Shape Evolution of the Compound Nucleus in the Superheavy Element Synthesis Reaction via the BUU Model *

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By taking the BUU model, we simulate the superheavy element synthesis reaction. With the rotation effect being included in the BUU model, the effect of the non-centrality of the reaction $^{48}Ca + {}^{238}U \to {}^{286}112$ is studied. It is shown that the promising impact parameter in the synthesis process can be released from zero to a value little smaller than the radius of the smaller nucleus involved in the reaction. Meanwhile, the compound nucleus may involve rich shape phases.

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Because of the interest in exploring the upper limit of heavy element on earth and even in the universe (how long is their lifetime, which character determines their stability, how to synthesize them and what are their properties, and so on), the study of superheavy element (SHE) synthesis has been one of the most significant areas in nuclear physics for many years. On the experimental side, hot fusion and cold fusion reactions have been taken to synthesize superheavy elements. Then the elements with $Z = 110$, 111, 112 have been produced in GSI, German[1−4] and those with $Z = 112$, 114, 115, 116, and 118 have been synthesized in DUBNA collaboration.[5−12] Since the experiments are subjects that take time and money drastically, it is essential to obtain information as much as possible from theoretical works. Especially,it should be theoretically predicted that combination of projectile and target nucleus is promising and incident energy is optimal. At the same time, the best excitation energy of compound nucleus and the maximum cross section to produce SHE should be calculated. Presently, there are mainly two kinds of approaches to study the SHE synthesis theoretically.[13] One is based on Bohr's reaction theory of compound nucleus, which says that the process of compound nucleus formation is separated to its decay. Some models have been included in this mechanism, such as the Smolanczuk model,[14,15] the Abe model[16−18] and the dinuclear model.[19−22] The shared view of these models is that the process of fusion is calculated firstly, through which the cross section of fusion may be given, then the de-excitation process of the compound nucleus is considered with neutron evaporation channels, through which one can obtain the survival probability.[23] The other approach is the full dynamical one with the BUU/VUU model,[24−27] QMD (quantum molecular dynamics) model[28,29] and others being implemented. As a nucleon transport theory, the complete process of reaction can be simulated with time in these models. However, in all the above-mentioned studies, even though the rotational effect has been taken into account in the dinuclear model,[19,20] the non-central effect in the collision has not yet been investigated thoroughly.[20] If the requirement of central collision can be released, the synthesis experiment will obtain more easier. In this Letter, we extend the BUU/VUU model to include the rotational effect, and then discuss the possibility to release the central collision requirement.

In the semiclassical scheme of the BUU/VUU model, the nuclear dynamics is described by the variation of the single particle phase-space density distribution via the BUU/VUU equation

$$\begin{aligned}&\frac{\partial f}{\partial t} + \frac{\boldsymbol{p}}{m}\cdot\nabla_R f - \nabla_R U(\boldsymbol{R})\cdot\nabla_p f \\ &= \frac{1}{(2\pi)^3}\int d^3\boldsymbol{p}_2 d^3\boldsymbol{p}_2' d\Omega \frac{d\sigma}{d\Omega}\nu_{12} \\ &\quad\cdot\{[ff_2(1-f_{1'})(1-f_{2'}) - f_{1'}f_{2'}(1-f)(1-f_2)] \\ &\quad\cdot\delta^3(\boldsymbol{p}+\boldsymbol{p}_2-\boldsymbol{p}_{1'}-\boldsymbol{p}_{2'})\}\,. \qquad (1)\end{aligned}$$

The right-hand side of Eq. (1) is a collision integral including Pauli principle. If the right-hand side is set equal to zero, one obtains the Vlasov equation,[24] which describes the movement of particles in a given mean field $U(\boldsymbol{R})$. Here f in the Vlasov term is the single particle density distribution function in the phase space. The f in the collision integral represents two-

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body collision probability between the particles.

The BUU equation can be solved numerically by using the test particle method. By simulating the movement trajectories of enough test particles, one can obtain the density distribution function f. The density distribution function f is given to be

$$f(\boldsymbol{r},\boldsymbol{p},t)=\frac{1}{\tilde{N}}\sum_i \delta(\boldsymbol{r}-\boldsymbol{r}_i(t))\delta(\boldsymbol{p}-\boldsymbol{p}_i(t))\,, \qquad (2)$$

where $\tilde{N}$ is the number of test particles per nucleon, $\boldsymbol{r}_i$, $\boldsymbol{p}_i$ represents the coordinate and momentum of particle i, respectively. The motion of each test particle is determined by the classical Hamilton equation

$$\dot{\boldsymbol{r}}=\frac{\boldsymbol{p}}{m}\,, \qquad (3)$$

$$\dot{\boldsymbol{p}}=-\nabla_r U\,. \qquad (4)$$

The mean field potential U is usually parametrized as Skyrme interaction

$$U(\rho)=A(\rho/\rho_0)+B(\rho/\rho_0)^\sigma\,, \qquad (5)$$

where the three parameters A, B and σ are usually fixed by fitting the values of nuclear compressibility $K=200\,\mathrm{MeV}$ (soft EOS) or $K=380\,\mathrm{MeV}$ (stiff EOS).

