Nondegenerate Four Wave Mixing in Rare-Earth Doped Fiber Under the Influence of Coherent Population Trapping *

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By applying a strong coupling field between a lower state and the upper state of an allowed transition to ground, an enhancement of nondegenerate four wave mixing (NDFWM) with a resonantly enhanced third-order nonlinear susceptibility in a rare-earth doped fiber can be obtained. We have derived an expression for the third-order nonlinear susceptibility using quantum theory and discussed the conversion efficiency for NDFWM. The theoretical result shows that the conversion efficiency has a maximum value while the third-order nonlinear susceptibility shows constructive interference at the Rabi frequency $\Omega_{23} = \pm\sqrt{\Gamma_2\Gamma_3}$. On account of phase-mismatch, the frequency conversion efficiency has a maximum value over a certain small range of frequency separation (such as $0 < \Delta f < 30\,GHz$ at the wavelength $\lambda_c = 1.55\,\mu m$), and the frequency conversion range increases with increase of the third-order nonlinear susceptibility.

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Since Imamoğlu and Harris proposed the concept of electromagnetically induced transparency (EIT),[1] it has been widely used to enhance the efficiency of nonlinear optical processes.[2–7] Briefly, when two resonant laser beams interact with a Λ-type three-level system, the atoms can be trapped in a coherent superposition state, which is decoupled from the excited state so that the laser beam is no longer absorbed by the medium. In particular, Ham *et al.*[7] demonstrated suppressed absorption and enhanced nondegenerate four-wave mixing (NDFWM) based on EIT in Pr^{3+}-doped Y_2SiO_5 crystal (Pr: YSO) at a temperature of 6 K with a cw laser. Our previous papers[9,10] show that Er^{3+} doped fiber and Pr^{3+} doped fiber have similar energy levels to Pr^{3+}-doped Y_2SiO_5 crystal. We predict that enhanced NDFWM based on EIT may well occur in some rare-earth highly doped fibers with a suitable substrate. In this paper we show how it is possible to generate NDFWM with resonantly enhanced susceptibility under the influence of coherent population trapping in rare-earth doped fiber. Further, it is possible to discuss light pulse suppression, stimulated Raman scattering, and optical parametric conversion with the resonantly enhanced third-order nonlinear susceptibility.

An energy-level diagram for a rare-earth doped fiber is shown in Fig. 1. We apply a strong electromagnetic coupling laser field of frequency ω_c between the lower energy state $|2\rangle$ and upper energy state $|3\rangle$. The laser field of frequency ω_d is resonant with the transition between the ground state $|1\rangle$ and upper state $|3\rangle$, and the laser field of frequency ω_a is input to create a signal field of frequency $\omega_b = \omega_a + \omega_d - \omega_c$. The laser field of frequency ω_R acts as a repump field which is used to limit the absorption of the signal field.

Since the power of the generated waves in NDFWM processes is determined by the third-order nonlinear susceptibility which exhibits the symmetry property, we first consider the process of the laser field ω_c, ω_a and ω_b inputting and calculate the total dipole moment at $\omega_d = \omega_c + \omega_b - \omega_a$. The pertinent field and polarization are defined, respectively, as

$$\begin{aligned} E(t) &= \mathrm{Re}\sum_{k=a}^{d} E(\omega_k)\exp(\mathrm{i}\omega_k t), \\ P(t) &= \mathrm{Re}\sum_{k=a}^{d} P(\omega_k)\exp(\mathrm{i}\omega_k t), \\ P(\omega_d) &= \varepsilon_0\chi_D^{(1)}(-\omega_d,\omega_d)E(\omega_d) + \frac{3}{2}\varepsilon_0 \\ &\quad\cdot\chi_D^{(3)}(-\omega_d,-\omega_a,\omega_b,\omega_c)E(\omega_a)^*E(\omega_b)E(\omega_c). \end{aligned} \tag{1}$$

