One- and Two-Dimensional Arrays of Double-Well Optical Traps for Cold Atoms or Molecules *

JI Xian-Ming(纪宪明)[1,2], YIN Jian-Ping(印建平)[1**]

[1]*Key Laboratory for Optical and Magnetic Resonance Spectroscopy, East China Normal University, Shanghai 200062*
[2]*Department of Physics, Nantong University, Nantong 226007*

(Received 25 June 2004)

We propose a novel scheme to form one- and two-dimensional arrays of double-well optical dipole traps for cold atoms (or molecules) by using an optical system composed of a binary π-phase grating and a lens illuminated by a plane light wave, and study the relationship between the maximum intensity $I_{\max}$ of each optical well (or the maximum trapping potential $U_{\max}$ for ^{85}Rb atoms) and the relative aperture β $(= a/f)$ of the lens. We also calculate the intensity gradients of each optical well and their curvatures, and estimate the spontaneous photon-scattering rate of trapped atom in each well, including Rayleigh and Raman scattering rates. Our study shows that the proposed 1D and 2D arrays of double-well traps can be used to prepare 1D and 2D novel optical lattices with cold atoms (or molecules), or form an all-optically integrated atom optical chip, or even to realize an array of all-optical double-well atomic (or molecular) Bose–Einstein condensates by optical-potential evaporative cooling, and so on.

PACS: 32. 80. Pj, 32. 80. Lg, 33. 80. Ps, 42. 50. Vk

In 1986, Chu *et al.* first demonstrated an optical dipole trap for cold Na atoms by using a focused red-detuned Gaussian laser beam output from a dye laser.[1] In their experiment, the optical trapping potential for Na atoms was 10 mK, and the photon scattering rate was about $2600\,s^{-1}$. Afterwards, researchers developed various optical dipole traps for cold atoms, and successfully realized laser trapping of neutral atoms. For example, Miller *et al.* realized the optical trapping of cold ^{85}Rb atoms by using a single-beam-focused far-red-detuned Gaussian beam.[2] In 1995, Takeoshi *et al.* proposed a new scheme to trap cold ^{133}Cs atoms by using a focused CO_2 laser beam with a giant red-detuning.[3] In the same year, Adams *et al.*[4] demonstrated the laser trapping of cold ^{133}Cs atoms by means of two crossed and focused red-detuned YAG laser beams, and implemented the experimental study of optical-potential evaporative cooling for the trapped ^{133}Cs atoms. In 1998, Yin *et al.* proposed a new gravito-optical trap (GOT) scheme to trap cold atoms by using a focused blue-detuned dark hollow beam (DHB).[5] Recently, researchers used a red-detuned YAG or CO_2 laser to realize quantum degenerate Fermi gases, all-optical trapped atomic or molecular Bose–Einstein condensates (BECs), and all-optical output of atom laser.[6−11] In addition, ones used current-carrying wires to form some controllable double-well magnetic traps[12] and 1D and 2D arrays of magnetic or magneto-optical traps,[13,14] even to prepare 1D and 2D magnetic or magneto-optical lattices, and to realize 1D and 2D arrays of BECs,[13] and the recent progress on double-well magnetic microtrap and its array (or lattices) as well as atom chip can be found in Ref. [15].

In this Letter, we propose a novel scheme to construct 1D and 2D arrays of double-well optical traps for cold atoms (or molecules) on the substrate, study and analyse the relationships between the characteristic parameters of each optical well (including geometric parameters, intensity distribution, intensity gradients and their curvatures) and the parameters of the optical system. We also calculate the optical potential of each optical well for ^{85}Rb atoms, and estimate the spontaneous photon-scattering rate (including Rayleigh and Raman scattering rates) of a two-level ^{85}Rb atom in each CO_2 laser well, and discuss the potential applications of our array of double-well traps in the fields of atom, molecule and quantum optics.

Figure 1 shows a novel scheme to form a 1D array of double-well optical traps for cold atoms (or molecules) by using an optical system composed of a 1D binary π-phase grating and a 1D array of spherical microlenses illuminated by a plane light wave. Figure 1(a) shows the side view of our optical system to generate 1D array of double-well optical traps, and Fig. 1(b) shows the corresponding top view, while Fig. 1(c) depicts the generated 1D array of double-well traps on the focal plane of the microlens array. As shown in Fig. 1(b), the phase values of the phase grating are divided into two ones: 0 and π, which have the same area, that is, the widths of 0 and π phase zones in the x direction both are $2a$, and the side width of each square microlens with a focal length f is $2a$. If the centre of each microlens is located at each phase-jumped line between the 0 and π phase zones, and

* Supported by the National Natural Science Foundation of China under Grant Nos 10174050 and 10374029, Shanghai Priority Academic Discipline and the 211 Foundation of the Education Ministry of China, and the Natural Science Foundation of College of Jiangsu Province under Grant No 04kjb140109.

