta} = \Gamma_1\xi_1(x(t-d)). \tag{24}$$

Similar to the proof of theorem 1, we can obtain that controller (23) can render the closed-loop system asymptotically stable.

In the following we make the simulations of synchronization on the Mackey–Glass system and the logistic system. For simplicity, we choose fuzzy basis $\xi_1(x) = \xi_2(x) = \xi(x)$, $\xi(x)$ is defined by Eq. (8), and the membership functions are selected as follows:

$$\begin{aligned} &\mu_{F^1}(x) = \exp[-(x+1)^2], \\ &\mu_{F^2}(x) = \exp[-(x+0.5)^2], \\ &\mu_{F^3}(x_i) = \exp[-(x)^2], \\ &\mu_{F^4}(x) = \exp[-(x-0.5)^2], \\ &\mu_{F^5}(x) = \exp[-(x-1)^2]. \end{aligned}$$

Therefore we can further obtain the controller as Eq. (13) and the adaptive law is chosen as

$$\begin{aligned} &\dot{\widehat{a}} = -2ex, \qquad \dot{\eta}_0 = 2|e|, \\ &\dot{\theta} = \xi_1(x(t-d))e, \qquad \dot{\vartheta} = -0.5\xi_2(y)e. \end{aligned} \tag{25}$$

We can see that the above controller is not related to a, $F(x(t-d))$ and $G(y)$. When they are changed, the controller is also successful in synchronization. Here we test the effectiveness of our main results when a, $F(x(t-d))$ are changed and $G(y)$ is unknown. For simulation, we assume $G(y) = \cos(y)$. In the following, we investigate the synchronization simulations of the Mackey–Glass system and the logistic system with different parameters, but the controller is not changed.

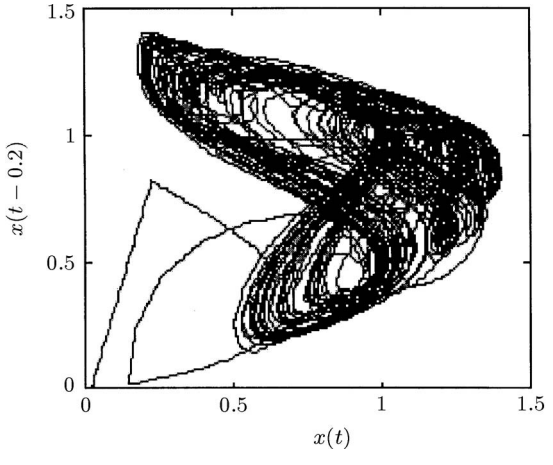


Fig. 1. Chaotic behaviour of the Mackey–Glass system.

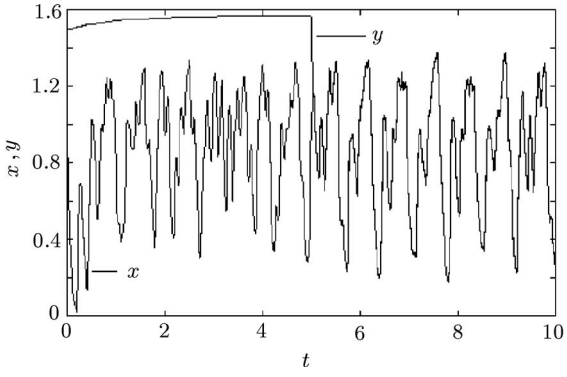


Fig. 2. Synchronization result of the Mackey–Glass system.

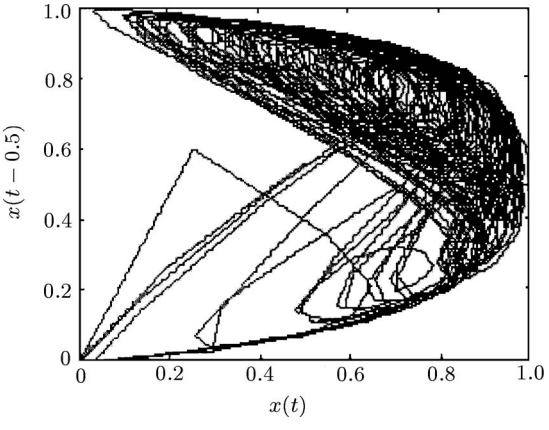


Fig. 3. Chaotic behaviour of the logistic system.

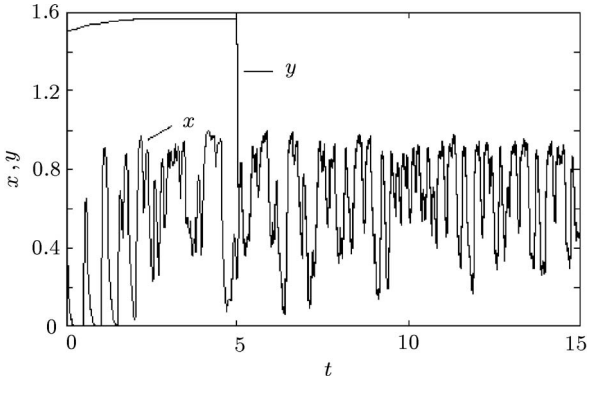


Fig. 4. Synchronization result of the logistic system.

1. Mackey–Glass time-delay chaotic system.

When $k_1 = 20$, $k_2 = 40$, $k_3 = 10$ and $\tau = 0.2$, the system is chaotic.[11] The chaotic behaviour is depicted in Fig. 1, in which the sample time $T = 0.01$. Using controller (13) with adaptive law (25), we have the following simulation result. The controller is not added into the system at the beginning, and we use the controller in 5 min. The simulation result is given in Fig. 2. From Fig. 2 we can see that the response system synchronizes the driving system quickly.

2. Logistic time delay chaotic system.

For system (2), when $\lambda = 26$, $\gamma = 104$ and $d = 0.5$, the system is chaotic.[11] The chaotic behaviour is shown in Fig. 3, in which sample time $T = 0.002$. The synchronization simulation result is given in Fig. 4.

When the controller is used in 5 s, we can see that state y of the response system quickly tracks state x of the driving system.

From the simulation results, we can see that whatever the structures and parameters of a and $F(x(t-d))$ are, the controller (13) with the corresponding adaptive law (25) is successful in the synchronization.

In conclusion, we have obtained that time-delay chaotic systems are high-dimensional and have many positive Lyapunov exponents, which present high safety in applications of secure communication. Also, we have investigated the synchronization problem of a time-delay chaotic system. Adopting the adaptive fuzzy control method, we use fuzzy logic systems to approximate time-delay continuous nonlinear functions, so that the precise mathematical models of systems need not be known. The controllers are constructed to render the closed-loop error system asymptotically stable. We carry out simulations of synchronizing Mackey–Glass and logistic systems, and the results are reasonable.

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