Responses of a Noisy Excitable System to External Signals with Different Periods*

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The behaviour of an excitable system under Gaussian white noise and external periodic forcing is systematically studied. In a large range of noise intensity, the n:1 phase locking patterns are obtained for certain ranges of the input periods, where n input periods give one spike. In the phase locking regimes, the system presents low noise-to-signal ratios and shows better regularities. Out of the regimes the system behaves less regularly and the relations between the noise-to-signal ratio and the noise intensity exhibit typical stochastic resonance phenomena. At a higher noise level, the system shows the characteristic behaviour of the noise.

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Excitable dynamics underlie the behaviour of many systems ranging from kinetics of chemical reactions to biological processes.[1] In an excitable system, an external periodic forcing can dramatically change its behaviour. The study of the response of excitable systems to external periodic forcing has been well documented. The phase locking, quasi-periodicity, period doubling, and chaos were observed.[2] Noise is another important factor in the dynamical behaviour of excitable systems. It is found to induce characteristic oscillations without any periodic forcing. The concept of coherence resonance was proposed to explain this phenomenon.[3−5]

The behaviour of an excitable system in the presence of both noise and subthreshold external forcing has attracted much attention because of the interests in biological nerve cells. In the line of these researches, periodic signals[6,7] or a periodic input[8−10] were used as the external stimuli. The studies showed that the responses of the excitable systems present two characteristic time scales, due to noise and external signal respectively. Theoretical analysis and simulations were first focused on Stochastic resonance phenomena with low frequency signals, and the investigation of the static threshold properties, which are independent of signal frequency, were carried out. Later, investigations were shifted to the effect of high frequency periodic signals on excitable systems, and the resonance phenomena of frequency-dependence are studied.[10,11] However, the study of the dynamical behaviour of excitable systems with signal frequency and noise intensity is still not complete. For example, how the system responses to noise in and out of a phase locking regime is unclear. In this Letter, we show that at low noise intensity, the time scale of the signal frequency dominates the behaviour of the system. In this case, there exist phase locking phenomena for certain frequency ranges, where the system presents the n:1 phase locking patterns and the noise-to-signal ratio is relatively low. Between these regimes, the noise-to-signal ratio is quite high and the different behaviour of the system can be observed. At higher noise intensity, noise is more important and determines the system behaviour.

The Fitz–Hugh–Nagumo (FHN) model is a simple but representative sample of excitable systems. The equations we used is similar to Ref. [3], except that a periodic signal is added:

$$\varepsilon\frac{dx}{dt} = x - \frac{x^3}{3} - y, \tag{1}$$

$$\frac{dy}{dt} = x + a + D\xi(t) + F\sin\left(\frac{2\pi}{T_{\rm in}}t\right), \tag{2}$$

where $x(t)$ and $y(t)$ are the fast and slow (recovery) variables respectively; $\varepsilon \ll 1$ is the time scale ratio of these two variables. The parameter a is chosen to be 1.05, as the same as in Ref. [3]. The parameter D governs the amplitude of the noisy external force $\xi(t)$, which is Gaussian white noise of zero mean ($\langle\xi(t)\rangle = 0$) and no time correlation ($\langle\xi(t)\xi(t')\rangle = \delta(t-t')$). The parameter F represents the strength of external periodic forcing, and $T_{\rm in}$ is the period of external signal. Numerical simulation indicates that, without noise, namely $D = 0$, the system has a threshold $F = 0.061$. Under the threshold the system just gives small oscillation around the stable fixed point; beyond the threshold the output will show a large excursion. Therefore, we choose $F = 0.058$ to put the system in a sub-excitable regime without noise.

We integrate the systems (1) and (2) numerically using Euler's method[12] for the parameter $\varepsilon = 0.001$,

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with time step $\Delta t = 5 \times 10^{-4}$. D and $T_{\rm in}$ are the control parameters.