** Email: To whom correspondence should be addressed. Email: jpyin@phy.ecnu.edu.cn

©2004 Chinese Physical Society and IOP Publishing Ltd

when a plane light wave (or quasi-plane wave) passes through the binary π-phase grating and is focused by the lens, an optical dipole trap with two wells for cold atoms (or molecules) will be produced near the focal plane of each lens because of the complete destructive interference effect at the light axis of each lens. Similarly, an optical system composed of a 1D π-phase grating and a 2D array of spherical microlenses is illuminated by a plane wave, we can obtain a 2D array of double-well optical traps near the focal plane of the microlens array, which is shown in Fig. 2(b), and the top view of our 2D optical system is shown in Fig. 2(a).

Since a 1D array of double-well traps is a special case of 2D array of double-well ones, we only need to discuss the intensity distribution of the 2D double-well array. Setting the grooving direction of the π-phase grating along the y direction, the amplitude transmission function of the grating is given by

$$g(x,y) = \sum_{m=-(N+1)/2}^{(N+1)/2} (-1)^{m+1} \mathrm{rect}\Big[\frac{x-(2m-1)a}{2a}\Big], \tag{1}$$

where $\mathrm{rect}(x)$ is the rectangle function,[16] and N is the number of the grating period. When N is odd, if the centre of the 2D array of $N \times N$ lenses is set at the origin $(0,0)$ of the x and y coordinates, the coordinates of the centre of the mnth lens in the 2D lens array are $x_m = 2ma$ and $y_n = 2na$, respectively, and then the amplitude transmission function of the mnth lens can be written as (as the constant phase factors are neglected)[16]

$$t_{mn}(x,y) = \mathrm{rect}\Big(\frac{x-x_m}{2a}, \frac{y-y_n}{2a}\Big) \cdot \exp\Big\{i\frac{\pi}{\lambda f}[(x-x_m)^2 + (y-y_n)^2]\Big\}. \tag{2}$$

When the condition $a^2 \gg f\lambda$ is satisfied, the diffraction intensity of each lens is mainly distributed near the focal point of the lens itself, and the interference effect between the diffraction light waves from different lenses can be neglected. In this case, when a plane wave is used to illuminate the optical system composed of a 1D π-phase grating and 2D lens array, the intensity distribution of each lens is identical, so we only need to discuss the intensity distribution of a single lens, and the diffracted field distribution of the mnth lens in the 2D lens array in the x_0oy_0 plane is given by[16]

$$U_{mn}(x_0,y_0,z) = \frac{A}{i\lambda z}\iint_{-\infty}^{+\infty} t_{mn}(x,y)g(x,y) \cdot \exp\Big[\frac{i\pi}{\lambda z}(x-x_0)^2 + (y-y_0)^2\Big]dxdy, \tag{3}$$

and the corresponding intensity distribution is $I_{mn}(x_0,y_0,z) = |U_{mn}(x_0,y_0,z)|^2$. In particular, the distribution of the diffracted intensity in the focal plane of the lens can be written as

$$I_{mn}(x_0,y_0) = 4P_n(\beta/\lambda)^2 \frac{\sin^4[\pi(x_0-2ma)\beta/\lambda]}{[\pi(x_0-2ma)\beta/\lambda]^2} \cdot \frac{\sin^2[2\pi(y_0-2na)\beta/\lambda]}{[2\pi(y_0-2na)\beta/\lambda]^2}, \tag{4}$$

where β is the relative aperture of the lens system, which is defined as $\beta \equiv a/f$.

Since the geometry of each optical well can be approximated as an ellipsoid, the trap volume of each well for cold atoms (or molecules) can be estimated by

$$\Delta V_{1/e^2} = 0.494\frac{\lambda^3}{\beta^4}, \tag{5}$$

and the corresponding trapped atomic number can be estimated by $N = n\Delta V_{1/e^2}$, where n is the density of atoms in the optical trap. Equation (5) shows that the trap volume of each optical well is inversely proportional to the fourth power of the relative aperture β of the lens system (a/f). That is, we can effectively change the trap volume of each well for cold atoms (or molecules) by adjusting the relative aperture of the lens system, which can be used to change the number of trapped atoms (or molecules) or to control the density of atoms (or molecules) in each well.

