# Shell Model Analysis of $\Xi^-$ Hypernuclei Spectra for Mass $A = 12$ and $A = 16$ *

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*The excitation spectra for ($^{12}_{\Xi^-}$Be) and ($^{16}_{\Xi^-}$C) are obtained in the frame of a shell model. The experimental values of the ground state binding energies of the $\Xi^-$ hyperon in ($^{12}_{\Xi^-}$Be) and ($^{16}_{\Xi^-}$C) are used to determine the well depth of the $\Xi^-$-nucleus potential. The information on the residual interaction is emphasized. It is found that the residual interaction does not have much effect on the spectra of the $\Xi^-$-hypernucleus.*

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One of the important subjects in hypernuclear physics is to study the spectra of the hypernuclei. The main reason is that the information we obtained from this studying can provide some knowledge about the hyperon(Y)−nucleon(N) interactions. Although a considerable body of information exists in the production and decay properties of the strangeness $S = -1$ hypernucler,[1,2] comparatively little is known about $S = -2\Xi^-$-hypernuclei. Until recent, only a few events were attributed to the formation of the $\Xi^-$-hypernuclei.[3] At the Koh-Ene-Ken(KEK), an active experimental program is underway to study the double strangeness exchange reaction on nuclear targets.[4−8] At the Brookhaven Alternating Gradient Synchrotron (AGS), a more intense separated $1-2\,\mathrm{GeV}/c$ $\mathrm{K}^-$ beam is now available for ($\mathrm{K}^-$, $\mathrm{K}^+$) studies.[9] Its application to more general studies of $S = -2$ spectroscopy is strongly indicated. A missing mass spectrum in the reaction $^{12}$C($\mathrm{K}^-$, $\mathrm{K}^+$)$^{12}_{\Xi^-}$Be has been recently shown from the Brookhaven AGS experiment E885, in which some events are candidates for $\Xi$-hypernculei, but no analysis of these data has been reported yet.[10]

The spectra of the $\Lambda$-hypernuclei and $\Sigma$-hypernuclei have been studied extensively. But little have been done for $\Xi^-$-hypernuclei. In this letter, the spectra of $^{12}$C($\mathrm{K}^-$,$\mathrm{K}^+$)$^{12}_{\Xi^-}$Be and $^{16}$O($\mathrm{K}^-$,$\mathrm{K}^+$)$^{16}_{\Xi^-}$C will be examined in the frame of the shell model with suitable single particle potentials and with a residual interaction. The analysis of the two $\Xi^-$-hypernuclei has been carried out with some treating as did in the case of $\Lambda$-hypernuclei: only one proton in the target nucleus is replaced by a $\Xi^-$ hyperon, while the rest of the nucleus remains unchanged. We describe the ground state of $^{12}$C and $^{16}$O as a closed shell, and the states of $^{12}_{\Xi^-}$Be and $^{16}_{\Xi^-}$C as 1-particle−1-hole excitation built upon this closed shell. Spin flips are excluded, so that the parity of the $\Xi^-$-particle proton-hole state $\left|\left[(n_{\mathrm{N}}l_{\mathrm{N}}j_{\mathrm{N}})^{-1}(n_{\mathrm{Y}}l_{\mathrm{Y}}j_{\mathrm{Y}})\right]\right\rangle^{J^\pi}$ is "natural" with $P = (-1)^{l_{\Xi^-}+l_P}$. For simplicity, we assume that the $\Xi^-$ particle can be in any one of the possible bound orbits and the proton hole in $0p_{3/2}$ orbit for the $^{12}$C case and $0p_{1/2}$, $0p_{3/2}$orbits for the $^{16}$O case. Therefore the configurations for $^{12}_{\Xi^-}$Be are $|(\mathrm{P}\Xi^-)0p^{-1}_{3/2}0s_{1/2}\rangle^{1^-}$, $|(\mathrm{P}\Xi^-)0p^{-1}_{3/2}0p_{3/2}\rangle^{0^+,2^+}$ and $|(\mathrm{P}\Xi^-)0p^{-1}_{3/2}0p_{1/2}\rangle^{2^+}$. The configurations for $^{16}_{\Xi^-}$C are similar. The Hamiltonian is

$$H = H^0_{\Xi^-} + H^0_{\mathrm{N}} + V_{\Xi^-\mathrm{N}}(\boldsymbol{r}_1 - \boldsymbol{r}_2), \tag{1}$$

where $H^0_{\Xi^-}$ and $H^0_{\mathrm{N}}$ are the single-particle Hamiltonian for $\Xi^-$ hyperon and nucleons, respectively.

