CHIN.PHYS.LETT. Vol. 19, No. 6 (2002) 832

Influence of Combination of Casimir Force and Residual Stress on the Behaviour of Micro- and Nano-Electromechanical Systems *

ZHENG Mao-Sheng(郑茂盛), ZHOU Gen-Shu(周根树), ZHAO Wen-Zhen(赵文轸), GU Hai-Cheng(顾海澄)
State Key Laboratory for Mechanical Behavior of Materials, Xi'an Jiaotong University, Xi'an 710049

(Received 14 January 2002)

Casimir force and residual stresses actually appear in over-layers or films simultaneously. The study of the behaviour of micro- and nano-electromechanical systems in the presence of Casimir force and residual stress is of significance to the design of the relevant devices. We derive analytical expressions of the deflection of a bridge-shaped device under the mutual actions of Casimir force and residual stress in films. It is shown that the tensile residual stress enhances wavy behaviour of the deflection, while the compressive residual stress increases the deflection value and reduces the wavy behaviour.

PACS: 62. 20. −x, 62. 20. Dc, 62. 20. Fe, 62. 20. Mk, 83. 50. −v

The Casimir force is one of the important predictions of modern quantum electrodynamics (QED), which was found by Casimir in 1948.[1] The theory shows that two closely spaced parallel conducting plates will be mutually attracted due to the Casimir force between them. The resource of this force is the quantum vacuum fluctuations of the electromagnetic field.

According to quantum theory, the vacuum contains virtual particles that are in a continuous fluctuation state. The magnitude of Casimir force per unit surface area S is

$$\frac{F(a)}{S} = \frac{\pi^2}{240}\frac{\hbar c}{a^4} = 0.016\frac{1}{a^4}\,(\mu\mathrm{m})^4\,\mathrm{dyn/cm^2}, \qquad (1)$$

where a (μm) is the distance between the plates, c is the light speed, and $\hbar$ is the Planck constant. Equation (1) shows that Casimir force varies proportionally to the inverse fourth power of the separation between the plates, and it will increase rapidly in submicrometre and nanometre regimes.

Nowadays, many devices are fabricated to be smaller and smaller in order to reduce their weight and volume, which, in turn, inevitably induces the Casimir force to affect their performances. As a consequence, when a 2μm thick highly conductive mono-crystalline silicon membrane (with a doping density greater than $10^{20}\mathrm{cm}^{-3}$) is only $0.4\,\mu$m away from a parallel metal plate, the Casimir force will approximately equal the weight of the membrane.[1]

In actual micro-laminate devices, besides the Casimir force, there are residual stresses in the film due to the mismatches of thermal expansion coefficient and the arrangement of atoms, as well as electronic parameters of the component materials.[2] The mismatching of atom arrangement along the interface of two materials has been observed by means of high-resolution electron microscopy (HREM). Modern electron theory shows that electrons immigrate across the interface because of the differences of electron densities and Fermi energy levels of the two materials. These two kinds of mismatching will inevitably induce residual stresses in the relevant materials. In addition, due to the differences of thermal expansion coefficients between the film and the substrate, residual stress forms at heating and cooling during device operation or from its fabrication procedures.

It has been found that over-large tensile residual stress could make the film break and over-large compressive residual stress results in buckling. Both breaking and buckling will plague many micro-/nano-electromechanical systems (MEMS/NEMS). The measured results show that residual stresses can even reach a few GPa in some film/substrate systems.[3,4] Some theoretical evaluations have estimated the internal stress in films, and a remarkable value of stress was also predicted.[5] Suo and Zhang[6] and Zhang *et al.*[7] studied the effect of the sole action of long-range forces on the stability of over-layers on semiconductor substrates.

The above outlines indicate that residual stresses and the Casimir force appear actually in over-layers or films simultaneously. Therefore, the consequent question is how the combination of the Casimir force and the residual stress affects the behaviour of MEMS/NEMS, e.g., this force induces failure of MEMS/NEMS or results in a new rule to the design of this kind of device. In order to clarify this question, the theory of plate and shell is employed to study the deflection behaviour of a micro-strip or bridge under the combined actions of Casimir force and residual stresses in the following sections.