The susceptibilities are calculated from the equations for the time-varying probability amplitudes of a single atom,

$$\begin{aligned} &\frac{\mathrm{d}b_1}{\mathrm{d}t} = \frac{\mathrm{i}\Omega_{12}}{2}b_2 + \frac{\mathrm{i}\Omega_{13}}{2}b_3, \\ &\frac{\mathrm{d}b_2}{\mathrm{d}t} + \mathrm{i}\Delta\bar{\omega}_{21}b_2 = \frac{\mathrm{i}\Omega_{12}^*}{2}b_1 + \frac{\mathrm{i}\Omega_{23}}{2}b_3, \\ &\frac{\mathrm{d}b_3}{\mathrm{d}t} + \mathrm{i}\Delta\bar{\omega}_{31}b_3 = \frac{\mathrm{i}\Omega_{13}^*}{2}b_1 + \frac{\mathrm{i}\Omega_{23}^*}{2}b_2, \end{aligned} \tag{2a}$$

$$\Delta\bar{\omega}_{21} = \Delta\omega_{21} - \frac{\mathrm{i}\Gamma_2}{2}, \quad \Delta\bar{\omega}_{31} = \Delta\omega_{31} - \frac{\mathrm{i}\Gamma_3}{2},$$

$$\Omega_{12} = \sum_i \frac{\Omega_{1i}\Omega_{i2}}{2}\left(\frac{1}{\omega_i + \omega_a} + \frac{1}{\omega_i - \omega_b}\right). \tag{2b}$$

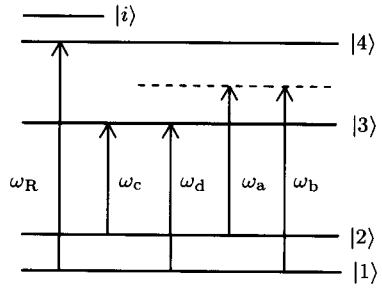


Fig. 1. Energy-level diagram for rare-earth doped fiber.

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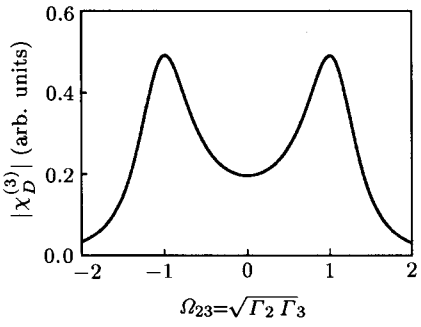


Fig. 2. $|\chi_D^{(3)}|$ as a function of normalized Rabi frequency Ω_{23}.

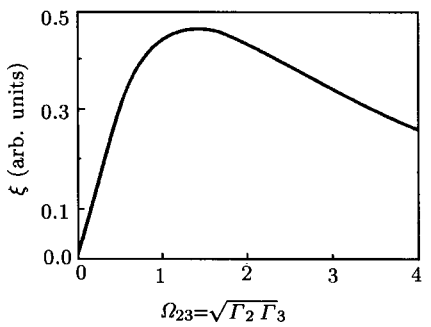


Fig. 3. Frequency conversion efficiency ξ as a function of normalized Rabi frequency Ω_{23}.

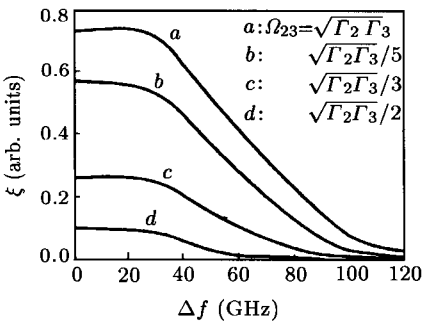


Fig. 4. Dependence of the frequency conversion efficiency ξ on frequency separation Δf for different Rabi frequencies Ω_{23}.

The Ω_{ij} are the respective Rabi frequencies; Ω_{12} is an effective Rabi frequency which is obtained by summing over intermediate states $|i\rangle$; Γ_2 and Γ_3 are noninterfering decay rates of energy states $|2\rangle$ and $|3\rangle$, respectively. The quantities $\Delta\omega_{21} = \omega_2-(\omega_b-\omega_a)-\omega_1$ and $\Delta\omega_{31} = \omega_3 - \omega_d - \omega_1$. For use below, $\mu_{lm} = \langle l|er|m\rangle$ are the pertinent matrix elements of the dipole.

