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Symmetric Mass Generation Transition and its Nonequilibrium Critical Dynamics in a Bilayer Honeycomb Lattice Model

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Symmetric mass generation (SMG) transitions defy the conventional Landau–Ginzburg–Wilson paradigm by opening a many-body gap without spontaneous symmetry breaking or topological order, attracting intense interest across particle physics and condensed matter physics. Here, we utilize unbiased quantum Monte Carlo simulations to investigate the equilibrium and nonequilibrium critical dynamics of the SMG transition in a bilayer honeycomb lattice model. We unambiguously confirm the existence of an SMG transition at $J_c = 2.584(4)$ that separates the Dirac semimetal phase from a symmetry-preserving SMG phase. High-precision extraction of the critical exponents reveals a novel universality class that profoundly departs from mean-field theory. We then extend our study to the nonequilibrium regime, exploring the driven dynamics of the SMG transition. Notably, despite the breakdown of the prerequisites for the celebrated Kibble–Zurek mechanism, the nonequilibrium SMG transition still follows the generalized finite-time scaling. By bridging equilibrium criticality and nonequilibrium dynamics, our work uncovers the universal critical properties of SMG transitions, providing a solid theoretical basis for future experimental studies of SMG physics.

Introduction

The origin of mass ranks among the core issues in fundamental physics. Within the framework of traditional theories, spontaneous symmetry breaking plays a pivotal role: bosons gain mass through the well-known Higgs mechanism, whereas fermions acquire mass via the spontaneous symmetry breaking of bosonic fields whose nonzero expectation value imparts mass to fermions through Yukawa coupling [1]. Symmetric mass generation (SMG) offers a novel mechanism for fermions to acquire mass without breaking any symmetries [2]. Furthermore, the SMG phase transition represents a fermionic version of the deconfined quantum critical point [3,4], wherein the dominant fluctuations are fractionalized degrees of freedom, transcending the conventional Landau–Ginzburg–Wilson (LGW) paradigm of phase transitions [5,6]. Accordingly, SMG and its critical behavior have garnered growing attention in recent years, particularly within the fields of particle physics and condensed matter physics [2–4,7–25].

Lattice models provide powerful platforms for investigating SMG transitions [7–11]. However, despite extensive research, the existence and precise nature of these transitions remain controversial in microscopic models. In particular, based on the variational Monte Carlo (VMC) method, the bilayer honeycomb lattice model was predicted to host a SMG transition between the Dirac semimetal (DSM) phase and the symmetric insulating phase [7]. However, the VMC approach relies strongly on the choice of the variational trial state, which may introduce unquantified systematic biases into the resulting predictions. Furthermore, the universal critical exponents governing this putative SMG transition remain largely unresolved, as illustrated in Fig. 1. Therefore, an unbiased and systematic investigation of this model is imperative to advance our fundamental understanding of SMG physics, potentially catalyzing the establishment of a generalized theory of quantum criticality beyond the LGW paradigm.

Meanwhile, nonequilibrium critical dynamics not only serves as a key research topic in fundamental statistical physics but also plays a vital role in the preparation and realization of novel quantum states of matter [26–29]. A typical kind of nonequilibrium dynamics involves linearly tuning a parameter to cross the critical point. In conventional symmetry-breaking phase transitions, the celebrated Kibble–Zurek mechanism (KZM) [30,31] and its full scaling form generalization, known as finite-time scaling (FTS) [32–36], have established a general mechanism for driven dynamics and sparked extensive research and generalization [37–72]. However, because the original KZM and FTS were historically formulated within the paradigm of spontaneous symmetry breaking (SSB), their generalization to SMG transitions, which lack a conventional order parameter, remains an open and important question. Therefore, given the fundamental significance of both SMG