From the theory of plate and shell, under the condition of small deflection the differential equation for a deflected plate is

$$D\nabla^4 W + N\nabla^2 W = q, \qquad (2)$$

where W is the deflection of the plate, $D = Eh^3/[12(1-\nu^2)]$ is the flexural rigidity, N is the force

*Supported by the National Natural Science Foundation of China under Grant No 59831040.
©2002 Chinese Physical Society and IOP Publishing Ltd

on the central plane of the plate, q is the load applied to the plate, and E, h and ν indicate the elastic modulus, thickness and Poisson ratio of the plate, respectively.

For a rather rigid plate, the central plate force N is negligible, thus Eq. (2) reduces to

$$D\nabla^4 W = q. \tag{3}$$

In order to investigate the deflection of a strip conveniently, a rectangular coordinate system is employed (see Fig. 1). The origin of the coordinate system coincides with the centre point of the strip, and the x-axis is along the direction of the strip length.

In general, the Casimir force between two flat plates, separated by distance a, is described by Eq. (1) theoretically. However, due to the action of the Casimir force, the strip will deflect to some amount, especially for a very thin strip such as a membrane. As a result the plate will obtain an inducing deflection $W(x)$, which in turn may inevitably change the Casimir force acting on it (see Fig. 1).

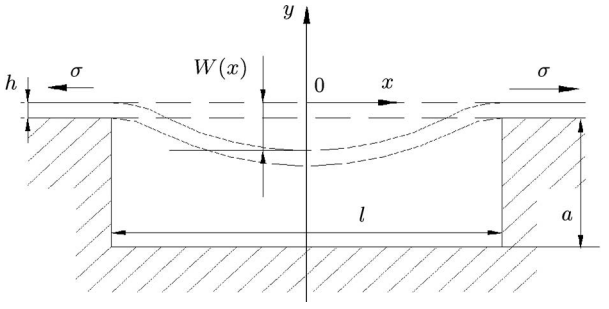

Fig. 1. Cross section of a MEMS/NEMS bridge device.

Finally, one assumes that the Casimir force becomes inhomogeneous on the whole strip and varies from point to point due to the variation of the deflection along the strip. Thus we have

$$q = -\frac{\pi^2}{240}\frac{\hbar c}{(a + W(x))^4},$$

which is the so-called local value approach proposed by Serry *et al.*[1] Consequently, the Casimir force leads to a positive feedback. By considering small deflection condition only, the Casimir force can be further approximated as

$$q = -\left[\frac{\pi^2}{240}\frac{\hbar c}{a^4} - \frac{\pi^2}{60}\frac{\hbar c}{a^5}W(x)\right].$$

Thus, Eq. (3) becomes

$$D\frac{\mathrm{d}^4 W(x)}{\mathrm{d}x^4} = -\left[\frac{\pi^2}{240}\frac{\hbar c}{a^4} - \frac{\pi^2}{60}\frac{\hbar c}{a^5}W(x)\right], \tag{4}$$

$$\frac{\mathrm{d}^4 W(x)}{\mathrm{d}x^4} - \omega^4 W(x) = -\beta, \tag{5}$$

where

$$\omega^4 = \frac{\pi^2}{60}\frac{\hbar c}{Da^5}, \quad \beta = \frac{\pi^2}{240}\frac{\hbar c}{Da^4}.$$

Furthermore, when residual stress σ appears in the film, Eq.(5) becomes

$$\frac{\mathrm{d}^4 W(x)}{\mathrm{d}x^4} + \alpha\frac{\mathrm{d}^2 W(x)}{\mathrm{d}x^2} - \omega^4 W(x) = -\beta, \tag{6}$$

where $\alpha = \sigma h/D$.

Under the residual stress free condition, the governing equation is Eq. (5).

