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# Certifying Quantum States with Uniform Measurements

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**Qubit-resolved operations and measurements are required for most current quantum information processing schemes. However, these operations can be experimentally costly due to the need for local addressing, demanding significant classical control. A more resource-efficient alternative to extract information is uniform measurement, where a site-independent rotation of qubits is performed before measuring in the computational basis. This operation can be performed in parallel, or globally, in atom- and ion-based platforms, reducing resource cost and increasing fidelity. In this work, we initiate the exploration of the utility of this operation in quantum information processing. In particular, we demonstrate that uniform measurements can certify certain graph states, a family of highly entangled and broadly useful quantum states. We provide a sample-efficient certification algorithm with a proved performance guarantee, together with an experimental scheme based on analog-mode Rydberg atom arrays. Uniform measurements, therefore, allow direct and efficient characterization of quantum states on quantum platforms in a hitherto unexplored manner. More broadly, our work establishes "uniformity" as a meaningful and practically motivated resource rubric for quantum information processing, and offers new insights into the architectural design of quantum computing devices.**

## Introduction

Recent years have witnessed significant progress in the control of large quantum systems, ranging from large-scale simulation of quantum many-body systems [1–6] to the early demonstration of quantum error correction [7–9]. Leveraging the advantages of controllability, flexibility and tunable interactions, the atom- [10–14] and ion-based [15–17] platforms feature compelling application in quantum simulation, as well as serving as promising candidates for quantum computation.

However, challenges remain in extracting classical data from these platforms. While performing measurements is vital in quantum information processing, most known protocols are demanding and limited by experimental capabilities. When considering many practical quantum information tasks, such as observable estimation [18–22], state certification or verification [23–31] or Hamiltonian learning [32–37], the state-of-the-art protocols still require randomized measurements [38], demanding qubit-resolved operations. These operations require significant classical control over individual atoms or ions, as illustrated in Fig. 1(a). On both platforms, one needs to either engineer the control pulse spatial distribution or change the atom positions, hence inevitably increasing resource cost and infidelity. This motivates us to consider more efficient protocols that take advantage of atomic or ionic platforms on a hardware level.

In this paper we propose a novel protocol for quantum state certification, leveraging the experimental feasibility of parallel operations on atomic platforms. Our protocol utilizes only *uniform measurements*, i.e., single-qubit projection measurements along an arbitrary but qubit-independent direction. Experimentally, this is tantamount to first applying a uniform single-qubit rotation prior to a computational basis measurement, which can be achieved by applying a global laser pulse on all the atoms or ions simultaneously, see Fig. 1(b).

Despite its immediate experimental relevance [39,40], this setup has not been extensively explored. Here we find that uniform measurements are capable of certifying certain quantum states. Among them, an especially interesting example is a family of *graph states*, which serve as the resource for measurement-based quantum computation [41,42]. Preparation and verification of such states have been a central target of current experiments, as demonstrated on platforms including the photon–quantum dot system [43], the Rydberg atom array [44], and the photon–atom system [45]. In these and related works,

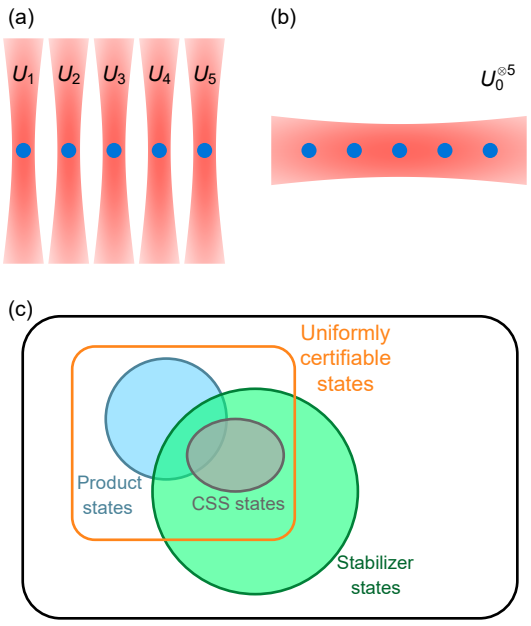


**Fig. 1.** Quantum state measurements and hierarchy of states. (a) General quantum state measurements. Depending on the scheme, one first applies some unitary operations before measuring in the $Z$-basis. (b) Uniform measurements. In this case, we restrict the unitary operations to be uniform. Uniformly certifiable states are those that can be verified from uniform measurements. (c) The relation of uniformly certifiable states with several important types of states. We show that both product states and Calderbank–Shor–Steane (CSS) states are uniformly certifiable. Furthermore, there are interesting stabilizer states beyond the CSS type that are also uniformly certifiable.