Aiming to investigate the formation and shape evolution of the compound nucleus in superheavy element synthesis reaction and to release the restriction of central collision, the case of non-zero impact parameter ($b\neq 0$) must be considered. In this case, a collective rotational effect cannot be ignored for the formation of compound nucleus and its stability. With an initialization for the test particles, the spacial distribution of them in any time t can be obtained. Then the moment of inertia $I(t)=\sum_{i=1}^{N_{\mathrm{total}}} m(x_i^2+y_i^2+z_i^2)$ can be calculated easily, for the rotation around the centre of mass. Here N_{total} is all the test particles in projectile and target. According to the conservation of angular momentum, the compound nucleus obtains an angular frequency $\omega(t)=(P_0 b)/I(t)$, where P_0 represents the incident momentum in laboratory frame, $I(t)$ refers to the moment of inertia of the compound nucleus at time t. Naturally, a particle travelling in the rotational potential should have action of centrifugal force $m\omega^2\boldsymbol{r}$ and Coriolis force $2\boldsymbol{p}\times\boldsymbol{\omega}$. Therefore, it is necessary to correct Eq. (4) as

$$\dot{\boldsymbol{p}}=-\nabla_r U+2\boldsymbol{p}\times\boldsymbol{\omega}+m\omega^2\boldsymbol{r}\,. \qquad (6)$$

Due to ω corresponding to b, we can investigate the influence of b to the cross section of reaction and the lifetime of compound nucleus.

So far, the BUU model with rotational effect included has been introduced. To study the influence of the non-centrality of the reaction (or the rotation) effect on the formation and shape evolution of compound nucleus, we take the synthesis reaction of superheavy element 112[11] $^{48}\mathrm{Ca}+{}^{238}\mathrm{U}\rightarrow{}^{286}112$ as an example and simulate the time evolution of the nuclear matter density distribution firstly. In the numerical calculation, we take incident beam energy from 231 to 238 MeV (just the same as those in the experiment) with an interval of 1 MeV, and the b from 0.0 fm to 10.0 fm with an interval 1.0 fm. The calculations are performed in the lattice space with $51\times51\times61$ mesh points and $\tilde{N}=1000$ test particles for each nucleon. The time evolution of density distribution function $f(\boldsymbol{r},\boldsymbol{p})$ is made out with time step $\delta t=0.05\,\mathrm{fm}/c$. The parameters of the mean field U are taken as those of the stiff EOS ($A=-0.124$, $B=0.0705$, $\sigma=2.0$). The numerical calculations show that the simulated results do not depend on the incident energy in the range 231 to 238 MeV and the projectile and the target reaches the maximal fusion after a time $450\,\mathrm{fm}/c$. We illustrate the contour plots of the density distribution in the process of the reaction $^{48}\mathrm{Ca}+{}^{238}\mathrm{U}\rightarrow{}^{286}112$ with incident energy $E_k=231\,\mathrm{MeV}$ and several impact parameters $b=0.0\,\mathrm{fm}$, 3.0 fm, 6.0 fm and 9.0 fm in Figs. 1–4, respectively. The figures show obviously that, if the impact parameter $b\leq 6.0\,\mathrm{fm}$, a compound nucleus can be formed. If the impact parameter $b\leq 3.0\,\mathrm{fm}$, the compound nucleus can be stable. Meanwhile the compound nucleus may appear in a spherical, axially deformed and triaxially deformed shape. However, if the impact parameter $b=9.0\,\mathrm{fm}$, it is almost impossible for a compound nucleus to be formed.