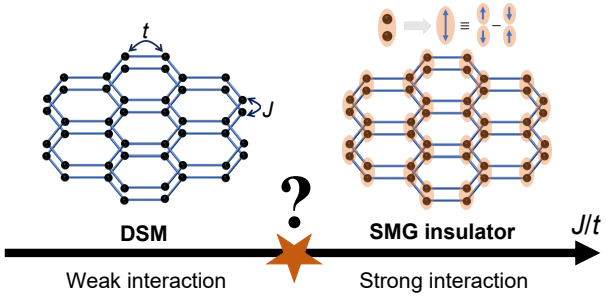

Fig. 1. Sketch of the phase diagram. In the weak-interaction regime, the system is in the DSM phase and hosts gapless Dirac fermions; while in the strong-interaction limit, the fermions acquire an energy gap, driving the system into the SMG phase, in which the spin interaction causes interlayer spin singlets (orange ellipse), without breaking the symmetry of the system. However, whether a direct SMG phase transition truly exists between these two phases was an open question prior to our unbiased DQMC calculations.

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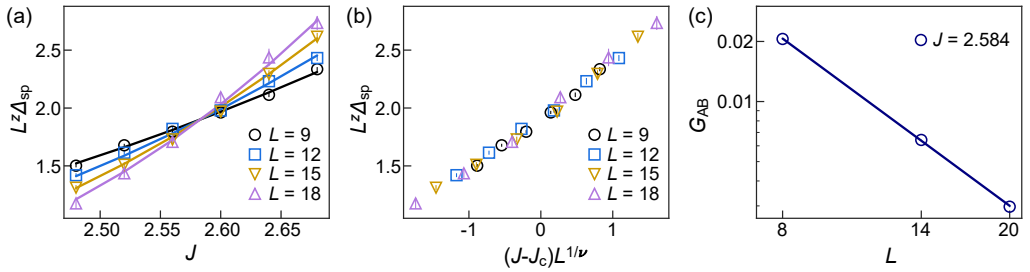

Fig. 2. Determination of critical point and critical properties. (a) The crossing point of $L^z\Delta_{sp}$ as a function of J for various L determines the critical point $J_c = 2.584(4)$. (b) The correlation-length exponent ν is estimated to be $\nu = 1.025(59)$ from the scaling collapse of the rescaled curves of $L^z\Delta_{sp}$ versus $(J - J_c)L^{1/\nu}$. (c) The fermion correlation function G_{AB} at $J_c = 2.584(4)$ is shown as a function of system size L on a log-log scale. Power-fitting gives the anomalous dimension η as $\eta = 0.11(2)$.

transitions and the KZM, an in-depth investigation into the nonequilibrium dynamics of SMG transitions is highly demanded.

In this work, we employ unbiased determinant quantum Monte Carlo (DQMC) simulations [73–76] to systematically study the bilayer honeycomb lattice model with interlayer spin coupling at half-filling. By performing comprehensive numerical calculations, we determine the ground-state phase diagram of the model and identify a phase transition between the Dirac semimetal (DSM) phase and a symmetric gapped phase. Crucially, we find that this transition proceeds without any signature of spontaneous symmetry breaking, as explicitly illustrated in Fig. 1, allowing us to unambiguously infer that the transition is an SMG transition. In addition, we determine the correlation length exponent ν and the anomalous dimension of the fermion η as $\nu = 1.025(59)$ and $\eta = 0.11(2)$, respectively. We find that these exponents differ significantly from previous theoretical predictions, calling for further investigations of the corresponding analytical theories.

Beyond equilibrium studies, we further extend our investigation to the nonequilibrium regime by exploring the driven critical dynamics of the SMG transition. Note that the conventional KZM was historically formulated within the symmetry-breaking paradigm, wherein the key dynamical scale is set by the separation of topological defects at the intersections of different broken-symmetry domains [30,31]. However, we find that FTS persists across the SMG phase transition, even in the complete absence of spontaneous symmetry breaking. On this basis, we generalize the KZM/FTS framework to nonequilibrium processes associated with SMG phase transitions for the first time. This extension not only validates the universality of KZM/FTS beyond conventional symmetry-breaking scenarios but also provides a novel theoretical frame work for characterizing nonequilibrium SMG dynamics in lattice systems.