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verification typically relies on stabilizer measurements that are technically demanding. Our protocol replaces them with uniform global operations, providing an immediately friendlier alternative to these platforms.

In the following, we first characterize the capability of uniform measurements. After a few illuminating simple examples, we discuss how to certify certain graph states. We provide a sample-efficient certification algorithm with a proved performance guarantee, using only uniform measurements. We also propose an experimental scheme based on the Rydberg atom array operated in analog mode. We conclude with a discussion of the broader architectural relevance and possible further extensions.

## Quantum State Certification

Given a quantum state $\rho$ synthesized in the lab, quantum state certification is the task of certifying whether the fidelity $\langle\psi|\rho|\psi\rangle$ to a target state $|\psi\rangle$ is sufficiently close to one. For a generic $|\psi\rangle$, the projector $|\psi\rangle\langle\psi|$ is a highly complicated non-local operator, which makes it difficult to directly measure its expectation value. In modern certification protocols designed for a fully-controllable quantum computer, global unitary operations or qubit-resolved random measurements are usually necessary [23–31].

In this paper, we deal with a special scenario: $|\psi\rangle$ is the ground state of some sparse Hamiltonian with a known energy [46–49], such as the ground states of physically local Hamiltonians or the stabilizer states. For these states, the parent Hamiltonian $H$ is a sum of local (or low complexity) terms $H=\sum_i h_i$. Then $|\psi\rangle$ can be efficiently certified by measuring each $h_i$ and comparing the results to the ground state energy. Note that despite its simplicity, estimating $h_i$ still typically requires qubit-resolved operations and measurements [50–52]. We will see how uniform measurements help to certify a class of these states in the following.

## Uniform Measurements

Before demonstrating the utility of uniform measurements for certification, we now characterize their capacity.

**Definition 1** (Uniform measurements). *We call a quantum measurement a uniform measurement if it applies a global parallel single-qubit rotation $\bigotimes_{i=1}^{n} U$ before doing the computational basis measurement.*

For the sake of illustration, we first restrict ourselves to uniform measurements of qubits in the $x-z$ plane. Generalization to an arbitrary direction on the Bloch sphere is straightforward. Uniform measurement in the $x-z$ plane can be performed by measuring in the $Z$ basis after a site-independent rotation in the $x-z$ plane $U=\prod_{i=1}^{N}\exp(-\mathrm{i}Y_i\theta/2)$ on all the $N$ qubits, and reading measurement outputs from any subset $I\subseteq\{1,...,N\}$. This gives the expectation value

$$\left\langle \bigotimes_{i\in I}(\cos\theta Z_i+\sin\theta X_i)\right\rangle. \tag{1}$$

Expanding the expression, we get a linear combination of expectation values of Pauli strings, each of which is a symmetrized summation of a product of $\alpha$ $Z$ operators and $(|I|-\alpha)$ $X$ operators,

$$E(\alpha;I)=\left\langle \sum_{\substack{J\subseteq I\\ |J|=\alpha}}\bigotimes_{\substack{j\in J\\ k\in I\setminus J}} Z_jX_k\right\rangle,\quad \alpha=0,...,|I|, \tag{2}$$

weighted by a factor $\cos^{\alpha}\theta\sin^{|I|-\alpha}\theta$. Then by repeating with $|I|+1$ different $\theta$'s, one can obtain all the expectation values $E(\alpha;I)$. Taking $I=\{1,2,3\}$ as an example, what can be measured is $E(0;I)=\langle X_1X_2X_3\rangle$, $E(1,I)=\langle X_1X_2Z_3+X_1Z_2X_3+Z_1X_2X_3\rangle$, $E(2;I)=\langle X_1Z_2Z_3+Z_1X_2Z_3+Z_1Z_2X_3\rangle$ and $E(3;I)=\langle Z_1Z_2Z_3\rangle$.

Taking into account all three directions $x,y,z$, one can estimate the expectation values of all the symmetrized operators with uniform measurements.

**Definition 2** (Symmetrized operators). *An operator $O$ is called a symmetrized operator if on its support $S\subseteq[n]$, $O$ commutes with all the symmetric group elements $\pi\in\mathcal{S}_{|S|}$.*

For any size-$k$ subset $I$ of $N$ qubits, the number of independent symmetric operators is $\binom{k+2}{2}$. So the total number of independent operators one can evaluate is

$$\mathcal{N}(N)=\sum_{k=1}^{N}\binom{N}{k}\binom{k+2}{2}=2^{N-3}(N^2+7N+8)-1. \tag{3}$$

Note that $\mathcal{N}(N)<4^N-1$ except for $N=1$. So we do not expect enough information for a full tomography of a generic density matrix. However, we will see that much can be learned from only uniform measurements.

