Metric of Accelerating, Radiating, Rotating, and Charged Black Hole in de Sitter Space

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(Received 1 April 1998)

Metric of accelerating, radiating Kerr-Newman de Sitter black holes is derived in this letter. The metric involves (7+10) independent parameters. The volume density of space-time, and the electromagnetic potential in accelerating, radiating cases are the same as in nonaccelerating, nonradiating cases. The space-time related to acceleration is not asymptotic Minkowski flat. So the energy momentum tensor is not positive definite.

PACS: 04.20.Jb, 97.60.Lf

The physical properties of nonstationary or dynamical black holes are important problems in astrophysics. In reality, all astronomical bodies are situated in accelerating and radiating states. On the other hand, the thermodynamical properties of the stars are closely dependent on its radiation and absorption. Perhaps the acceleration also effects the thermodynamical states of the black holes. In this letter we derive the general metric of the black holes which have acceleration and radiation.

First we construct the null tetrad

$$\begin{aligned} l^\alpha =&\{0,1,0,0\},\\ n^\alpha =&\Big\{\frac{1}{\sqrt{A}}\frac{1}{\Sigma}[(r^2+a^2)-\Sigma ga\sin\theta],\\ &-\frac{1}{2\Sigma}\Delta_\lambda+\Delta_\theta dr\cos\theta+\Delta_\theta ga\sin\theta,\\ &\Delta_\theta f,\frac{1}{\sqrt{A}}\Big[\frac{a}{\Sigma}-\frac{g}{\sin\theta}\Big]\Big\},\\ m^\alpha =&\frac{1}{\sqrt{2}\rho\bar{\sigma}}\Big\{\frac{1}{\sqrt{A}}\mathrm{i}\,a\sin\theta,0,\Delta_\theta,\frac{1}{\sqrt{A}}\frac{\mathrm{i}}{\sin\theta}\Big\}, \end{aligned} \tag{1}$$

where

$$\begin{aligned} &\Delta_\lambda=r^2+a^2-2Mr+Q^2-\frac{1}{3}\Lambda r^2(r^2+a^2),\\ &\Delta_\theta=1+\frac{1}{3}\Lambda a^2\cos^2\theta,\\ &\rho=r+\mathrm{i}\,a\cos\theta,\\ &\sigma=1+\mathrm{i}\Big(\frac{1}{3}\Lambda\Big)^{1/2}a\cos\theta,\\ &f(u)=-d(u)\sin\theta+b(u)\sin\varphi+c(u)\cos\varphi,\\ &g(u)=b(u)\cos\theta\cos\varphi-c(u)\cos\theta\sin\varphi,\\ &A=\Big[1+\frac{1}{3}\Lambda a^2\Big]^{-2}, \end{aligned} \tag{2}$$

$d(u)$ is the acceleration and u is the retarded time. The definition of g in Eq. (1) is different from that of Kinnersley,[1] but our definition makes the formulas involving f and g to be more symmetric and benefit to evaluate the differentiation of $g(u,\theta,\varphi)$ with respect to θ.

According to the formula

$$g^{\alpha\beta}=l^\alpha n^\beta+l^\beta n^\alpha-m^\alpha\bar{m}^\beta-m^\beta\bar{m}^\alpha, \tag{3}$$

one can derive $g^{\alpha\beta}$:

$$\begin{aligned} g^{00}=&-\frac{1}{A}\frac{1}{\Delta_\theta\Sigma}a^2\sin^2\theta,\\ g^{01}=&\frac{1}{\sqrt{A}}\Big[\frac{1}{\Sigma}(r^2+a^2)-ga\sin^2\theta\Big],\\ g^{02}=&0,\\ g^{03}=&-\frac{1}{A}\frac{a}{\Delta_\theta\Sigma},\\ g^{11}=&-\frac{\Delta_\lambda}{\Sigma}+2\Delta_\theta rd\cos\theta+2\Delta_\theta ga\sin\theta,\\ g^{12}=&\Delta_\theta f,\\ g^{13}=&\frac{1}{\sqrt{A}}\Big(\frac{a}{\Sigma}-\frac{g}{\sin\theta}\Big),\\ g^{22}=&-\frac{\Delta_\theta}{\Sigma},\\ g^{23}=&0,\\ g^{33}=&-\frac{1}{A}\frac{1}{\Delta_\theta\Sigma\sin^2\theta}, \end{aligned} \tag{4}$$

and

$$g_d^{-1}=\det g^{\alpha\beta}=-\frac{1}{A^2\Sigma^2\sin^2\theta}. \tag{5}$$

From Eq. (4) one gets the metric $g_{\alpha\beta}$ as follows:

$$\begin{aligned} g_{00}=&A\Big[\frac{\Delta\lambda}{\Sigma}-\frac{\Delta_\theta}{\Sigma}a^2\sin^2\theta\\ &-2\Delta_\theta rd\cos\theta-\Delta_\theta\Sigma(f^2+g^2)\Big],\\ g_{01}=&\sqrt{A}, \end{aligned}$$