Model and Methods

We consider a bilayer honeycomb lattice model with antiferromagnetic interlayer spin interactions at half-filling. The Hamiltonian is expressed as follows [7]:

$$H = -t\sum_{\langle ij\rangle\sigma l}\left(c^\dagger_{i\sigma l}c_{j\sigma l} + \text{h.c.}\right) + J\sum_i S_{i,1}\cdot S_{i,2}, \tag{1}$$

where t and J denote the hopping amplitude and interaction strength, respectively. $c^\dagger_{i\sigma l}$ ($c_{i\sigma l}$) represents the creation (annihilation) operator with spin polarization $\sigma = \uparrow,\downarrow$ and layer index $l = 1,2$. $\langle ij\rangle$ denotes a pair of nearest-neighbor sites, while $S_{i,l}$ is defined as $S_{i,l} = (1/2)c^\dagger_{i\alpha l}\boldsymbol{\sigma}^{\alpha\beta}c_{i\beta l}$ with $\boldsymbol{\sigma} = (\sigma_x,\sigma_y,\sigma_z)$ being the Pauli matrices. In the following, t is fixed to be unity.

As shown in Fig. 1, in the limit $J/t \to 0$, the system resides in the DSM phase with gapless Dirac fermions. Conversely, as $J/t \to \infty$, the system enters an insulating phase in which the ground state is the direct product state of spin singlets with the wave function $|\psi\rangle = \prod_i(1/\sqrt{2})(c^\dagger_{i1\uparrow}c^\dagger_{i2\downarrow} - c^\dagger_{i1\downarrow}c^\dagger_{i2\uparrow})|0\rangle$. Since this state does not break any symmetry, it constitutes an SMG insulator. However, although the VMC results suggest that a direct SMG transition exists between these two phases, the VMC approach is inherently biased, as it relies strongly on the choice of the variational trial state [7]. Consequently, the presence of an intervening symmetry-breaking phase cannot be definitively ruled out. Specifically, the primary candidate suggested by mean-field analysis is the interlayer exciton condensation (EC) order [7,25], whose order parameter is defined as $\phi_{EC} \equiv \sum_{i\sigma}(-)^i c^\dagger_{i\sigma 1}c_{i\sigma 2}$, where i is even (odd) for sublattice A (B).

The model in Eq. (1) has a prominent feature that it is free of the fermion sign problem, permitting large-scale DQMC simulations to act as an unbiased numerical tool for directly accessing its critical properties [73–78]. Given the importance of the SMG phase transition and outstanding puzzles remaining for other theoretical approaches to this model, we adopt the DQMC approach to investigate both the equilibrium and dynamical critical properties therein.

SMG Transition and Its Equilibrium Criticality

Firstly, we explore the ground state properties of this model (1) via the DQMC method. In contrast to VMC used in Ref. [7], the QMC approach employed here is unbiased. If a SMG phase transition exists, there is no corresponding order parameter to characterize it. Thus, the single-particle energy gap Δ_{sp} becomes an important indicator for characterizing this phase transition. In the thermodynamic limit, $\Delta_{sp} = 0$ indicates that the system is in the DSM phase, whereas $\Delta_{sp} > 0$ signifies a transition to the SMG insulator phase.

For finite-size systems, one can construct the dimensionless quantity as $L^z\Delta_{sp}$ with $z = 1$ being the dynamic exponent to probe the critical point. According to the general scaling theory, near the critical point J_c, $L^z\Delta_{sp}$ should satisfy

$$L^z\Delta_{sp} = \mathcal{F}((J - J_c)L^{1/\nu}), \tag{2}$$

in which $\mathcal{F}$ is the scaling function and ν is the correlation length exponent. When $J = J_c$, $L^z\Delta_{sp}$ is a constant independent of L, allowing us to precisely pinpoint the location of the critical point.

To extract Δ_{sp}, we numerically compute the single-particle imaginary-time Green's function $G(K,\tau) = \langle c_K(\tau)c^\dagger_K(0)\rangle$, in which K denotes the momentum of the Dirac point. For sufficiently large τ, $G(K,\tau)$ satisfies $G(K,\tau) \propto e^{-\Delta_{sp}(K)\tau}$. Accordingly, Δ_{sp} can be obtained by performing an exponential fit to $G(K,\tau)$ [see details in Note 3 of the Supplementary Materials (SM)]. The numerical results are shown in Fig. 2(a), in which curves of $L^z\Delta_{sp}$ versus J for different L almost intersect at a single crossing point, demonstrating that a phase transition indeed occurs at the critical point $J_c = 2.584(4)$.

