Concept of Excessive Mesons in Nuclei*

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The concept of excessive mesons in nuclei is discussed. Since nucleons are identical particles, it is pointed out that the physical content of a meson distribution around a bound nucleon in nuclei is different from that of a meson cloud around a free nucleon, if the effect of the principle of identity cannot be neglected. The difference is enunciated in detail.

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Recently the question whether there are excessive pions in nuclei has been discussed with considerable interest.[1,2] This is a very important question for quantum hadrodynamics (QHD), since according to it excessive mesons must exist in the nuclear ground state (GS), otherwise the GS expectation value of the meson-nucleon interaction Hamiltonian will be zero and thus no adequate theory of nuclear structure can be constructed on the basis of QHD.[3] However, there are two different definitions of excessive mesons in nuclei. Let $|\Psi_0(A)\rangle$ denote the exact GS of a nucleus with A nucleons, $\langle O\rangle \equiv \langle\Psi_0(A)|O|\Psi_0(A)\rangle$ and $N_r(A) = \langle\tilde{N}_r\rangle$, where $\tilde{N}_r = \sum_{\boldsymbol{k}} a_r^+(\boldsymbol{k})a_r(\boldsymbol{k})$ is the meson number operator and r designates the kind of meson. In non-relativistic theory of nuclear structure it is assumed that nuclei are composed of protons and neutrons and the existence of mesons need not be considered, though the nuclear force is caused by the exchange of virtual mesons. For this reason we have called $N_r(A)$ the number of excessive r-mesons,[3] since it represents the excess resulting from the exchange. Another definition is given by

$$\Delta N_r(A) = N_r(A) - AN_r(1), \tag{1}$$

which represents the number of mesons in excess of that associated with free nucleons.[4] In order to calculate $N_r(A)$ as well as $\Delta N_r(A)$, we may consider the meson propagator $D_r(\boldsymbol{kl}, t = t_1 - t_2) = \langle T[\hat{a}_r(\boldsymbol{k}t_1)\hat{a}_r^+(\boldsymbol{l}t_2)]\rangle$, where an operator O with a hat means $\hat{O}(t) = \exp(iHt)O\exp(-iHt)$. The Fourier transform is defined as $D_r(\boldsymbol{kl};\omega) = i\int_{-\infty}^{\infty} dt D_r(\boldsymbol{kl};t)e^{i\omega t}$. Obviously one has $N_r(A) = D_r(\boldsymbol{kk};t\to 0^-) = (2\pi i)^{-1}\int_{-\infty}^{\infty} d\omega e^{i\omega 0^+} D_r(\boldsymbol{kk};\omega)$. As is well-known, $D_r(\boldsymbol{kl};t)$ can be calculated by the perturbation formula

$$D_r(\boldsymbol{kl};t) = \sum_{n=0}^{\infty}\frac{(-i)^n}{n!}\int\cdots\int_{-\infty}^{\infty} d\sigma_1\cdots d\sigma_n \langle\phi_0|T[H_1(\sigma_1)\cdots H_1(\sigma_n) \times a_r(\boldsymbol{k}t_1)a_r^+(\boldsymbol{l}t_2)]|\phi_0\rangle_C, \tag{2}$$

where all the operators are in the interaction representation, $|\phi_0\rangle$ is the zero-order approximation to $|\Psi_0\rangle$ and the subscript C denotes a sum over connected diagrams only. Friman *et al.*[4](FPW) suggested that $\Delta N_r(A)$ can also be calculated by means of $D_r(\boldsymbol{kk};t\to 0^-)$ and Eq. (2), if one

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further requires that mesons emitted and absorbed by the same nucleon should not be counted. Symbolically this is indicated by attaching an asterisk to C, i.e., $\Delta N_r(A)$ can also be obtained directly from Eq. (2) if one substitutes C^* for C. However, since nucleons are identical particles, according to the principle of identity (PI), it is impossible to ascertain whether in a many-nucleon nucleus mesons are emitted and absorbed by the same nucleon; in other words, without violating PI in some way, C^* cannot be realized, except that nucleons in nuclei can be regarded as almost free and the effect of PI is unimportant. Let us, for instance, consider a simple meson-nucleon system. Its interaction Hamiltonian is given by

$$H_1 = -\sum_{\eta\lambda kr}[f(\eta\lambda kr) : c_\eta^+ c_\lambda : a_r(k) + f^*(\eta\lambda kr) : c_\lambda^+ c_\eta : a_r^+(k)], \tag{3}$$

where $c_\eta^+(c_{\bar\eta})$ denotes a nucleon (antinucleon) creation operator with $\bar\eta$ referring to a negative energy state. If H_1 is substituted into Eq. (2), no way to carry out C^* can be found because no distinction between different nucleons is made in Eq. (3). FPW achieved C^* by labelling H_1, i.e., rewriting Eq. (3) as

$$H_1 = -\sum_j\sum_{\eta\lambda kr}[f_j(\eta\lambda kr) : c_\eta^+(j)c_\lambda(j) : a_r(k) + f_j^*(\eta\lambda kr) : c_\lambda^+(j)c_\eta(j) : a_r^+(k)], \tag{4}$$

where j runs from 1 to A and the fermion anticommutation relations read

$$\{c_\eta(i), c_\lambda(j)\} = \{c_\eta^+(i), c_\lambda^+(j)\} = 0, \quad \{c_\eta^+(i), c_\lambda(j)\} = \delta_{ij}\delta_{\eta\lambda}. \tag{5}$$

