Bright–Dark Vector Screening-Photovoltaic Spatial Solitons in Biased Photorefractive–Photovoltaic Crystals *

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We show that the vector beam evolution equations in properly oriented biased photorefractive–photovoltaic crystals can exhibit bright–dark vector solitons, which result from both the bulk photovoltaic effect and the spatially non-uniform screening of the external bias field. By adjusting the polarization of the incident beam to obtain the appropriate ratio of two orthogonal components, these vector solitons can be established. When the bulk photovoltaic effect is negligible, these vector solitons are bright–dark vector screening solitons. When the external field is absent, these vector solitons predict bright–dark vector photovoltaic solitons.

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Photorefractive (PR) spatial solitons have attracted more interest in recent years.[1−10] So far, two different types of steady-state PR spatial solitons are known, i.e. screening solitons[1] and photovoltaic (PV) solitons.[4] Because of the bulk PV effect, PV solitons can be supported in PR crystals with appreciable PV coefficients. While due to the non-uniform screening of the external bias field in PR–nonPV crystals, screening solitons can be built. Very recently, vector screening solitons[5,6] have also been predicted, which involve the two polarization components of an optical beam that are orthogonal to one another. Depending on the symmetry class of the appropriate crystal and its orientation, these solitary beams have been found to obey a self-coupled or a cross-coupled system of nonlinear evolution equations. To date, bright–bright and dark–dark, self-coupled or cross-coupled vector solitons, and bright–dark self-coupled vector solitons have been predicted for screening solitons in biased PR–nonPV crystals. Recently, we have shown theoretically that the application of an external field enables steady-state screening-photovoltaic (SP) solitons in PR–PV crystals, which result from both the bulk PV effect and the spatially non-uniform screening of the external bias field.[7−9] Of particular interest are SP solitons, which change into screening solitons when the bulk PV effect can be neglected, and PV solitons when the external field is absent. Thus, the screening solitons and PV solitons studied may be integrated in SP solitons.[7,8] Therefore, we should pay attention to vector SP solitons in biased PR–PV crystals.

In this Letter, we show that the self-coupled vector beam evolution equations in biased PR–PV crystals can exhibit bright–dark vector solitons under steady-state conditions, which result from both the bulk PV effect and the spatially non-uniform screening of the external bias field. These vector solitons are obtained provided that the intensities of the two optical beams are approximately equal. Moreover, our analysis indicates that when the bulk PV effect can be neglected, these vector solitons are bright–dark vector screening solitons, and that when the external field is absent, these vector solitons predict bright–dark vector PV solitons.

We consider an optical beam that propagates in a PR–PV crystal along the z-axis, and is permitted to diffract only along the x-direction. For illustration purposes, let the PR–PV crystal be $LINbO_3$, which is a good candidate for the observation of the self-coupled or cross-coupled vector solitons.[5] The orientation of the crystal is selected such that the optical c-axis of this crystal makes an angle $\theta \approx 11.9^\circ$ with respect to the z-axis. In this case, the permittivity changes of the extraordinary and ordinary beams in $LiNbO_3$ are equal,[6] i.e. $\Delta\varepsilon_{ee} = \Delta\varepsilon_{oo}$, where $\Delta\varepsilon_{ee}$ and $\Delta\varepsilon_{oo}$ are the changes of the diagonal elements of the relative permittivity tensor, while those of the off-diagonal elements are zero, i.e. $\Delta\varepsilon_{eo} = \Delta\varepsilon_{oe} = 0$. Furthermore, let us assume that the external bias electric field is applied along the x-direction. By associating slowly-varying envelopes with the extraordinary and ordinary polarizations, $\phi_e(x,z)$ and $\phi_o(x,z)$, we find the following self-coupled nonlinear evolution equations:[5,6]

$$2ik_e\frac{\partial\phi_e}{\partial z} + \frac{\partial^2\phi_e}{\partial x^2} + k^2\Delta\varepsilon\phi_e = 0, \tag{1}$$

$$2ik_o\frac{\partial\phi_o}{\partial z} + \frac{\partial^2\phi_o}{\partial x^2} + k^2\Delta\varepsilon\phi_o = 0. \tag{2}$$

Here, $k = 2\pi/\lambda$, λ is the free-space wavelength of the lightwave employed, $k_e = kn'_e$ and $k_o = kn_o$ are the wave vectors for the extraordinary and ordinary com-