$$H^0_{\Xi^-} = \frac{p^2_{\Xi^-}}{2M_{\Xi^-}} + U^{\Xi^-}, \tag{2a}$$

$$H^0_{\mathrm{N}} = \sum_i \left[\frac{p^2_i}{2M_i} + U^N(i)\right], \tag{2b}$$

and $V_{\Xi^-\mathrm{N}}(\boldsymbol{r}_1-\boldsymbol{r}_2)$ is a residual interaction. The single-particle potentials for N and $\Xi^-$ we adopted are in the Woods-Saxon(W-S) form

$$U_B = V^B_0 f(r) + V^B_{\mathrm{LS}}\left(\frac{\hbar}{m_\pi c}\right)^2(\boldsymbol{l}\cdot\boldsymbol{s})\frac{\mathrm{d}f(r)}{r\mathrm{d}r} + U_{\mathrm{coul}}(r), \tag{3}$$

where the index stands for N and $\Xi$, and the radial form factor reads

$$f(r) = \frac{1}{1+\exp[(r-R)/a]}, \quad R = r_0(A-1)^{1/3}. \tag{4}$$

For the nucleon, the values of the centeral and spin-orbit well depths are chosen as $V^{\mathrm{N}}_0 = -54\,\mathrm{MeV}$, $V^{\mathrm{N}}_{LS} = 7\,\mathrm{MeV}$ ($r_0 = 1.25\,\mathrm{fm}$, $a = 0.65\,\mathrm{fm}$), which can reproduce adequately the $0p$-state proton hole energies $E_{0p}$ known experimentally. The well depth for the $\Xi^-$ potential $V^{\Xi^-}_0$ is treated as a free parameter and will be determined from a comparison of the calculated ground state binding energies of the $\Xi^-$ hyperon in $^{12}_{\Xi^-}$Be and $^{16}_{\Xi^-}$C with the experimental data. The spin-orbit coupling in the $\Xi^-$-nucleus interaction can not be determined uniquely, since no experimental data are available. Using the model D of the Nijmegen group,[11] Dover and Gal predicted that $V^{\Xi^-}_{\mathrm{LS}} = 1$ MeV, so we adopt $V^{\Xi^-}_{\mathrm{LS}} = 1$ MeV ($r_0 = 1.1$ fm, $a = 0.65$ fm) in this work. As for the residual interaction, it seems premature for us to use a more complicated form. Therefore, for simplicity, we choose the residual interaction with only one parameter $\alpha$:

$$V_{\Xi^-\mathrm{N}}(\boldsymbol{r}_1 - \boldsymbol{r}_2) = -V_0\delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)(1 + \alpha\sigma_1\cdot\sigma_2). \tag{5}$$

For internal consistency of the shell model, the overall strength $V_0$ must be related to $V_{\Xi^-}$:

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Table 1. Single-particle energies of $\Xi^-$ and proton calculated for ${}^{12}_{\Xi^-}$Be and ${}^{16}_{\Xi^-}$C systems. $V_0^B$ ($B = \Xi^-$,N) is the potential depth in units of MeV and $E_{nl}(l = s$ or $p)$ is the energy with respect to the corresponding threshold.

| | ${}^{12}_{\Xi^-}$Be | | | | ${}^{16}_{\Xi^-}$C | | | |
|---|---|---|---|---|---|---|---|---|
| | $V_0^B$ | $E_{0s_{1/2}}$ | $E_{0p_{3/2}}$ | $E_{0p_{1/2}}$ | $V_0^B$ | $E_{0s_{1/2}}$ | $E_{0p_{3/2}}$ | $E_{0p_{1/2}}$ |
| $\Xi^-$ | −24 | −12.49 | −2.79 | −2.08 | −24 | −14.97 | −5.37 | −4.68 |
| P | −54 | | −11.72 | | −54 | | −14.94 | −10.66 |

$$V_0 = \frac{V_0^{\Xi^-}}{A} \int \mathrm{d}^3\boldsymbol{r} f(r) \qquad (6)$$

since $V_{\Xi^-\mathrm{N}}$ folded with the nuclear density should reproduce the average potential (3), at least its central part.[12] The spin dependence of $V_{\Xi^-\mathrm{N}}$ characterized by the parameter $\alpha$ is not established. Referring to the $\Lambda$ hypernuclei, a very weak spin dependence seems reasonable. Therefore, for $\Xi^-$, to see the effect of it on the final spectra, we chose $\alpha = 0, -0.05, -0.15, -0.5$.