According to Eq. (4), we can derive the analytical expressions of the maximum intensity of each optical well and its intensity gradients and curvatures in the x_0 and y_0 directions as follows:

$$I_{\max} = 8.401A^2a^2(\beta/\lambda)^2 = 2.1P_n(\beta/\lambda)^2, \tag{6}$$

$$\begin{cases} \Big|\dfrac{\partial I}{\partial x_0}\Big|_{\max} = 8.400(\beta/\lambda)^3 P_n, \\ \Big|\dfrac{\partial I}{\partial y_0}\Big|_{\max} = 7.127(\beta/\lambda)^3 P_n, \end{cases} \tag{7}$$

$$\begin{cases} \Big|\dfrac{\partial^2 I}{\partial x_0^2}\Big|_{\max} = 78.96(\beta/\lambda)^4 P_n, \\ \Big|\dfrac{\partial^2 I}{\partial y_0^2}\Big|_{\max} = 55.27(\beta/\lambda)^4 P_n, \end{cases} \tag{8}$$

where P_n is the illuminating laser power of each lens. The maximum intensity gradient and the curvature in the z_0 direction can only be obtained by numerical calculation. From Eqs. (7) and (8), we can see that the maximum intensity gradient of each optical well is proportional to the third power of the relative aperture β of the lens system, and the maximum curvature is proportional to the fourth power of the relative aperture β. Thus the motion of cold atoms (or molecules) in each optical well can be easily controlled and manipulated by adjusting the relative aperture of the lens system. When $a = 0.3\,\mathrm{mm}$, $f = 2\,\mathrm{mm}$, $\lambda = 10.6\,\mu\mathrm{m}$, and $P_n = 40\,\mathrm{W}$, we calculate the trapping volume of each optical well and its the maximum intensity gradients and curvatures in the x_0, y_0 and z_0 directions, and the results are shown in Table 1.

It can be seen from Table 1, when $\beta = 0.15$ $(a = 0.3\,\mathrm{mm},\ f = 2\,\mathrm{mm})$, and $\lambda = 10.6\,\mu\mathrm{m}$ and $P_n = 40\,\mathrm{W}$, we obtain $V_{1/e^2} \approx 1.16 \times 10^{-3}\,\mathrm{mm}^3$, and

Table 1. Calculated results of the geometrical and optical parameters of each optical well for $a = 0.3\,\mathrm{mm}$, $f = 2\,\mathrm{mm}$, $l = 10.6\,\mu\mathrm{m}$ and $P_n = 40\,\mathrm{W}$.

$\Delta V_{1/e^2}$ (mm^3)	$\lvert\partial I/\partial x_0\rvert_{\max}$ ($\mathrm{W/m}^3$)	$\lvert\partial I/\partial y_0\rvert_{\max}$ ($\mathrm{W/m}^3$)	$\lvert\partial I/\partial z_0\rvert_{\max}$ ($\mathrm{W/m}^3$)	$\lvert\partial^2 I/\partial x_0^2\rvert_{\max}$ ($\mathrm{W/m}^4$)	$\lvert\partial^2 I/\partial y_0^2\rvert_{\max}$ ($\mathrm{W/m}^3$)	$\lvert\partial^2 I/\partial z_0^2\rvert_{\max}$ ($\mathrm{W/m}^3$)
1.16×10^{-3}	9.52×10^{14}	8.08×10^{14}	4.69×10^{13}	1.27×10^{20}	8.87×10^{19}	2.7×10^{16}

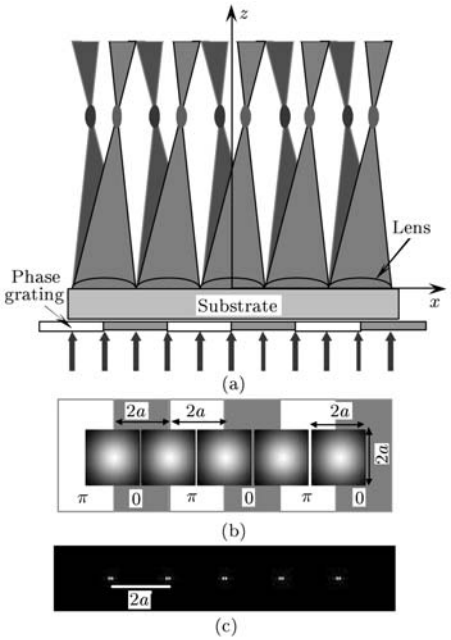

Fig. 1. Schematic diagram of a 1D array of controllable double-well optical traps: (a) side and (b) top views of 1D double-well array scheme; (c) intensity–density distribution of 1D array of double-well optical traps on the focal plane of the lens array.