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ponents, n_e' and n_o are the refractive indices of the extraordinary and ordinary components at $\theta = 11.9°$, respectively, and $\Delta\varepsilon = \Delta\varepsilon_{ee} = \Delta\varepsilon_{oo}$. When $\theta = 11.9°$, $\Delta\varepsilon = 235.85 \times 10^{-12} E_{sc}$, where $\boldsymbol{E}_{sc} = E_{sc}\boldsymbol{i}$ is the induced space-charge field inside the PR–PV crystal, and $\boldsymbol{i}$ is the unit vector pointing to the x-direction.[5] Moreover, under strong bias conditions and for relatively broad beam configurations, the induced space-charge field is approximately given by[2,8,9]

$$E_{sc} = E_0 \frac{I_\infty + I_d}{I + I_d} + E_p \frac{I_\infty - I}{I + I_d}, \tag{3}$$

where $I = I(x, z)$ is the total power density of the extraordinary and ordinary components, I_d is the so-called dark irradiance which phenomenologically accounts for the thermal generation of electrons in the conduction band, and I_∞ represents the total power density the vector pair attains away from the centre of the PR–PV crystal. The PV field constant E_p is defined as $E_p = \kappa\gamma_e N_A/q\mu$, where κ is the PV constant, γ_e is the recombination rate coefficient, N_A is the density of negatively charged acceptors, μ is the electron mobility, and q is the electron charge. The constant field E_0 can be computed from $V_0 = -\int_{-l/2}^{l/2} E_{sc}\mathrm{d}x + RJS$ yielded by Ohm's law, where V_0 is the electro-motive force of the source, S is the surface area of the electrodes, R is the external resistance, and l is the width of the crystal between the electrodes.[4,9] Thus, expression (3) yields the following result

$$E_{sc} = -(V_0\eta + E_p\tau\eta - RJS\eta)\frac{I_\infty + I_d}{I + I_d} + E_p\frac{I_\infty - I}{I + I_d}, \tag{4}$$

where $\eta = \left\{\int_{-l/2}^{l/2}(I_\infty + I_d)(I + I_d)^{-1}\mathrm{d}x\right\}^{-1}$, and $\tau = \int_{-l/2}^{l/2}(I_\infty - I)(I + I_d)^{-1}\mathrm{d}x$. For convenience, we adopt the following dimensionless coordinates and variables, $\phi_e = (2\eta_0 I_d/n_e')^{1/2}U$, $\phi_o = (2\eta_0 I_d/n_o)^{1/2}V$, $\xi = z/(k_o x_0^2)$, and $s = x/x_0$, where x_0 is an arbitrary spatial width. By employing these transformations and substituting Eq. (4) into Eqs. (1) and (2), we find that the normalized planar envelopes U and V satisfy

$$\mathrm{i}\left(\frac{n_e'}{n_o}\right)U_\xi + \frac{1}{2}U_{ss} + (\sigma - \alpha - \beta)(\rho + 1) \cdot \frac{U}{1 + |U|^2 + |V|^2} + \delta\frac{\lfloor\rho - (|U|^2 + |V|^2)\rfloor U}{1 + |U|^2 + |V|^2} = 0, \tag{5}$$

$$\mathrm{i}V_\xi + \frac{1}{2}V_{ss} + (\sigma - \alpha - \beta)(\rho + 1)\frac{V}{1 + |U|^2 + |V|^2} + \delta\frac{\lfloor\rho - (|U|^2 + |V|^2)\rfloor V}{1 + |U|^2 + |V|^2} = 0, \tag{6}$$

where

$$\rho = I_\infty/I_d, \quad \beta = 235.85 \times 10^{-12}(kx_0)^2(\eta/2)V_0,$$
$$\alpha = 235.85 \times 10^{-12}(kx_0)^2(\tau\eta/2)E_p,$$
$$\sigma = 235.85 \times 10^{-12}(kx_0)^2(JS\eta/2)R,$$
$$\delta = 235.85 \times 10^{-12}(kx_0)^2(1/2)E_p.$$

To simplify the analysis, we have neglected any loss effects in Eqs. (5) and (6).