The single-particle energies and wavefunction for the $\Xi^-$-particle and proton-hole are obtained by solving the Schrödinger equations with Hamiltonians (2a and 2b) numerically. For the wave function the harmonic oscillator (HO) wave function is employed as

$$\phi_{nlj}^{\mathrm{W-S}}(\boldsymbol{r}) = \sum_m a_m \phi_{mlj}^{\mathrm{HO}}(\boldsymbol{r}). \qquad (7)$$

In comparison with the experimental ground state binding energies of the $\Xi^-$ hyperon in ${}^{12}_{\Xi^-}$Be and ${}^{16}_{\Xi^-}$C we found that a reasonable value for the well depth of the $\Xi^-$-nucleus potential is −24 MeV, which is in agreement with the results of Dover and Gal.[13] In Ref. 14, the well depth of the $\Xi^-$ potential was determined as −23.9 MeV, also close to the present result. So, we choose $V_0^{\Xi} = -24$ MeV in our following discussion. The single-particle energies for both $\Xi^-$ and proton we got are listed in Table 1.

By diagonalizing the residual interaction (5) in the $\Xi^-$-particle proton-hole basis ($\Xi^-\mathrm{P}^{-1}$), the excitation spectra and the wavefunctions are obtained for both ${}^{12}_{\Xi^-}$Be and ${}^{16}_{\Xi^-}$C. As an example, the spectra for ${}^{12}_{\Xi^-}$Be with $\alpha$ changing from 0 to 0.5 MeV are shown in Fig. 1. The results for ${}^{16}_{\Xi^-}$C are similar. It is seen that the energies of hypernucleus experience a downward shift with $\alpha$, but the speed with which they decrease is very slow, which indicates that unlike the NN case, a rather weak spin-dependence is indicated for the $\Xi^-$-hypernuclei. We also found that the spectra consist of two well-separated groups of states: the ground $1^-$ state, composed of $|(\Xi^-\mathrm{P})0s_{1/2}0p_{3/2}^{-1}\rangle^{1^-}$ configuration, and three states at about 10 MeV above the ground state mainly of the $\Xi^- p_{3/2}^{-1}$ configurations.

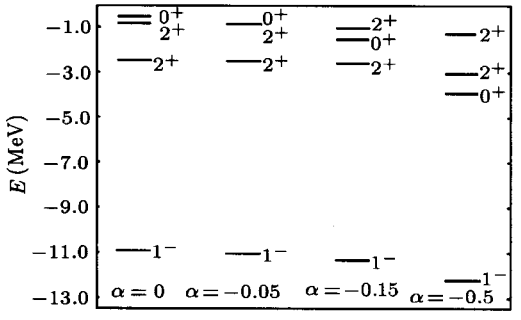



**Fig. 1.** Excitation spectrum of ${}^{12}_{\Xi^-}$Be-hypernucleus.

Our calculation shows that the admixtures in the $2^+$ states induced by the residual interaction are very small, they are basically (more than 0.95 in amplitude) $|(\Xi^-\mathrm{P})0p_{3/2}0p_{3/2}^{-1}\rangle^{2^+}$ and $|(\Xi^-\mathrm{P})0p_{1/2}0p_{3/2}^{-1}\rangle^{2^+}$ configurations, respectively. The positions of the excitation states are close to energies of the unperturbed $\Xi^-\mathrm{P}^{-1}$ states, that is to say, the results are not very sensitive to the residual interaction. The reason for the weak dependence on the residual interaction is because the $\Xi^-$(particle)-proton(hole) matrix elements of the residual interaction are typically of the order of 1 MeV or less, smaller on the average than the corresponding NN force. For light hypernuclei, since the model space is composed of only a few given $\Xi^-\mathrm{P}^{-1}$ states, the $V_0^{\Xi^-}$ cannot induce significant configuration mixing, the diagonalized wave functions should be dominated by a single unperturbed $\Xi^-\mathrm{P}^{-1}$ configurations, and the positions of the diagonalized $\Xi^-\mathrm{P}^{-1}$ states should remain within one or two MeV of the unperturbed energies. Since the $\Xi^-\mathrm{P}^{-1}$ states of a given $J$ are only weakly admixed by the residual interaction (at least for the light closed shell hypernuclei), it will enable one to assign the experimental data to the dominant $\Xi^-\mathrm{P}^{-1}$ configurations. However, the AGS E885 experiment at present stage only conforms some events for $\Xi$-hypernuclei,[10] one could not identify discrete $\Xi$-hypernuclear states yet, before the data are improved. So, we have to compare our results with theoretical ${}^{12}\mathrm{C}(\mathrm{K}^-, \mathrm{K}^+){}^{12}_{\Xi^-}$Be double-differential cross sections of Ref. 15. Basically the excitation spectra we obtained agree with the positions of cross section peaks of Ref. 15. Finally, it is worth to note that, since a $\Xi^-$-particle can interact with a proton to form two $\Lambda$-particles, the admixtures of $\Lambda\Lambda$ hypernculear states into a $\Xi$ hypernculear state may take place. The influence of such state-mixing on the level position and width may be complicated and it remains to be further studied.

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