We consider a bridge-shaped MEMS/NEMS (see Fig. 1). We let l express the length of the strip, and both ends of the strip are clamped. The general solution for Eq. (5) is

$$W(x) = A\cos(\omega x) + Be^{(\omega x)} + Be^{(-\omega x)} + \beta/\omega^4. \tag{7}$$

Applying the boundary conditions for the clamped strip, i.e., $W(x)|_{x=\pm l/2} = 0$ and $\mathrm{d}W(x)/\mathrm{d}x|_{x=\pm l/2} = 0$, we obtain

$$A = -\frac{\beta}{\omega^4}\frac{1}{\sin\dfrac{\omega l}{2}(\operatorname{ctan}\dfrac{\omega l}{2} + \coth\dfrac{\omega l}{2})},$$

$$B = -\frac{\beta}{2\omega^4}\frac{1}{\sinh\dfrac{\omega l}{2}(\operatorname{ctan}\dfrac{\omega l}{2} + \coth\dfrac{\omega l}{2})}. \tag{8}$$

In Eq. (7), the first term related to parameter A decays with the coordinate x while the second and third terms related to B increase with respect to coordinate x. Thus Eq. (7) may reflect an oscillatory shaped deflection.

It is apparent from Eq. (7) that the value for the deflection at the centre point $x = 0$ is $W(0) = A + 2B + \beta/\omega^4$. Although the first derivative of function $W(x)$ in Eq.(7) with respect to x at the centre point $x = 0$ always remains zero. The sign of the second derivative of this function with respect to x at the same point, $2B - A$, is undefined. Obviously it depends on the exact values of both parameters A and B. If $|A| > 2|B|$, the centre point $x = 0$ gives the only maximum deflection. There is no oscillation on the deflected film, otherwise two symmetrical maximum deflections occur at both sides of the centre point $x = 0$, which indicates an oscillation appearing on the deflected film.

Since the governing equation is rather complicated, the deflection behaviour varies significantly with the controlling factor $\omega l/2$. It is found that, under the residual stress free condition, the absolute value of parameter A is slightly greater than that of parameter B, and thus the centre point gives a maximum deflection in this case. If $\omega l/2$ is no greater than *0.5*, both parameters A and $2B$ are not far from $-0.5\beta/\omega^4$, and the absolute value of the maximum deflection is less than

$0.0007a$. However, as $\omega l/2$ reaches 0.6 and 1.5708, the deviations of both parameters A and $2B$ increase from $-0.5\beta/\omega^4$. This leads to the corresponding absolute values of the maximum deflection approaching $0.0014a$ and $0.08a$, respectively, where a indicates the original distance between the two flat plates.

When residual stress appears, one has to solve Eq. (6) to obtain the solutions. The general solution is

$$W(x) = A\cos(\omega_A x) + B\mathrm{e}^{(\omega_B x)} + B\mathrm{e}^{(-\omega_B x)} + \beta/\omega^4, \tag{9}$$

where

$$\omega_A = \sqrt{\frac{\alpha + \sqrt{\alpha^2 + 4\omega^4}}{2}}, \omega_B = \sqrt{\frac{-\alpha + \sqrt{\alpha^2 + 4\omega^4}}{2}}, \tag{10}$$

$$A = -\frac{\beta\omega_B}{\omega^4\omega_A}\Big[\sin\frac{\omega_A l}{2}\Big(\frac{\omega_B}{\omega_A}\mathrm{ctg}\frac{\omega_A l}{2} + \coth\frac{\omega_B l}{2}\Big)\Big]^{-1}, \tag{11}$$

$$B = -\frac{\beta}{2\omega^4}\Big[\sinh\frac{\omega_B l}{2}\Big(\frac{\omega_B}{\omega_A}\mathrm{ctan}\frac{\omega_A l}{2} + \coth\frac{\omega_B l}{2}\Big)\Big]^{-1}. \tag{12}$$

Again, from Eq. (9), it can be seen that the deflection at the centre point $x = 0$ is $W(0) = A+2B+\beta/\omega^4$. The only difference from that in the last paragraph shows that at this time the residual stress may affect the behaviour of the deflected film.

For very small residual stress, either tensile or compressive, the deviations of ω_A and ω_B from ω are also small, so the influence of residual stress on the deflection behaviour is not sensitive.