To find the bright–dark solitary pair solutions of Eqs. (5) and (6), the normalized envelopes U and V are expressed in the forms

$$U = r^{1/2}f(s)\exp[\mathrm{i}(n_o/n_e')\mu\xi],$$
$$V = \rho^{1/2}g(s)\exp(\mathrm{i}\nu\xi),$$

where $f(s)$ and $g(s)$ correspond to bright and dark beam envelopes, respectively. Hence, $f(0) = 1$, $f'(0) = 0$, $f(s \to \pm\infty) = 0$, $g(0) = 0$, $g(s \to \pm\infty) = \pm 1$. Moreover, all the derivatives of $f(s)$ and $g(s)$ are assumed to vanish at infinity. The positive variables r and ρ represent the ratios of their maximum power density with respect to the dark irradiance I_d. Substitution of these forms of U and V into Eqs. (5) and (6) yields

$$f'' = 2\Big[\mu - \frac{(\sigma - \alpha - \beta)(\rho + 1)}{1 + rf^2 + \rho g^2} - \frac{\delta(\rho - rf^2 - \rho g^2)}{1 + rf^2 + \rho g^2}\Big]f, \tag{7}$$

$$g'' = 2\Big[\nu - \frac{(\sigma - \alpha - \beta)(\rho + 1)}{1 + rf^2 + \rho g^2} - \frac{\delta(\rho - rf^2 - \rho g^2)}{1 + rf^2 + \rho g^2}\Big]g, \tag{8}$$

where $f'' = \mathrm{d}^2 f/\mathrm{d}s^2$. We now look for particular solutions that also satisfy the condition $f^2 + g^2 = 1$. Using appropriate boundary conditions, μ and ν can be readily found[6,8] to be $\mu = [(\sigma - \alpha - \beta - \delta)/\vartheta]\ln(1 + \vartheta) - \delta$ and $\nu = \sigma - \alpha - \beta$, where $\vartheta = (r - \rho)/(1 + \rho)$. Equations (7) and (8) can then be solved provided that $|\vartheta| \ll 1$, i.e. when the peak intensities of the two vector components are approximately equal. In this case, the approximate vector pair solutions of Eqs. (7) and (8) are given by[6,8]

$$U = r^{1/2}\mathrm{sech}\{[(\beta + \alpha - \sigma - \delta)\vartheta]^{1/2}s\}\exp\{\mathrm{i}(n_0/n_e') \cdot [\sigma - \beta - \alpha) + (\beta + \alpha - \sigma - \delta)\vartheta/2]\xi\}, \tag{9}$$

$$V = \rho^{1/2}\tanh\{[(\beta + \alpha - \sigma - \delta)\vartheta]^{1/2}s\} \cdot \exp[\mathrm{i}(\sigma - \beta - \alpha)\xi]. \tag{10}$$

The above equations clearly show that these solutions are possible only when $(\beta + \alpha - \sigma - \delta)\vartheta$ is a positive quantity. Under the $\vartheta < 0$ and $|\beta| < |\alpha - \sigma - \delta|$ conditions, these vector solitons will be possible for $\alpha < \sigma + \delta$ no matter whether β (i.e. V_0) is positive or negative (so as $(\beta + \alpha - \sigma - \delta)\vartheta > 0$). To illustrate our results, we take the calculation with $\lambda_0 = 0.633\,\mu$m, $l = 1\,$cm, $x_0 = 40\,\mu$m, $R = 0$, and $E_p = 40\,$kV/cm. Figure 1 depicts the normalized intensity profiles of the bright–dark vector pairs under these conditions for $V_0 = \pm 20000\,$V, $\rho = 10$, and $\vartheta = -0.05$. When $\vartheta > 0$, β is positive quantity bounded $\beta > |\alpha - \sigma - \delta|$ so that $(\beta + \alpha - \sigma - \delta)\vartheta > 0$. The bright–dark vector soliton obtained at $V_0 = +50000\,$V, $\rho = 5$ and $\vartheta = 0.01$ is shown in Fig. 2.

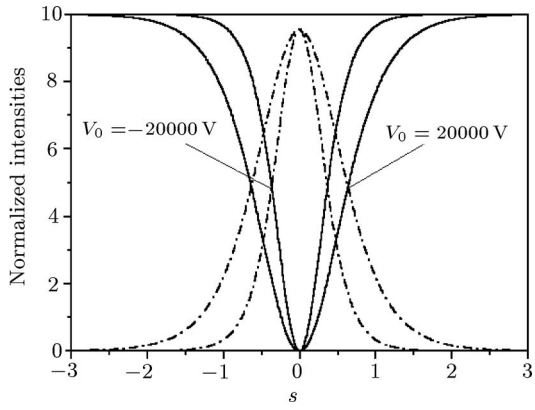

Fig. 1. Soliton components $|U|^2$ (dot-dashed curve) and $|V|^2$ (solid curve) for the bright–dark vector pairs when $R = 0$, $V_0 = \pm 20000$ V, $\rho = 10$, and $\vartheta = -0.05$.