In the presence of both periodic signal and noise, the system gives quasi-periodic spikes. Figure 1 shows three examples of such output signals for different periods of the external forcing. We can easily obtain the interspike interval sequence $T_{{\rm out},i}$, which is defined as the time between ith and $(i+1)$th pulse, and the noise-to-signal ratio R of the this sequence is defined as in Ref. [3],

$$R = \frac{\sqrt{\langle T_{\rm out}^2 \rangle - \langle T_{\rm out} \rangle^2}}{\langle T_{\rm out} \rangle}. \qquad (3)$$

Obviously, a low value of R means better periodicity of the sequence.

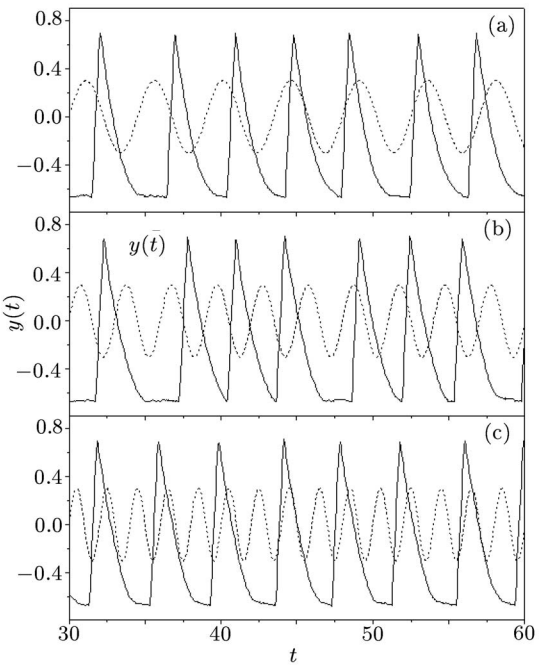

Fig. 1. Outputs of the system (solid lines) as a function of time for a low noise intensity $D = 0.014$. For comparison, the input signals of the external periodic forcing ($F = 0.3$) are plotted with the dashed lines. The periods of the external forcing are taken: (a) $T_{\rm in} = 4.0$, (b) $T_{\rm in} = 2.4$, (c) $T_{\rm in} = 2.0$.

First, we study the relations between the noise-to-signal ratio R, the mean period $\langle T_{\rm out} \rangle$ and noise intensity D. The results are summarized in Fig. 2.

One can see from Fig. 2(a) that at higher noise intensity ($D > 0.07$) the response of the system to noise intensity is independent of the external forcing period, indicating that noise dominates the behaviour of the system. Contrarily, at lower noise intensity ($D = 0 \sim 0.04$), the system behaves differently with different external forcing periods. For some values of the period, e.g. $T = 4.0$ and $T = 2.0$, the outputs of the system have very low noise-to-signal ratios and keep better regularities. For other values of the period, e.g. $T = 2.8$, the system exhibits worse regularity. Figure 2(b) shows the mean period of these spikes ($\langle T_{\rm out} \rangle$) as a function of the noise intensity D. One observes that for $T = 4.0$ or $T = 2.0$, there is a flat stage where the noise-to-signal ratio R changes slightly. The system is in the $1:1$ and $2:1$ phase locking states respectively, as demonstrated in Figs. 1(a) and 1(c), respectively. When the system is out of the phase locking regime, as shown in Fig. 1(b), the flat stage disappears (Fig. 2(b), $T = 2.8$). One can notice from Fig. 2 that when the noise intensity is in the phase locking range, the value of noise-to-signal ratio is near the minimum. We thus conclude that in the phase locking regimes, the system shows better regularity, but out of the regimes the system behaves less regularly.

