Chinese Physics Letters

RESEARCH HIGHLIGHT

# Breakthrough in a Century-Old Paradigm: Multipolar Anisotropy of the Anomalous Hall Effect

Cite as: Lei Wang *et al*. 2026 *Chin. Phys. Lett.* **43** 070703

DOI CSTR

## Recommended Articles

**Pressure-Tunable Large Anomalous Hall Effect in Ferromagnetic Metal $LiMn_6Sn_6$**

Lingling Gao *et al*.
*Chin. Phys. Lett.* **41**, (2024)

**Layer-Valley Hall Effect under Inversion and Time-Reversal Symmetries**
Jiaojiao Zhao, Gui-Bin Liu, Peng Chen, Yugui Yao, Guangyu Zhang, and Luojun Du
*Chin. Phys. Lett.* **41**, (2024)

**Unveiling of Terahertz Emission from Ultrafast Demagnetization and the Anomalous Hall Effect in a Single Ferromagnetic Film**
Zhiqiang Lan, Zhangshun Li, Haoran Xu, Fan Liu, Zuanming Jin, Yan Peng, and Yiming Zhu
*Chin. Phys. Lett.* **41**, (2024)

**Quantum Anomalous Hall Effect with Tunable Chern Numbers in High-Temperature 1T-$PrN_2$ Monolayer**

Xu-Cai Wu, Shu-Zong Li, Jun-Shan Si, Bo Huang, and Wei-Bing Zhang
*Chin. Phys. Lett.* **41**, (2024)

**Large Anomalous Hall Conductivity in Quasi-One-Dimensional Geometrically Frustrated Metal $Pr_3MnBi_5$**

Xinyao Li, Feihong Guan, Xinhang Shan, Xiangde Zhu, Wei Ning, and Mingliang Tian
*Chin. Phys. Lett.* **43**, 040702 (2026)

**Pressure-Enhanced Intrinsic Anomalous Hall Conductivity in the Kagome Ferrimagnet $TbMn_6Sn_6$**

Hao Sun *et al*.
*Chin. Phys. Lett.* **43**, 030704 (2026)

A series journal of Chinese Physical Society distributed by IOP Publishing

Chinese Physics Letters

*Chin. Phys. Lett.* **43** 070703 (2026)

RESEARCH HIGHLIGHT

DOI: 10.1088/0256-307X/43/7/070703
CSTR: 32039.14.0256-307X.43.7.070703

# Breakthrough in a Century-Old Paradigm: Multipolar Anisotropy of the Anomalous Hall Effect

**Lei Wang(王蕾) and Ke Xia(夏钶)***

Key Laboratory of Quantum Materials and Devices of Ministry of Education, School of Physics, Southeast University, Nanjing 211189, China

*Corresponding author. Email: kexia@seu.edu.cn

For nearly a century, textbooks have taught us a simple picture of the anomalous Hall effect (AHE): the Hall voltage is always perpendicular to the magnetization direction. Since Edwin Hall's discovery of the Hall effect in 1880 [1], this concise dipole framework has served as the standard description of the AHE in ferromagnetic materials. A collaborative work [2] by the groups of Dazhi Hou, Yang Gao, and Qian Niu at the University of Science and Technology of China (USTC) has now fundamentally overturned this century-old paradigm. They show that even in ordinary ferromagnets such as bcc Fe and fcc Ni, the anomalous Hall conductivity contains a previously overlooked octupole contribution, which can generate a measurable Hall voltage parallel to the magnetization direction.

This discovery marks a genuine paradigm shift in our understanding of the AHE—from the dipole framework to a multipolar framework. It represents the first fundamental revision of the conceptual structure underlying AHE phenomenology in a hundred years.

The modern understanding of the AHE began with the Berry-phase formulation of Sundaram and Niu (1999) [3], which linked the Hall conductivity directly to the Berry curvature of electronic bands. Yet as experimental techniques advanced, puzzling observations began to accumulate. Over the past two decades, unconventional Hall signals have been reported across diverse systems: As early as 2001, Friedland *et al.* observed an in-plane AHE in conventional bcc Fe [4], a result that defied explanation within the dipole framework for over twenty years. More recently, a giant planar Hall effect in the Weyl semimetal $Co_3Sn_2S_2$ [5], anomalous responses from localized Berry curvature in $Fe_3Sn_2$ [6], and strong-correlation effects in $SrRuO_3$ that appear to break the conventional dipole symmetry constraint [7] have been observed. Each observation was explained within its own system-specific context, and a unified understanding remained elusive.

In 2025, Zheng Liu *et al.* proposed a radically new perspective [8]: treating the AHE as a phenomenon of spin-group symmetry breaking, and performing a multipolar expansion of the anomalous Hall conductivity in magnetization space, with the spin–orbit coupling strength as a small perturbative parameter. This framework predicts that the leading correction to the dipole picture is an octupole term, whose angular dependence is dictated by the crystal symmetry of the ferromagnet.

Parallelly, the USTC experimental group grew epitaxial Fe(103) thin films on MgO(113) substrates and Ni(111) thin films on MgO(111). The distinct crystal symmetries endow the octupole term with characteristic angular signatures: a $\sin^3\theta$ dependence for Fe(103) and a $\cos 3\theta$ dependence for Ni(111). These distinct behaviors serve as unambiguous experimental fingerprints of the multipolar framework. Notably, both patterns are third-harmonic in character—this three-fold symmetry is the universal signature of the octupole contribution, being the lowest-order multipole term capable of generating a Hall signal that the dipole forbids. The particular functional form ($\sin^3\theta$ versus $\cos 3\theta$) reflects how the same cubic octupole tensor is projected onto different crystallographic film planes.