We proceed to determine the critical exponents associated with this quantum phase transition. First, the correlation length exponent ν is obtained via a finite-size scaling analysis of the single-particle gap, Δ_{sp}. By rescaling the data according to the ansatz $L^z\Delta_{sp} = \mathcal{F}((J - J_c)L^{1/\nu})$, we tune ν to achieve the best data collapse for different system sizes L (see details in Note 4 of the SM). As illustrated in Fig. 2(b), this procedure yields $\nu = 1.025(59)$. Second, we determine the anomalous dimension η by computing the fermion correlation

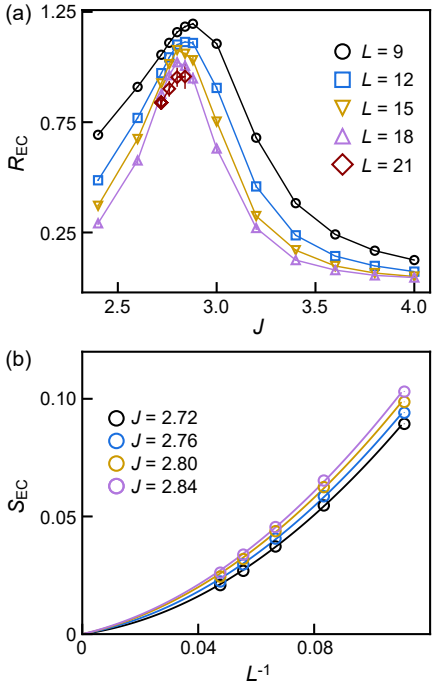

Fig. 3. Exclusion of the exciton order. (a) The correlation-length ratio for the exciton order R_{EC} as a function of J is shown for different system sizes. The lack of crossings in these curves suggests that no phase transition to long-range EC order occurs. (b) The extrapolated results of the exciton structure factor S_{EC} show that the structure factor tends to zero for $J > J_c$.

function at the critical point, $J_c = 2.584(4)$. This function, defined as $G_{AB}(L) \equiv L^{-2}\sum_{r'}\langle c^{\dagger}_{B,r'+r_m} c_{A,r'} + \text{h.c.}\rangle$ for sublattices A and B at maximum separation $r_m = (L/2, L/2)$ [79], is expected to decay as a power law. A fit of our numerical data to the scaling relation $G_{AB}(L) \propto L^{-(2+\eta)}$, shown in Fig. 2(c), gives an anomalous dimension of $\eta = 0.11(2)$.

With the critical properties established, the key open question is the nature of this phase transition and whether it corresponds to the SMG transition. Mean-field theory suggests that the leading alternative to the SMG transition is the EC transition [7]. To elucidate the mechanism of the observed phase transition, we examine the correlation-length ratio of the EC order parameter, defined as $R_{EC} \equiv S_{EC}(q)/S_{EC}(q+\Delta q) - 1$, where S_{EC} is the structure factor (the detailed definition is given in Note 2 of the SM), $q = 0$ and $\Delta q = (2\pi/L, 2\pi/L)$. If the observed transition were an EC transition, the dimensionless ratio R_{EC} would exhibit a crossing at J_c for different system sizes L. However, as shown in Fig. 3(a), R_{EC} displays no such crossing behavior. Rather, R_{EC} monotonically decreases with increasing L, providing compelling evidence for the absence of EC long-range order. This result clearly demonstrates that the observed phase transition is not an EC transition. Additionally, figure 3(b) shows the S_{EC} for $J > J_c$. We find that all extrapolated S_{EC} for $L \to \infty$ converge to zero, further suggesting the lack of EC order within this region. In addition, in Note 2 of the SM, we exclude other possible orders by using a similar procedure. These results demonstrate that the observed quantum phase transition is not driven by spontaneous symmetry breaking and is therefore an SMG transition.