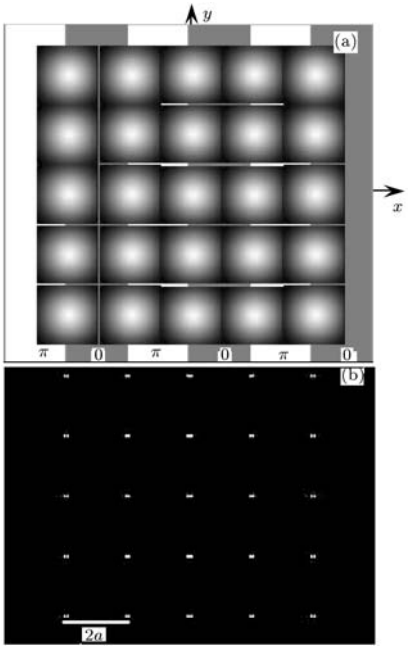

Fig. 2. Schematic diagram of a 2D array of controllable double-well optical traps: (a) top view of 2D double-well array scheme; (b) intensity–density distribution of 2D array of double-well optical traps on the focal plane of the lens array.

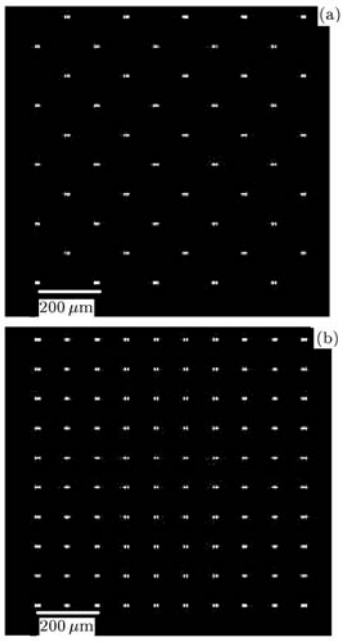

Fig. 3. Two-dimensional double-site optical lattices formed by using (a) two incident beams and (b) four beams to illuminate the optical system for Nd:YAG laser ($\lambda = 1.06\,\mu\mathrm{m}$).

have the maximum intensity gradient of greater than $4.7 \times 10^{13}\,\mathrm{W/m}^3$ and the maximum curvature of greater than $2.7 \times 10^{16}\,\mathrm{W/m}^4$. When the density of atoms in each optical well is $n = 10^{13} \sim 10^{15}/\mathrm{cm}^3$ (which is the typical density range to realize an all-optical BEC), the number of trapped particles can reach $N = 10^7 \sim 10^9$. Therefore, such an array of double-well optical traps is suitable to load and study an array of atomic (or molecular) double-well BECs, even to realize an array of all-optical atomic (or molecular) double-well BECs by optical-potential evaporative cooling.

It is well known that when a two-level atom (or molecule) moves in an inhomogeneous light field, because of the ac Stark effect, it will experience an action from the optical dipole force. When the light field is red-detuned (the laser frequency ω_l is smaller than the atomic resonance frequency ω_a, i.e. $\delta = \omega_l - \omega_a < 0$), the dipole potential $U_D(r)$, induced by the interaction of light field with atoms, is an attractive potential, and the atoms are attracted to the place of the maximum intensity. For $\omega_a - \omega_l \approx \omega_l$, the optical potential $U_D(r)$ is given by[17]

$$U_D(r) = -\frac{1}{2}\frac{\alpha_s}{[1-(\omega_l/\omega_a)^2]}|E_{rms}(r)|, \qquad (9)$$

where $|E_{rms}(r)|^2 = \sqrt{\mu_o/\varepsilon_o}I(r)$, where $I(r)$ is the intensity distribution of the light field, α_s is the static scalar atomic polarizability of the ground state.