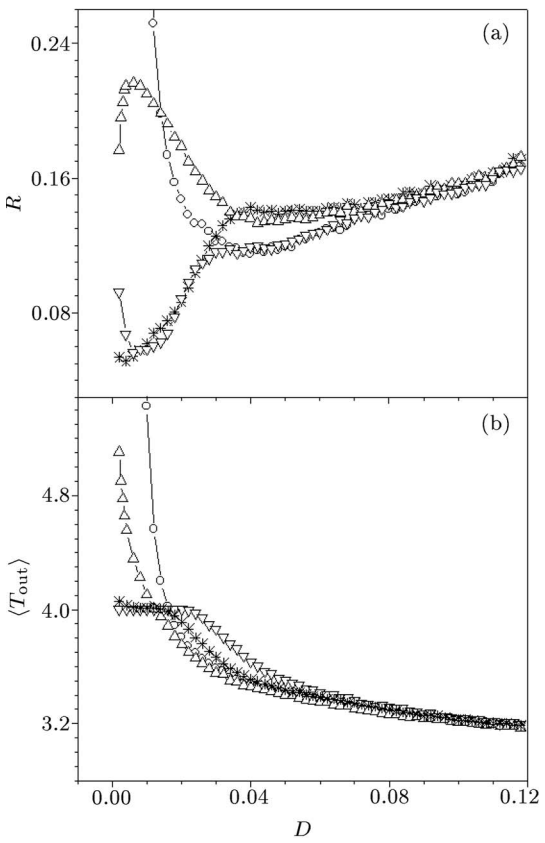

Fig. 2. Response of the system as a function of noise intensity with different input period $T_{\rm in}$. Circles: without periodic forcing $F = 0$. Stars: $T_{\rm in} = 4.0$. Triangles: $T_{\rm in} = 2.8$. Diamonds: $T_{\rm in} = 2.0$. (a) The relation between R and D, (b) the relation between $\langle T_{\rm out} \rangle$ and D.

Next, we set $D = 0.014$ and study the phase locking and unlocking behaviour of the system with different input period $T_{\rm in}$. Figure 3 shows the mean period $\langle T_{\rm out} \rangle$ and the most probable period $T_{\text{out-max}}$ of the spike sequences as functions of the input period $T_{\rm in}$.

One observes that with the increase of input period, the response of the noisy excitable system to the

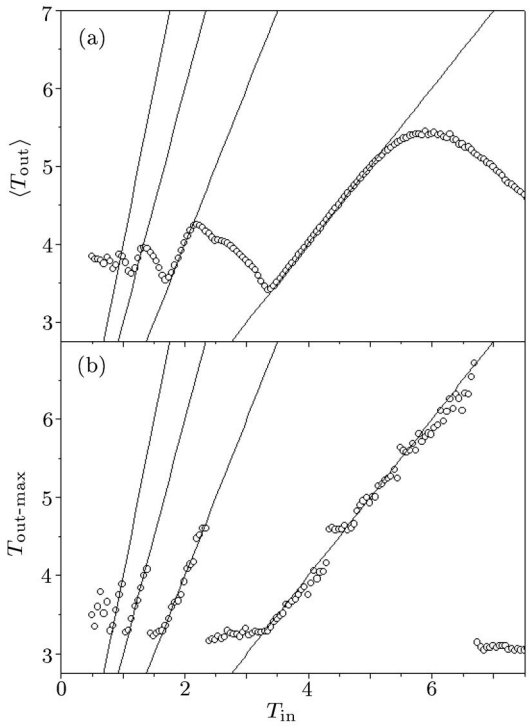

Fig. 3. The relations between mean duration $\langle T_{\rm out}\rangle$, most probability duration $T_{\rm out\text{-}max}$ and period of input signal T. The solid lines are the fitting linear function of $nT_{\rm in}$, n = 1, 2, 3, 4.

the external periodic forcing displays an alternation of phase locking and unlocking. In the phase locking regimes, the n:1 phase locking patterns appear, where n input periods give one spike. Out of the phase locking regime, the average period of output signal decreases with the increase of the forcing period, and the response of the system is an intermediate state between the neighboring phase locking patterns. The range of phase locking regime is largest for the 1:1 locking; it decreases as n increases. For larger input periods ($T_{\rm in} > 6.7$), the system is insensitive to the change of the forcing period.