In the language of quantum field theory, this means that we are considering a system of A different fermion fields interacting with the same meson fields as considered in the original system, namely Eq. (3). Hereafter this system will be referred to as SII and the original one as SI. We should like to show that PI is still violated even though the wavefunctions used in C^* satisfy the Pauli principle and $AN_r(1) = N_r(A) - \Delta N_r(A)$ calculated according to C^* is actually not the same as it is defined in Eq. (1). Clearly every baryon number operator $\tilde B_j = \sum_\eta : c_\eta^+(j)c_\eta(j) :$ commutes with the total Hamiltonian. Thus each baryon number $B_j = A_j - \bar A_j$ is a motion constant, though neither the particle number A_j nor the antiparticle number $\bar A_j$ is a good quantum number. Following FPW, we shall neglect the contribution of the negative energy states. Then the normal ordering of $c_\eta^+ c_\lambda$ is unnecessary and A_j may be assigned a definite value. Let us set each $A_j = 1$. From Eqs. (4) and (5) it is seen that SII may describe a system of $A = \sum_j A_j$ particles, whose wavefunction is antisymmetric, while each particle is distinguishable from the other only by the attached label. Since $c_\eta^+(i)c_\eta^+(j) = -c_\eta^+(j)c_\eta^+(i) \neq 0$ if $i \neq j$, two particles in SII are still allowed to occupy the same state. However, the Pauli principle can be satisfied by further requiring that no two particles should be in the same state. Thus, the wavefunctions in SII can take account of the Pauli principle properly, though SI and SII remain different. Its difference lies in the fact that now in SII all fermion operators anticommute, while for SI the anticommutation relations are given by Eq. (5) with $i = j$. Let us calculate $N_r(A)$ by perturbation theory. For our present purpose we only need to consider the lowest order non-zero approximation to $N_r(A)$, which is given by

$$N_r^{(2)}(A) = \sum_{i,j}\sum_{\eta\mu k} f_i^*(\eta\eta kr)\frac{n_\eta(i)}{\omega_r(k)}\frac{n_\mu(j)}{\omega_r(k)}f_j(\mu\mu kr)$$
$$+\sum_{j}\sum_{\eta\lambda k} f_j^*(\eta\lambda kr)\frac{n_\eta(j)[1-n_\lambda(j)]}{[\varepsilon_\eta(j)-\varepsilon_\lambda(j)-\omega_r(k)]^2}f_j(\eta\lambda kr)\,, \qquad (6)$$

where n_η denotes the occupation number in the η state. According to the prescription of C^* $\Delta N_r(A)$ is also given by Eq. (6) if we require $i \neq j$ in its first term and discard the second term. However, $AN_r(1) = N_r(A) - \Delta N_r(A)$ calculated in this way is actually not the same as it is defined in Eq. (1). For this reason quantities calculated according to C^* will be indicated hereafter by an overhead bar. Let us consider nuclear matter as an example and calculate $N_\sigma(A)$ for the scalar σ meson. The matrix élement $f(\eta\lambda k\sigma)$ has the explicit form : $f = [(2\pi)^3 2\omega_\sigma(k)]^{-1/2}\bar{u}(\boldsymbol{ps})u(\boldsymbol{qz})F(k)\delta(\boldsymbol{p}-\boldsymbol{k}-\boldsymbol{q})$, where $\eta \equiv \boldsymbol{ps}$ and $\lambda \equiv \boldsymbol{qz}$, s and z stand for the spin-isospin variable and $F(k)$ is a form factor. Clearly Eq. (6) also applies to S I if indices i, j, $\cdots$ and their relevant summations are omitted. Though A may be assigned any number, to treat $A=1$ as nuclear matter is a sheer artifact. Nevertheless it is adequate for our purpose to consider $A=1$ and $A>1$ on the same footing, as it suffices to demonstrate the difference between S I and S II. In the case of $A=1$ one need not consider the Pauli principle and the particle correlation. This says that for $A=1$, $u(\boldsymbol{qz})$ is simply given by the free particle Dirac spinor, and $n_\alpha = 1$ for $\varepsilon_\alpha = M$, while $n_\eta = 0$ with $\varepsilon_\eta = (p_\eta^2 + M^2)^{1/2}$ for $\eta \neq \alpha$. For a many-particle system, besides the residual interaction an optimal sp potential, for instance, the relativistic Hartree-Fock self-consistent potential[5] has to be introduced. The relevant sp spinor and energy will be denoted by $\hat{u}(\boldsymbol{qz})$ and $\hat{\varepsilon}_\lambda$, respectively. The sp states are filled up to the Fermi level $k_F = \alpha A^{1/3}$, where $\alpha = (6\pi^2/\gamma V)^{1/3}$, γ denotes the spin-isospin degeneracy and V the volume of box normalization. From Eq. (6) we get