On the other hand, when a two-level atom is moving in the light field, it will scatter a spontaneous-emission photon (including a Rayleigh or a Raman photon), and the corresponding spontaneous-scattering photon rate can be estimated by[17]

$$\begin{aligned} S_{sp} &= \sum_f \frac{8\pi\alpha^2\omega^3 I(r)}{3\hbar c^2}\Big|\sum_{m,q}\frac{\langle f|r_a^{(1)}|m\rangle\langle m|r_Q^{(1)}|i\rangle}{\omega_m-\omega} \\ &\quad + \frac{\langle f|r_Q^{(1)}|m\rangle\langle m|r_q^{(1)}|i\rangle}{\omega_m+\omega}\Big|^2 \\ &\equiv S_{\mathrm{Rayleigh}} + S_{\mathrm{Raman}}, \end{aligned} \qquad (10)$$

where

$$S_{\mathrm{Rayleigh}} = \frac{8\pi r_0^2 I(r)\omega^3}{3\hbar}\Big[\sum_e \frac{f_{eg}}{\omega_{eg}^2}\Big]^2 = \frac{8\pi r_0^2 I(r)\omega_l^3}{3\hbar\omega_a^4}, \qquad (11)$$

$$S_{\mathrm{Raman}} = \frac{8}{9}\Big(\frac{\Delta_{fs}\omega_l}{\omega_a^2}\Big)^2 \cdot S_{\mathrm{Rayleigh}}, \qquad (12)$$

where r_0 is the classical radius of an electron ($= 2.818 \times 10^{-15}\,\mathrm{m}$), and Δ_{fs} is the frequency splitting of atomic fine structure. For the D_2 line of ^{85}Rb atom,

$\alpha_s = 5.3 \times 10^{-39}\,\mathrm{m^2/J}$,[18] $\omega_a = 2\pi \times 3.85 \times 10^{14}\,\mathrm{Hz}$, and $\Delta_{fs} = 2\pi \times 7.1 \times 10^{12}\,\mathrm{Hz}$. From Eqs. (6) and (9)–(12), we calculate the maximum trapping potential of each optical well for cold ^{85}Rb atoms and its spontaneous-scattering rate, including Rayleigh and Raman scattering rates, and the results are listed in Table 2.

Table 2. Calculated results of the optical potential, spontaneous-scattering rate, Rayleigh and Raman scattering rates of trapped ^{85}Rb atom in each CO_2 and YAG laser well.

P_n (W)	λ_l (μm)	$I_{\max}$ (GW/m^2)	$U_{\max}$ (mK)	S_{Sp} (s^{-1})	S_{Rayleigh} (s^{-1})	S_{Ramam} (s^{-1})
40	10.6	16.8	1.24	1.75×10^{-3}	1.75×10^{-3}	2.86×10^{-9}
0.1	1.06	7.48	1.20	0.78	0.78	1.27×10^{-4}

It is clear from Table 2 that when $a = 0.3\,\mathrm{mm}$, $f = 2\,\mathrm{mm}$, and $\lambda = 10.6\,\mathrm{mm}$, and $P_n = 40\,\mathrm{W}$, the maximum optical potentials for ^{85}Rb atom is about 1.24 mK, the spontaneous-scattering rate (including Rayleigh and Raman scattering rates) can be completely neglected for the CO_2 laser well. For the YAG laser trap, we have $U_{\max} \approx 1.2\,\mathrm{mK}$, and $S_{Sp} \approx S_{\mathrm{Rayleigh}} \approx 0.78/\mathrm{s}$, which is smaller than 1/s. Therefore, every optical well we discuss can be used to trap cold atoms with a temperature of 20–50 μK from a standard optical molasses, or to trap cold molecules generated by quantum degenerate atomic gases,[19−22] even to realize all-optically trapped atomic BECs[23−25] (or molecular BECs[10,11,26]) by the optical-potential evaporative cooling.

In addition, we can see from Figs. 1 and 2 that when cold atoms (or cold molecules) are loaded into 1D and 2D arrays of double-well YAG or CO_2 laser traps, novel 1D and 2D double-site optical lattices with a larger lattice constant, which is similar to the 1D and 2D standing-wave optical lattices,[27] can be formed, but the lattice constant ($2a$) is far larger than that of the standing-wave optical lattice ($\lambda/2$). This is the case of single beam illumination. However, if a few incoherent laser beams with different incident angles are used to illuminate our optical system composed of a 1D π-phase grating and a 1D (or 2D) microlens array, some new double-site optical lattices with a smaller lattice constant and different lattice types can be obtained near the focal plane of the lens array. For example, if using two incoherent YAG laser beams ($a = 100\,\mu\mathrm{m}$, $f = 500\,\mu\mathrm{m}$, $\lambda = 1.06\,\mu\mathrm{m}$, and $P_n = 100\,\mathrm{mW}$) with an incident angle $\pm\theta_i = \pm a/(2f)$ to illuminate symmetrically our optical system, and when the angle between the incident plane of two laser beams and the xoz (or yoz) plane is 45°, a 2D triangular optical lattice as shown in Fig. 3(a) will be formed. In Fig. 3(b), a square optical lattice is generated by the illumination of using four incoherent beams where the incident angles of two incoherent beams in the xoz plane are $\theta_x = \pm a/(2f)$, and the angles of other two beams in the yoz plane are $\theta_y = \pm a/(2f)$, respectively. In Fig. 3, the parameters of the used 2D microlens array are $f = 500\,\mu\mathrm{m}$ and $a = 100\,\mu\mathrm{m}$. It can be seen from Fig. 3 that if using more incoherent incident beams to illuminate our optical system, we can obtain many new optical lattices with a smaller lattice constant and a more complex lattice structure.