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$$
\begin{aligned}
g_{02} =&\sqrt{A}f\Sigma,\\
g_{03} =&A\Big\{\frac{1}{\Sigma}[\Delta_\theta(r^2+a^2)-\Delta_\lambda]\\
&+2\Delta_\theta rd\cos\theta-\frac{\Delta_\theta\Sigma g}{a\sin\theta}\\
&+\Delta_\theta\Sigma(f^2+g^2)\Big\}a\sin^2\theta,\\
g_{11} =&0,\\
g_{12} =&0,\\
g_{13} =&-\sqrt{A}a\sin^2\theta,\\
g_{22} =&-\frac{\Sigma}{\Delta_\theta},\\
g_{23} =&-\sqrt{A}\Sigma fa\sin^2\theta,\\
g_{33} =&-A\Big\{\frac{1}{\Sigma}[\Delta_\theta(r^2+a^2)^2-\Delta_\lambda a^2\sin^2\theta]\\
&+2\Delta_\theta rda^2\sin^2\theta\cos\theta-2\Delta_\theta\Sigma ga\sin\theta\\
&+\Delta_\theta\Sigma(f^2+g^2)a^2\sin^2\theta\Big\}\sin^2\theta. \qquad (6)
\end{aligned}
$$

From Eqs. (1) and (6) we have

$$g_d=\det g_{\alpha\beta}=-A^2\Sigma^2\sin^2\theta, \qquad (7)$$

and

$$
\begin{aligned}
l_\alpha =&\Big\{\sqrt{A},0,0,-\sqrt{A}a\sin^2\theta\Big\},\\
n_\alpha =&\Big\{\sqrt{A}\Big[\frac{\Delta_\lambda}{2\Sigma}-\Delta_\theta ga\sin\theta-\Delta_\theta rd\cos\theta\Big],\ 1,\ 0,\\
&\sqrt{A}[-\frac{\Delta_\lambda}{2\Sigma}+\Delta_\theta rd\cos\theta+\Delta_\theta ga\sin\theta]a\sin^2\theta\Big\},\\
m_\alpha =&\Big\{\sqrt{A}\frac{\sigma}{\sqrt{2}\rho}[\mathrm{i}\,a\sin\theta+\Sigma f-\mathrm{i}\,\Sigma g],0,\\
&-\frac{1}{\sqrt{2}\rho\bar\sigma}\Sigma,-\sqrt{A}\frac{\sigma}{\sqrt{2}\rho}[(r^2+a^2)\mathrm{i}\sin\theta\\
&-\Sigma fa\sin^2\theta+\mathrm{i}\,\Sigma ga\sin^2\theta]\Big\}. \qquad (8)
\end{aligned}
$$

It is easy to show that the null tetrad satisfies

$$
\begin{aligned}
&l_\alpha l^\alpha=n_\alpha n^\alpha=m_\alpha m^\alpha=l_\alpha m^\alpha+n_\alpha m^\alpha=0,\\
&l_\alpha n^\alpha=-m_\alpha\overline{m}^\alpha=1. \qquad (9)
\end{aligned}
$$

The electromagnitic potential A_a is a null vector and must be proportional to l_α or n_α. Here we choose that it is proportional to l_α

$$A_\alpha\mathrm{d}x^\alpha=\frac{\sqrt{A}Qr}{\Sigma}(\mathrm{d}u-a\sin^2\theta\mathrm{d}\varphi), \qquad (10)$$

which is the same as Kerr-Newman metric case.

Discussions:

1. The metric involves (7+10) parameters. The mass M, specific angular momentum a, electric charge Q, cosmological constant Λ, acceleration $d(u)$, rate change of acceleration direction $b(u)$, $c(u)$ belong to the first class; $\partial M/\partial u$, $\partial^2M/\partial u^2$, $\partial Q/\partial u$, $\partial^2Q/\partial u^2$, $\partial d/\partial u$, $\partial^2d/\partial u^2$, $\partial b/\partial u$, $\partial^2b/\partial u^2$, $\partial c/\partial u$, $\partial^2c/\partial u^2$ belong to the second class.

2. When $d=b=c=0$ the metric (6) reduces to that in Ref. 2.

3. When $Q=0$ and $\Lambda=0$, the metric reduces to that in Ref. 3.

4. When $a=0$ the metric reduces to that in Ref. 4.

5. Strictly speaking, the metric of accelerating, radiating charged black holes in de Sitter space is not derived by Wang *et al.* in Ref. 4, but be guessed by them, since, initially, they did not know the energy momentum tensor related to the acceleration, etc. Besides these, they did not know the real or concrete form or structure of the electromagnetic potential.

6. We guess that Jing jiliang *et al.*[3] have not derived the energy momentum tensor related to the accelerating, radiating Kerr black hole which ought to be put on the right-hand side of general Einstein's equations.

7. The volume density of space-time in accelerating, radiating cases, i.e., $\sqrt{-g_d}=A\Sigma\sin\theta$ is the same as in nonaccelerating, nonradiating cases.

8. The space-time related to acceleration is not asymptotic Minkowski, so the energymomentum tensor is not positive definite. These problems will be studied and be discussed elsewhere.

9. The calculation of the energy momentum tensor to be added on the right-hand side of the general Einstein-Maxwell equations is very tedious, the study of which will be given elsewhere.

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