Nonequilibrium Criticality

Then, we turn to the driven critical dynamics of the SMG transition in model (1). The original KZM was historically developed for symmetry-breaking phase transitions, where the scaling of topological defect density provides a natural observable [30,31]. As a generalization, the FTS theory shows that the general scaling behaviors exist for other quantities in the whole driven process [32–35]. Although the original KZM and FTS are based on the adiabatic-impulse scenario, which typically requires a gapped initial state to ensure the existence of an adiabatic stage before the system enters the impulse regime near the critical point, it has recently been shown that in symmetry-breaking phase transitions in Dirac systems, the KZM and FTS still hold when the initial state is the gapless DSM phase [54,55]. Here, we investigate whether the FTS remains applicable to the SMG transition.

We focus on the dynamic scaling behaviors of the fermion correlation function $G_{AB}(L)$. In the initial DSM phase, the dimension of $G_{AB}(L)$ is $[G_{AB}] = 2$ and $G_{AB}(L) \propto L^{-2}$; while at the critical point, the dimension $[G_{AB}] = 2 + \eta$. For driven dynamics to the critical point, when the driving rate R is large, the initial state information should still be preserved at the critical point, making $G_{AB}(L) \propto L^{-2}$. Therefore, the residual anomalous dimension η in $[G_{AB}]$ should manifest as the dependence on the driving rate R. Accordingly, for large R, the dynamic scaling relation of $G_{AB}(L)$ at the critical point J_c should satisfy

$$G_{AB}(R, L) \propto L^{-2} R^{\eta/r}, \tag{3}$$

where $r = z + 1/\nu$ is the dimension of R. In addition, for small R, the equilibrium scaling $G_{AB}(L) \propto L^{-2-\eta}$ must be restored. This requires that $G_{AB}(R, L)$ should obey the scaling form

$$G_{AB}(R, L) = L^{-2-\eta} \mathcal{F}_1(RL^r), \tag{4}$$

in which $\mathcal{F}_1(RL^r)$ should satisfy $\mathcal{F}_1(RL^r) \propto (RL^r)^{\eta/r}$ for large R.

Figure 4(a) shows the numerical results of the curve of $G_{AB}(R, L)$ versus R for different L at J_c. Fit with a power function on the curve for $L = 20$ shows that $G_{AB}(R, L) \propto R^{0.048(6)}$, in which the exponent is close to $\eta/r = 0.055(10)$ with $\eta = 0.11$ and $\nu = 1.025$ determined above as inputs. In addition, for a large R, the inset shows that $G_{AB}(L) \propto L^{-2}$. Combining all these results, we thereby establish the validity of Eq. (3). Then, by rescaling G_{AB} and R as $G_{AB}L^{2+\eta}$ and RL^r, respectively, we find that the rescaled curves match with each other, as shown in Fig. 4(b), confirming Eq. (4). In addition, for large R, from Fig. 4(b), we find the scaling function $\mathcal{F}_1(RL^r)$, which is just the rescaled curve, satisfies a power law with the exponent close to η/r, confirming that $\mathcal{F}_1(RL^r) \propto (RL^r)^{\eta/r}$.