In conclusion, we have proposed a novel scheme to construct 1D (or 2D) arrays of double-well optical traps by using a single plane wave to illuminate the optical system composed of a 1D binary π-phase grating and a 1D (or 2D) array of microscopic lenses, and calculated the intensity distribution of each optical well and its intensity gradient and curvature, and also discussed some potential applications of our array of double-well trap array in the fields of atom, molecule and quantum optics. Our study shows that when a Gaussian laser beam with a far-red-detuned and a high power is used to illuminate our optical system, a desired array of double-well optical traps with an enough high optical potential and a lower spontaneous-scattering and Rayleigh scattering rates as well as an ignorable Raman scattering rate can be formed. Therefore, such an array of double-well optical traps is very suitable to trap cold atoms (or molecules), even to realize an array of all-optical atomic (or molecular) double-well BECs by the optical-potential evaporative cooling. Moreover, our proposed 1D and 2D arrays of double-well traps have some important and wide applications in preparation of novel optical lattices, all-optical integrated atom optical chip,[28] quantum computer and quantum information processing [29,30] and so on.

References

[1] Chu S et al 1986 *Phys. Rev. Lett.* **57** 314
[2] Miller J D et al 1993 *Phys. Rev.* A **47** R4567
[3] Takekoshi T et al 1996 *Opt. Lett.* **21** 77
[4] Adams C S et al 1995 *Phys. Rev. Lett.* **74** 3577
[5] Yin J et al 1998 *Opt. Commun.* **152** 421
[6] Granade S R et al 2002, *Phys. Rev. Lett.* **88** 120405-1
[7] Loftus T et al 2002 *Phys. Rev. Lett.* **88** 183201-1
[8] Barrett M D et al 2001 *Phys. Rev. Lett.* **97** 010404-1
[9] Weber T et al 2003 *Science* **299** 232
[10] Jochim S et al 2003 *Science* **302** 2101
[11] Greiner M et al 2003 *Nature* **426** 537
[12] Hu J et al 2002 *J. Opt. Soc. Am.* **19** 2844
[13] Yin J et al 2002 *Opt. Commun.* **206** 99
[14] Grabowski A et al 2003 *Eur. Phys. J.* D **22** 347
[15] Folman R et al 2002 *Adv. At. Mol. Opt. Phys.* **48** 263
[16] Gaskill J D 1978 *Linear Systems, Fourier Transforms, and Optics* (New York: Wiley) pp 45 350
[17] Takekoshi T et al 1995 *Opt. Commun.* **114** 421
[18] Friebel S et al 1998 *Appl. Phys.* B **67** 699
[19] Herbig J et al 2003 *Science* **301** 1510
[20] Jochim S 2003 *Phys. Rev. Lett.* **91** 240402-1
[21] Cubizolles J et al 2003, *Phys. Rev. Lett.* **91** 240401-1
[22] Regal C A et al 2003 *Nature* **424** 47
[23] Barrett M D et al 2001 *Phys. Rev. Lett.* **87** 010404-1
[24] Weber T et al 2003 *Science* **299** 232
[25] Takasu Y et al 2003 *Phys. Rev. Lett.* **91** 040404-1
[26] Zwieriein M W et al 2003 *Phys. Rev. Lett.* **91** 250401-1
[27] Jessen P S et al 1996 *Adv. At. Mol. Opt. Phys.* **37** 95
[28] Birkl G et al 2001 *Opt. Commun.* **191** 67
[29] Julsgaard B et al 2001 *Nature* **413** 400
[30] Mandel O et al 2003 Nature **425** 937