$$AN_\sigma^{(2)}(1) = Ag_s^2 V\Big\{\frac{1}{2m_\sigma^3}\Big[\frac{\gamma}{(2\pi)^3}\int_0^{k_F(1)}d^3p\,\frac{M}{E_p}\Big]^2$$
$$+\frac{1}{(2\pi)^3}\sum_{sz}\int_{k_F(1)}^{\infty}d^3q\int_0^{k_F(1)}d^3p\,\frac{\bar{u}(\boldsymbol{qz})u(\boldsymbol{ps})\bar{u}(\boldsymbol{ps})u(\boldsymbol{qz})}{[\varepsilon_p-\varepsilon_q-\omega_\sigma(|\boldsymbol{p}-\boldsymbol{q}|)]^2}F(|\boldsymbol{p}-\boldsymbol{q}|)^2\Big\}\,, \qquad (7)$$

while according to C^* we should have

$$\overline{AN_\sigma^{(2)}(1)} = g_s^2 V\Big\{\frac{\gamma}{(2\pi)^6 2m_\sigma^3}\int_0^{k_F(A)}d^3p\,\Big(\frac{M_p^*}{E_p^*}\Big)^2$$
$$+\frac{1}{(2\pi)^3}\sum_{sz}\int_{k_F(A)}^{\infty}d^3q\int_0^{k_F(A)}d^3p\,\frac{\hat{\bar{u}}(\boldsymbol{qz})\hat{u}(\boldsymbol{ps})\hat{\bar{u}}(\boldsymbol{ps})\hat{u}(\boldsymbol{qz})}{[\hat{\varepsilon}_p-\hat{\varepsilon}_q-\omega_\sigma(|\boldsymbol{p}-\boldsymbol{q}|)]^2}F(|\boldsymbol{p}-\boldsymbol{q}|)^2\Big\}\,, \qquad (8)$$

where M_p^* is the effective mass and $E_p^* = (M^{*2} + p^{*2})^{1/2}$ (see Ref. 5). Clearly $\overline{AN_\sigma^{(2)}(1)}$ and $AN_\sigma^{(2)}(1)$ are not the same. It is easily found that this conclusion also holds for higher order calculations, i. e., the term subtracted from $N_r(A)$ in Eq. (1) according to C^* is actually not $AN_r(1)$. By definition $N_r(1)$ in Eq. (1) has nothing to do with a bound nucleon in nuclei, except when the latter may be regarded as almost free. Besides the quantities related to the sp states, this is expressed in Eq. (7) by the fact that the integration limit is $k_F(1)$. Obviously, for an A-nucleon nucleus PI requires that the limit should be $k_F(A)$. For this reason one might think that Eq. (8) took a better account of the meson cloud around bound nucleons. But we ob-

serve that the second term on the righthand side of Eq. (6) is included completely in $\overline{AN_r(1)}$, because each term in the summation over j is contributed by the same particle. However, this is not in accord with PI, since the corresponding expression for S I only says that a nucleon is annihilated and a nucleon created at the relevant vertex. It does not assert that they are the same. Thus, assuming that the transition involves only the same particle as advocated by S II violates PI, it is not adequate to subtract such terms completely from $N_r(A)$ in order to obtain $\Delta N_r(A)$. Certainly there are mesons distributed around each nucleon in a nucleus. In the above we have demonstrated that owing to PI such meson distribution should be understood as the excess resulting from exchange of mesons among nucleons and it is impossible to separate from it unambiguously a part as emitted and absorbed by the same nucleon. It has been pointed out by a model calculation in Ref. 6 that $N_r(1)$ is independent of the state of the free nucleon. Indeed, this can be seen generally and easily by calculating $N_r(1)$ in the nucleon rest frame and making use of the rotational invariance in the ordinary as well as isospin space. Since $N_r(1)$ is a constant and can be calculated independently, both definitions of excessive mesons actually ask for the calculation of the same $N_r(A)$, which is A-dependent and can be used to reflect the medium effect on nuclear meson content. As $N_r(A)$ is just the expectation value of the meson number operator and can be referred to simply as the meson number in nuclei, the adjective "excessive" may be reserved for $\Delta N_r(A)$. Mathematically $\Delta N_r(A)$ is well defined according to Eq. (1). However, if the effect of PI is important, Eq. (1) is not very useful and even somewhat misleading physically. Let $K(A)$ denote the expectation value of a physical observable which depends on mesons in nuclei. If PI must be observed, we suggest that one calculates $K(A)$ directly from the meson distribution, since generally $K(A) \neq K_\Delta(A) + AK(1)$, where $K_\Delta(A)$ denotes the contribution from the excessive mesons defined in Eq. (1). Numerical results for $N_r(A)$ and a more detailed study of the effect of PI will be discussed elsewhere.

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