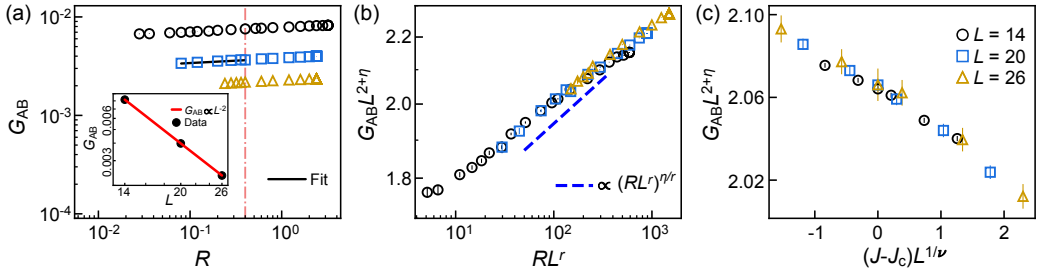

Fig. 4. Nonequilibrium dynamic scaling of the SMG transition. (a) Log-log plots of G_{AB} versus the driving rate R for different L at $J_c = 2.584(4)$. The inset in (a) shows $G_{AB} \propto L^{-2}$ at $R = 0.4$, as indicated by the dash-dotted line. For $L = 20$, the power-law fit (black solid line) gives $G_{AB} \propto R^{0.048(6)}$, with an exponent close to $\eta/r = 0.055(10)$. (b) In log-log coordinates, the rescaled curves of $G_{AB}L^{2+\eta}$ versus RL^r collapse onto a single curve, which has a slope close to η/r (dashed line). (c) Rescaled curves of $G_{AB}L^{2+\eta}$ versus $(J - J_c)L^{1/\nu}$ for a fixed $RL^r = 463.92$ and different L collapse onto a single curve.

To further reveal the dynamic scaling, we now turn to the off-critical-point effects in the whole driven process. For G_{AB}, the FTS analysis predicts that the scaling form should obey

$$G_{AB}(R, L, J - J_c) = L^{-2-\eta}\mathcal{F}_2[RL^r, (J - J_c)L^{1/\nu}]. \quad (5)$$

To examine Eq. (5), we fix RL^r to be a constant and calculate the fermion correlation function for different $(J - J_c)$. By rescaling G_{AB} and $(J - J_c)$ by $L^{2+\eta}$ and $L^{1/\nu}$ respectively, as shown in Fig. 4(c), we find that the rescaled curves collapse onto each other, thereby confirming Eq. (5). These results are twofold in their significance. First, they successfully generalize the KZM/FTS framework to the SMG phase transition, which lacks a symmetry-breaking order parameter. Second, they provide decisive and independent support for the validity of the critical point and exponents derived from equilibrium analyses.

Discussion and Conclusion

The conventional KZM was originally developed in the context of spontaneous symmetry breaking. In fact, the driving-induced length scale $\xi_d \propto R^{-1/r}$, a key quantity in the KZM, just quantifies the typical spacing of topological defects at the intersections of distinct broken-symmetry domains. However, although no symmetry breaking is present in the SMG phase transition, the dynamic scaling behaviors shown above demonstrate that ξ_d nonetheless still exists. This poses a key conceptual question: what is the physical meaning of ξ_d?

To address this question, we resort to the putative field theory describing the SMG phase transition. In usual symmetry-breaking phase transitions, the order-parameter field provides the fundamental critical fluctuations. In contrast, it was shown that the fundamental building blocks of the field theory describing the SMG phase transition are the fractionalized bosonic and fermionic partons, arising from the decomposition of physical fermions, together with the emergent gauge field [3,4]. This suggests that ξ_d may characterize the typical fluctuation scales of these fractionalized excitations under external driving. By identifying this scale, we thereby develop a nonequilibrium approach for probing the hidden characteristic length associated with fractionalization.

In summary, we utilize QMC to comprehensively investigate the properties of the bilayer model (1) on the honeycomb lattice. Our results provide unambiguous evidence for the existence of an SMG phase transition in model (1), ruling out the possibility of a phase transition associated with any symmetry-breaking phase. Additionally, we determine the critical exponents $\nu = 1.025(59)$ and the anomalous dimension $\eta = 0.11(2)$. Notably, these values deviate significantly from those obtained via VMC methods, highlighting the need for further investigation within a field-theoretic framework. Moreover, by studying the driven critical dynamics of the SMG phase transition, our work firmly establishes that FTS provides a dynamic approach to quantify the fluctuation scales of fractionalized excitations, constituting a notable generalization of the traditional KZM. Consequently, our findings represent a fundamental advance in the nonequilibrium dynamics of phase transitions beyond the LGW framework.

Note Added

Upon completing this work, we became aware of interesting related papers by He, You, and Xu that focus on the equilibrium criticality in the same model [80,81]. Consistent equilibrium results are obtained therein.

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