This understanding naturally connects to theoretical studies of interface and multilayer AHE [9,10]: the breaking of lattice continuous rotational symmetry is the core mechanism behind the emergence of multipolar anisotropy, and interfaces provide a particularly clear realization of this symmetry principle.

These angular dependence patterns are the "fingerprints" of the octupole theory: different crystal symmetries produce distinct multipolar anisotropy patterns, and matching these predictions to experiment constitutes the core verification of the framework.

This work contributes on five levels. (i) Quantitative verification: it represents the first experimental extraction of multipolar coefficients and direct comparison with first-principles calculations, completing the full cycle of theory–experiment–quantitative agreement [8]. (ii) Unified framework: the seemingly disparate unconventional Hall effects reported in recent years—from Weyl semimetals to oxide interfaces—are unified under the single language of magnetization multipolar anisotropy. The octupole term is not a peculiarity of specific materials. (iii) Symmetry connection: traditional interface/multilayer AHE studies and multipolar AHE share the same symmetry language; differences across systems are essentially realizations of the same tensorial expansion under different symmetries. (iv) Paradigm shift: the century-old dipole picture is fundamentally incomplete. The proper tensorial expansion must include all symmetry-allowed terms—this is not a correction but an expansion of the descriptive language itself. (v) Applications: conventional AHE can only probe out-of-plane magnetization, but in thin films, magnetization is usually locked in-plane by shape anisotropy. The octupole AHE provides a new electrical mechanism for probing in-plane magnetization, with potential impact on magnetic sensors and read heads for magnetic storage.

It is worth noting that tensorial expansion schemes including cubic terms were already discussed in classic monograph Symmetry and Magnetism (1964) [11], though the explicit connection to octupolar anisotropy was not recognized at the time. The present work focuses on cubic ferromagnets with $O_h$ symmetry, where the octupole term is the lowest-order correction allowed. For low-symmetry ferromagnets (orthorhombic, monoclinic), the octupole term takes distinct forms and may yield even richer angular dependencies. Higher-order multipolar terms, strictly forbidden in cubic symmetry, become allowed in these lower-symmetry systems and at interfaces. Detecting these weak signals will require improved measurement sensitivity and finer angular resolution, but the theoretical landscape is now wide open. The present multipolar framework is conceptually distinct from a conventional Taylor expansion of the anomalous Hall conductivity in powers of magnetization components. Rather, it performs a multipolar decomposition as a function of magnetization direction on the unit sphere, with each term directly linked to Berry curvature multipoles in

Received 13 May 2026; accepted 2 June 2026; published 10 July 2026

© 2026 Chinese Physical Society and IOP Publishing Ltd. All rights, including for text and data mining, AI training, and similar technologies, are reserved.

Check for updates

momentum space. It is this physical grounding in Berry curvature topology that endows the framework with microscopic content and quantitative predictive power, going beyond a purely phenomenological power-series expansion.

Connections to Band Topology and Berry Curvature:

The intrinsic part of the AHE is driven by the Berry curvature—a topological geometric quantity in momentum space. Octupolar anisotropy fundamentally reflects the angular anisotropy of the Berry curvature near the Fermi surface. Relating the multipolar expansion coefficients to topological invariants such as the Chern number remains a deep and largely open theoretical question.

From Hall's first measurement in 1880, through the Berry-curvature formulation in the 1990s, to the discovery of multipolar anisotropy in 2025–2026, a new chapter has been added to the century-long story of the anomalous Hall effect. The dipole picture, long thought complete, is now revealed as merely the leading term in a richer multipolar expansion.

The core contribution of this work is not the discovery of a single new effect, but the demonstration that the descriptive language we have relied on for a century is incomplete. If our long-accepted description has been missing an entire class of terms, what else have we overlooked? That question, once asked, opens a vast landscape for future research.

## References


[1] Hall E H 1880 *Proc. Phys. Soc. London* **4** 325
[2] Peng W, Liu Z, Pan H, Wang P, Chen Y, Zhang J, Yu X, Shen J, Yang M, Han J, Niu Q, Gao Y, and Hou D 2026 *Chin. Phys. Lett.* **43** 060704
[3] Sundaram G, and Niu Q 1999 *Phys. Rev. B* **59** 14915
[4] Friedland K J, Nötzel R, Schönherr H P, Riedel A, Kostial H, and Ploog K H 2001 *Physica E: Low-dimensional Systems and Nanostructures* **10** 442
[5] Gopal S R K and Singh Y 2020 *J. Magn. Magn. Mater.* **502** 166547
[6] Wang Q, Sun S, Zhang X, Pang F, and Lei H 2016 *Phys. Rev. B* **94** 075135
[7] Nishihaya S, Matsuki Y, Kaminakamura H, Sugeno H, Jiang M C, Murakami Y, Arita R, Ishizuka H, and Uchida M 2025 *Adv. Mater.* **37** e02624
[8] Liu Z, Wei M, Peng W, Hou D, Gao Y, and Niu Q 2025 *Phys. Rev. X* **15** 031006
[9] Li L, Sun J, Wang L, Wang X R, and Xia K 2025 *Phys. Rev. B* **112** 014403
[10] Bai J, Wang L, and Xia K 2026 *Phys. Rev. B* **113** 085434
[11] Birss R R 1964 *Symmetry and Magnetism* (North-Holland, Amsterdam)