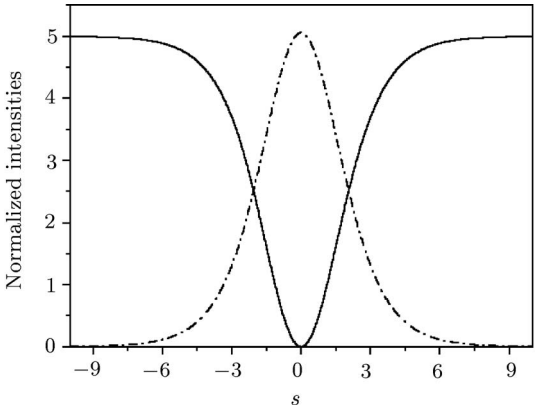

Fig. 2. Soliton components $|U|^2$ (dot-dashed curve) and $|V|^2$ (solid curve) for the bright–dark vector pairs when $R = 0$, $V_0 = +50000$ V, $\rho = 5$, and $\vartheta = 0.01$.

The bright–dark vector solitons for screening and PV solitons can be simply obtained from the bright–dark vector solitons in biased PR–PV crystals under appropriate conditions. When the bulk PV effect can be neglected for short circuits ($R = 0$), i.e. $\delta = \alpha = \sigma = 0$, Eqs. (5), (6), (9) and (10) yield

$$\mathrm{i}(n_e'/n_o)U_\xi + \frac{1}{2}U_{ss} - \beta(\rho+1)\frac{U}{1+|U|^2+|V|^2} = 0, \tag{11}$$

$$\mathrm{i}V_\xi + \frac{1}{2}V_{ss} - \beta(\rho+1)\frac{V}{1+|U|^2+|V|^2} = 0, \tag{12}$$

$$U = r^{1/2}\mathrm{sech}\lfloor(\beta\vartheta)^{1/2}s\rfloor \exp\{-\mathrm{i}\beta(n_o/n_e')(1-\vartheta/2)\xi\}, \tag{13}$$

$$V = \rho^{1/2}\tanh\lfloor(\beta\vartheta)^{1/2}s\rfloor \exp(-\mathrm{i}\beta\xi). \tag{14}$$

Equations (11)–(14) have the same dimensionless parameters and forms as Eqs. (3a), (3b), (7a) and (7b) in Ref. [6], in which the bright–dark vector screening solitons are discussed for short circuits.

When the external bias field is absent, i.e. $\beta = 0$, the physical system of SP solitons becomes the physical system of previously studied PV solitons.[7] Thus, when $\beta = 0$, the bright–dark vector SP solitons should become the bright–dark vector solitons for PV solitons. Substituting $\beta = 0$ into Eqs. (5), (6), (9) and (10) yields

$$\mathrm{i}(n_e'/n_o)U_\xi + \frac{1}{2}U_{ss} + (\sigma-\alpha)(\rho+1)\frac{U}{1+|U|^2+|V|^2} + \delta\frac{\lfloor\rho-(|U|^2+|V|^2)\rfloor U}{1+|U|^2+|V|^2} = 0, \tag{15}$$

$$\mathrm{i}V_\xi + \frac{1}{2}V_{ss} + (\sigma-\alpha)(\rho+1)\frac{V}{1+|U|^2+|V|^2} + \delta\frac{\lfloor\rho-(|U|^2+|V|^2)\rfloor V}{1+|U|^2+|V|^2} = 0, \tag{16}$$

$$U = r^{1/2}\mathrm{sech}\{[(\alpha-\sigma-\delta)\vartheta]^{1/2}s\} \cdot \exp\{\mathrm{i}(n_o/n_e')[(\sigma-\alpha) + (\alpha-\sigma-\delta)\vartheta/2]\xi\}, \tag{17}$$

$$V = \rho^{1/2}\tanh\{[(\alpha-\sigma-\delta)\vartheta]^{1/2}s\} \cdot \exp[\mathrm{i}(\sigma-\alpha)\xi]. \tag{18}$$

Equations (15)–(18) are just the bright–dark vector solitons for PV solitons.

We have shown that the self-coupled vector beam evolution equations in properly oriented biased PR–PV crystals can exhibit bright–dark vector solitons, which result from both the bulk PV effect and the spatially non-uniform screening of the external bias field. When the intensities of two optical beams with cross polarization are approximately equal, these vector solitons are established. We have found that when the bulk PV effect can be neglected, these vector solitons are bright–dark vector screening solitons, and that when the external field is absent, these vector solitons predict bright–dark vector PV solitons.

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