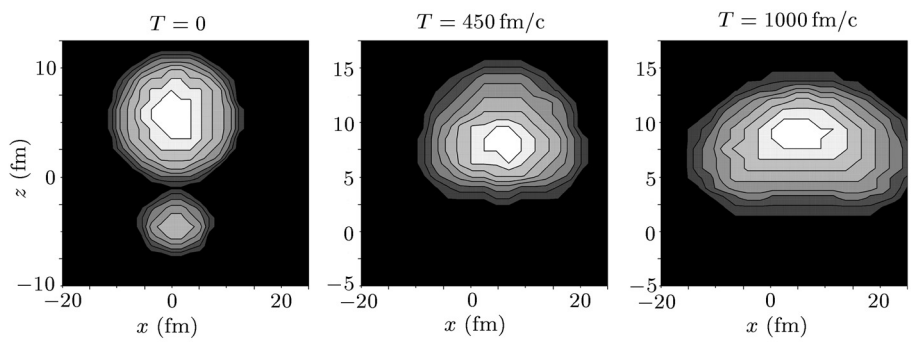


Fig. 1. Contour plots of the density distribution in the process of reaction $^{48}\mathrm{Ca}+{}^{238}\mathrm{U}\rightarrow{}^{286}112$ with $E_k=231\,\mathrm{MeV}$ and impact parameter = 0.0 fm. The black board shows the background without any nuclear matter. The brightness from white to dark indicates that the nuclear matter density becomes lower.

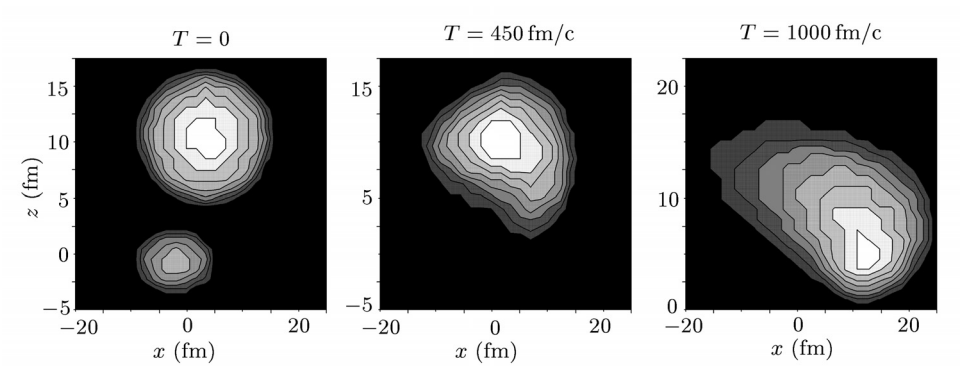


Fig. 2. The same as Fig. 1 but for that with impact parameter $b = 3.0$ fm.

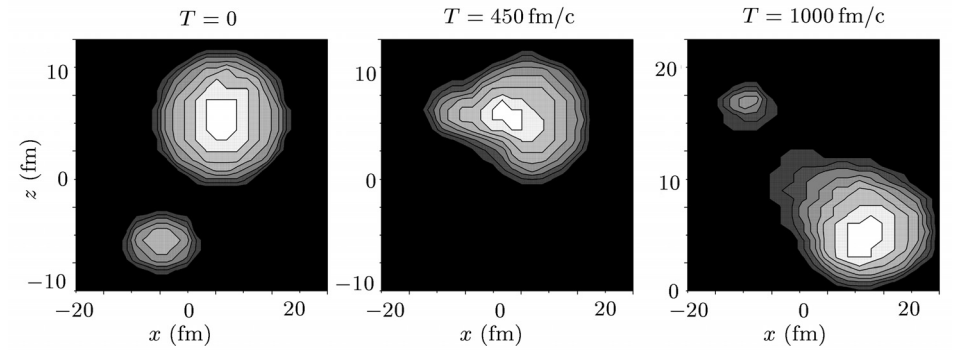


Fig. 3. The same as Fig. 1 but for that with impact parameter $b = 6.0$ fm.

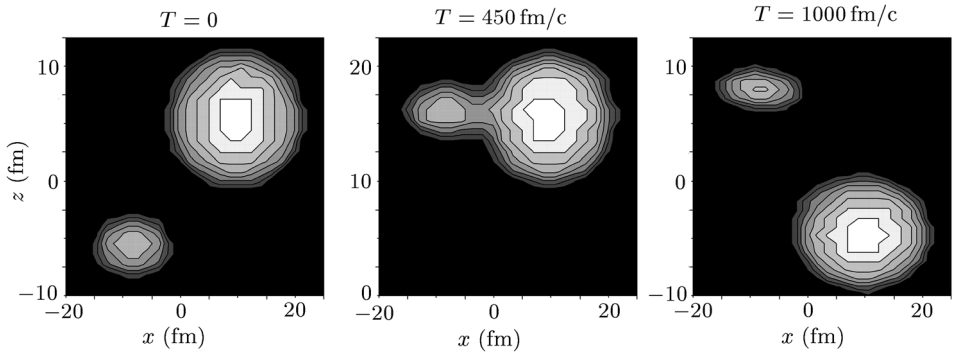


Fig. 4. The same as Fig. 1 but for that with impact parameter $b = 9.0$ fm.