However, when residual stress is rather great, e.g., $|\alpha| \approx \omega^2$, there is quite a difference and it varies with the residual stress state. For the tensile residual stress, the coefficient ω_A increases to $1.2720\,\omega$, while coefficient ω_B decreases to $0.7862\,\omega$. Taking $\omega l/2$ equal to 0.5 as an example, parameters A and $2B$ become $-0.29640\beta/\omega^4$ and $-0.70628\beta/\omega^4$, respectively. As a consequence, oscillatory behaviour occurs on the deflected film though the value of the deflection at the centre point $x = 0$ remains almost the same as that in the residual stress free condition. Inversely, for the compressive residual stress state, ω_A and ω_B become $0.7862\,\omega$ and $1.2720\,\omega$, respectively. Simultaneously, when $\omega l/2$ equals 0.5, parameters A and 2B become $-0.74356\beta/\omega^4$ and $-0.25898\beta/\omega^4$, respectively. As a consequence, no wavy behaviour appears on the deflected film though the maximum value of the deflection increases slightly with respect to that under the residual stress free condition.

Furthermore, when the residual stress is much greater, i.e. $|\alpha| \gg \omega^2$, the behaviour becomes more interesting. For the tensile stress, ω_A and ω_B become $\sqrt{\alpha}\left(1 + \omega^4/2\alpha^2\right)$ and $\omega^2/\sqrt{\alpha}$, respectively. For example, letting $\alpha = 10\,\omega^2$, the values for ω_A and ω_B reach $3.17790\,\omega$ and $0.31623\,\omega$, respectively. If $\omega l/2$ equals 0.5, the values of parameters A and $2B$ become $-0.01561\beta/\omega^4$ and $-0.98791\beta/\omega^4$, respectively; the absolute value of the deflection at the central point $(x = 0)$ is $0.0009a$. It is obvious that the wavy behaviour of the deflected film is enhanced apparently though the value of the deflection at the centre point increases slightly. Inversely, for compressive residual stress, ω_A and ω_B become $\omega^2\sqrt{|\alpha|}/|\alpha|$ and $\sqrt{|\alpha|}\left(1 + \frac{\omega^4}{2\alpha^2}\right)$, respectively. We assume that the value $|\alpha|$ equals $10\omega^2$ again, and the values of ω_A and ω_B thus approach $0.31623\,\omega$ and $3.17790\,\omega$, respectively. Letting $\omega l/2$ equal 0.5 once more, the values of parameters A and 2B become $-1.01621\beta/\omega^4$ and $-0.01340\beta/\omega^4$, respectively. This indicates that there is no wavy behaviour of the deflected film, whereas the deflection is enhanced significantly with the absolute value of the maximum deflection reaching $0.0074a$. This may also result in the failure of MEMS/NEMS to varying degrees.

It can be seen from the above analysis that the tensile residual stress in films stretches the bridge and thus promotes the wavy behaviour of the deflected films. The compressive residual stress enhances the deflection of bridge and reduces its wavy behaviour.

Since residual stresses and the Casimir force appearing in the film of MEMS/NEMS depend on the material parameters of the components and geometry, one should pay attention to the design of MEMS/NEMS to avoid failure. In addition, the operation process must also be included in the MEMS/NEMS design due to the thermal circling residual stress.

Owing to the positive feedback property of the Casimir force, this force could induce failure of MEMS and especially NEMS, and residual stresses influence the deflection behaviour of MEMS/NEMS together with the Casimir force. Thus, attention must be paid when designing bridge-shaped MEMS/NEMS.

References

[1] Serry F M, Walliser D and Maclay G J 1998 *J. Appl. Phys.* **84** 2501

[2] Thompson C V and Carel R 1996 *J. Mech. Phys. Solids* **44** 657

[3] Chiou S Y and Hwang B 1998 *J. Phys. D: Appl. Phys.* **31** 349

[4] Yu L, Bai C, Chen H, Xu K and He J 1993 *Acta Metall. Sin.* B **6** 393

[5] Cheng K J and Cheng S Y 1998 *Prog. Nat. Sci.* **8** 20

[6] Suo Z and Zhang Z 1998 *Phys. Rev.* **58** 5116

[7] Zhang Z, Niu Q and Shih C 1998 *Phys. Rev. Lett.* **80** 5381