We are interested in steady-state solutions and drop the derivatives in Eq. (2a). We take $|b_1|^2 = 1$, assume that Ω_{12} and Ω_{13} is very small as compared to Ω_{23}, and solve for b_2 and b_3. The per-atom moment at ω_d is $2\mathrm{Re}[\mu_{31}b_3^*\exp(\mathrm{i}\omega_d t)]$, and the third-order nonlinear susceptibility in Eq. (1) is

$$\chi_D^{(3)} = \frac{\mu_{23}\mu_{31}N}{6\varepsilon_0\hbar^3(\Delta\overline{\omega}_{21}\Delta\overline{\omega}_{31}^* - |\Omega_{23}|^2/4)^*} \cdot\sum_i \frac{\mu_{1i}\mu_{i2}}{2}\left(\frac{1}{\omega_i+\omega_a}+\frac{1}{\omega_i-\omega_b}\right), \quad (3)$$

where N is the atom density. The Rabi frequency Ω_{23} is expressed as $\Omega_{23} = \mu_{23}E_c/\hbar$.

Figure 2 shows the magnitude of $\chi_D^{(3)}$ under the condition of small detuning. It is resonantly enhanced and has a constructive interference at $\Omega_{23} = \pm\sqrt{\Gamma_2\Gamma_3}$.

We know that the time-averaged optical power P_b for the frequency component ω_b is written as[8]

$$P_b = \eta\frac{1024\pi^6}{n^4\lambda^2c^2}(D\chi_D^{(3)})^2\left(\frac{L_{eff}}{A_{eff}^2}\right)P_aP_cP_d\exp(-\alpha L), \quad (4)$$

$$\eta = \frac{\alpha^2}{\alpha^2+\Delta\beta^2}\frac{1+4\exp(-\alpha L)\sin^2(\Delta\beta L/2)}{[1-\exp(-\alpha L)]^2}, \quad (5)$$

$$\Delta\beta = \beta_b+\beta_c-\beta_a-\beta_d = \frac{2\pi\lambda_c^2}{c}\left[D_c\Delta f^2+\frac{\lambda_c^2}{2c}\frac{dD_c(\lambda_c)}{d\lambda}\Delta f^3\right], \quad (6)$$

$$\Delta f = f_b = f_c,$$

where L is the fiber length, n the refractive index of the core, λ the wavelength, c the light velocity in free space, D the degeneracy factor which can take the values $D = 1$, 3 or 6, and A_{eff} the effective area of the optical fiber. The input powers at the different frequencies are denoted by P_a, P_d and P_c, L_{eff} is the effective fiber length given by $L_{eff} = (1-e^{-\alpha L})/\alpha$, α is the attenuation coefficient and $\Delta\beta$ the propagation constant difference. We consider the case where the frequency ω_c is approximately equal to ω_d, and M is the fiber's chromatic dispersion value given by

$$D_c = -\frac{\omega_c^2}{2\pi c}\frac{d^2\beta(\omega_c)}{d\omega^2}.$$

We define $\xi = P_b/P_a$ as the frequency conversion efficiency from frequency ω_a to ω_b. The dependence of ξ as a function of the Rabi frequency Ω_{23} is shown in Fig. 3, for $\Delta\beta = 0$. As Ω_{23} increases from a small value, the conversion efficiency ξ first improves to a maximum value and then slowly decreases.

Figure 4 shows the dependence of the frequency conversion efficiency ξ on frequency separation Δf for various Rabi frequencies Ω_{23}. We consider the case where the attenuation coefficient α is small. The result shows that the nonlinear susceptibility plays an important role in the growth of the generated signal field ω_b. When Ω_{23} is large compared to the effective decay rate $\sqrt{\Gamma_2\Gamma_3}$, one obtains large frequency conversion efficiency and conversion range. For fixed Ω_{23}, the conversion efficiency can maintain a maximum value by increasing the frequency separation for small frequency separations ($\Delta f < 30$ GHz), and it decreases by increasing the frequency separation for large frequency separations. This offers the possibility of improved nonlinear device operation over all ranges of optical communications.

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