To show the stability of the compound nucleus, we investigate the time evolution of its quadrupole momentum, too. The quadrupole momentum is defined as $Q^{(2)} = \int \rho(\boldsymbol{r}) r^2 Y_{20} d\boldsymbol{r}$. The simulated result of the time evolution of the quadrupole momentum of the compound nucleus in the reaction to synthesize superheavy element $Z = 112$[11] is shown in Fig. 5. From Fig. 5, one may see that the varying behaviour of quadrupole momentum $Q^{(2)}(t)$ is quite different from the variation of the impact parameter b. For b being 0.0 fm to 2.0 fm (Fig.5(a)), the $Q^{(2)}(t)$ descends with time nearly linearly. This implies that the compound nucleus keeps a shape with smaller deformation. For the b being 3.0 fm (Fig. 5(b)), the quadrupole momentum changes very slowly. Then it may appear as a quite stable compound nucleus. For b being 4.0 fm to 5.0 fm (Fig. 5(b)), the curves reach a minimal point then rise up, which indicates that the compound nucleus may exist for a short time then have a trend to be deformed. It also shows that the compound nucleus may have rich shape phases. While for b from 6.0 fm to 9.0 fm (Fig. 5(c) and 5(d)), $Q^{(2)}$ rises up globally with time, which indicates that the compound nucleus is very unstable and will be deformed, even fissioned, soon. In the limit case of $b = 10.0$ fm, $Q^{(2)}$ is much smaller than the one of $b = 9.0$ fm. $Q^{(2)}$ may then only be that of the target. It shows that the projectile and the target nucleus touch too little to produce a

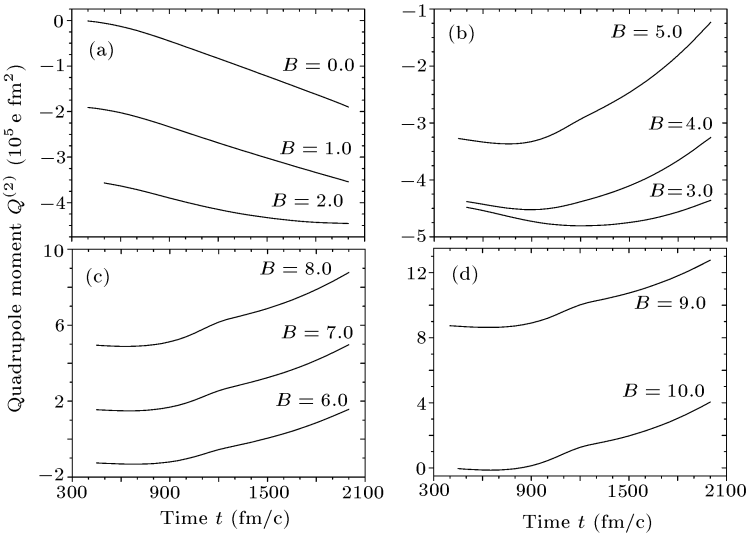


Fig. 5. Time evolution of the quadrupole momentum of compound nucleus in the reaction of ^{48}Ca + ^{238}U $\rightarrow$ ^{286}112 with various impact parameter b from 0.0 fm to 10.0 fm.

compound nucleus.

From the result mentioned above, we can recognize that, for the synthesis reaction of ^{48}Ca + ^{238}U $\rightarrow$ ^{286}112, the appropriate impact parameter b is less than 3.0 fm. Since 3.0 fm is smaller than the radius of the smaller nucleus in the reaction, then we can infer that the promising impact parameter in the superheavy element synthesis reaction should be smaller than the radius of the smaller nucleus involved in the reaction.

In summary, we have taken the BUU model to simulate the superheavy element synthesis reaction. With the rotation effect being included, the effect of the non-centrality of the reaction is studied. As the reaction ^{48}Ca + ^{238}U $\rightarrow$ ^{286}112 being taken as a concrete example, we show that the promising impact parameter in the synthesis reaction should be slightly smaller than the radius of the smaller nucleus involved in the reaction. Meanwhile, our simulation indicates that superheavy elements may involve spherical, axially deformed, triaxially deformed and other shape phases. With the BUU model, we can also study the cross section of superheavy element production, its decay rate and other information. Then the related investigation is in progress.

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