Finally, we study the transition from 1:1 to 2:1 phase locking state. The result of the study is presented in Fig. 4.

When $T_{\rm in} = 4.0$, the interspike interval histogram shows a peak, and the most probable interval of the spikes is coincident with the period of external forcing, so that the system shows the 1:1 phase locking pattern. This phase locking state persists when $T_{\rm in}$ decreases to 3.35. When the input period is further decreased, the value of the peak changes slightly with the change of input period. The 1:1 relation between input and output signal is not hold anymore. We thus enter into an unlocking state. When $T_{\rm in}$ is changed into 2.8, the second peak emerges. As the decrease of $T_{\rm in}$, the first peak shrinks while the second one inflates. At $T_{\rm in} = 2.2$, the first peak is lower than the second one, the system is thus entering into the 2:1 phase locking regime.

We investigate the response of a noisy excitable system to a subthreshold external periodic signal with different periods. Without noise, the system has no large spike. Without the periodic input, the noise-to-signal ratio R has the minimum value at an optimal noise level.[4] However, due to the competition between the noise and the period forcing, the response of the system to the subthreshold external forcing has

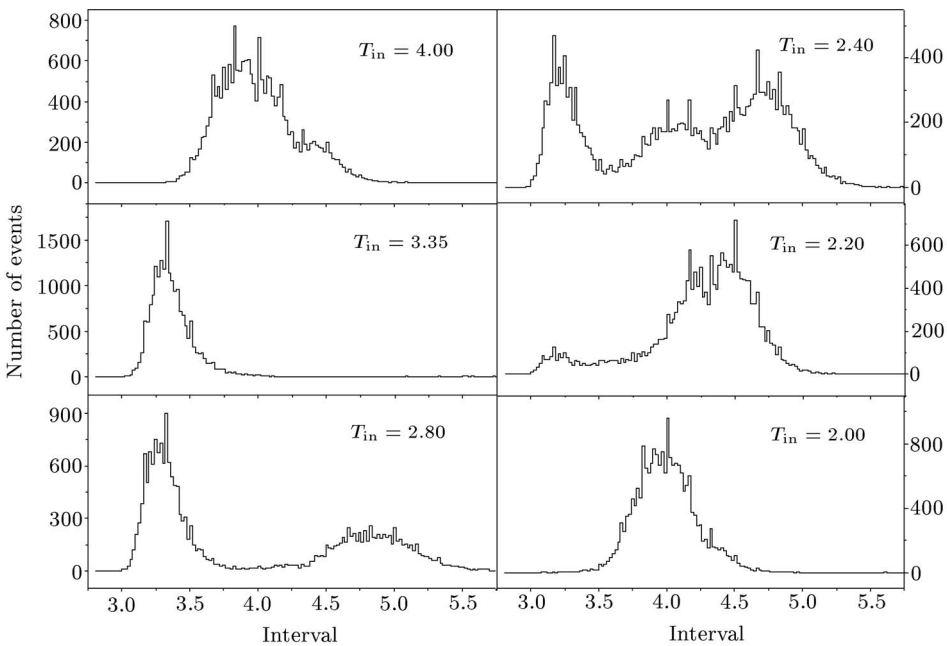

Fig. 4. Interspike interval Histogram for different signal period $T_{\rm in}$.

different behaviour with different input periods. This study shows that at high noise intensity, the system tends to display the characteristic behaviour under noise; while at low noise intensity, the system is highly sensitive to some forcing periods. In this situation, the strong resonances of the system with periodic forcing under the noise exist, giving the n:1 phase locking patterns in certain ranges of forcing period. In the meantime, the system exhibits very low noise-to-signal ratios. While under longer input periods and the periods between the phase locking patterns, the system is in unlocking states, where noise is more important and the system exhibits high noise-to-signal ratios.

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