## Elementary Examples

**Example 1**. Any direct product state can be certified with uniform measurements. For a product state $|\psi\rangle=\otimes_{i=1}^{N}|\psi_i\rangle$, we only need to estimate the fidelity for every $|\psi_i\rangle$. Because $|\psi_i\rangle\langle\psi_i|$ can be expanded with Pauli operators, $\langle\psi_i|\rho|\psi_i\rangle$ can be estimated by uniform measurements through $x,y,z$ directions and analyzing the results for each qubit separately.

Since we will mostly focus on stabilizer states below, we wish to point out first that uniform measurements have capability beyond such states. More generally, we have the following result.

**Proposition 1.** If two states $\psi_1,\psi_2$ defined on different qubits can each be uniformly certified, then so can the product state $\psi_1\otimes\psi_2$. This follows directly from the definition.

**Example 2.** Calderbank–Shor–Steane (CSS) states [53,54]. These states are stabilizer states where the generators can be chosen to be products of either only $Z$ or only $X$ operators. Such states lie at the heart of current quantum error correction schemes. The CSS states can be certified by first performing uniform measurements along $x$ and $z$ directions, and then estimating each stabilizer. This example shows that uniform measurements are also capable of capturing a rich entanglement structure.

## Graph States

In the above examples, the stabilizers of the states all have a symmetric structure on their supports. Surprisingly, the certification capability of uniform measurements goes beyond stabilizer states with only symmetric stabilizers. We find that all bipartite graph states with even degrees on one partition can be certified with uniform measurements. As an illustration, we demonstrate this with an example of one-dimensional (1D) graph state below.

The 1D graph state $|\psi_g\rangle$ is depicted in Fig. 2(a) For an $N$-qubit chain, $|\psi_g\rangle$ is stabilized by $Z_{i-1}X_iZ_{i+1}$ for $2\le i\le N-1$, together with $X_1Z_2$ and $Z_{N-1}X_N$ on the boundaries. Here we use $Z_i$ and $X_i$ to denote Pauli operators on the $i$th qubit. $N$ is chosen to be an odd number, so that any qubit $i\in$ even has an even degree. Note that the one-dimensional graph state without boundary stabilizers serves as a symmetry-protected topological phase [55] and a resource state for measurement-based quantum computation [56], and experimental realizations thereof have been actively pursued [57–59]. Our certification protocol easily generalizes to it. Here we keep the boundary terms for the sake of generality.

Assume that the synthesized state in the lab matches exactly the 1D graph state, $\rho=|\psi_g\rangle\langle\psi_g|$. One can verify it through the following procedures. First, note that $\rho$ has a $\mathbb{Z}_2$ symmetry generated by $U_X=\prod_{i\in\text{odd}}X_i$, whose existence can be verified by uniform measurements. Next, one estimates the expectation values of bulk stabilizers by measuring

$$M_i=Z_{i-1}X_iZ_{i+1}+Z_{i-1}Z_iX_{i+1}+X_{i-1}Z_iZ_{i+1} \tag{4}$$

for $2\le i\le N-1$, and boundary stabilizers by measuring

$$M_1=X_1Z_2+Z_1X_2,\quad M_N=X_{N-1}Z_N+Z_{N-1}X_N, \tag{5}$$

**Algorithm 1:** Certifying the 1D graph state

**Input:** $3T$ samples of unknown quantum state $\rho$.

**Output:** CERTIFIED or FAILED depending on the fidelity $\langle\psi_g|\rho|\psi_g\rangle$ (See Theorem 1).

1 Take uniform measurements along the basis $X^{\otimes N}$, $\left(\frac{X+Z}{\sqrt{2}}\right)^{\otimes N}$, and $\left(\frac{X-Z}{\sqrt{2}}\right)^{\otimes N}$ for $T$ times each. Collect the measurement outcomes $b_x^{i,t}, b_{xz+}^{i,t}, b_{xz-}^{i,t} = \pm 1$ of each measurement event of each qubit, for qubits $i = 1, 2, \cdots N$ and measurement events $t = 1, 2, \cdots T$.

2 Estimate $\langle U_X\rangle$ by the empirical average $\widetilde{\mathbb{E}}[U_X] = \frac{1}{T}\sum_{t=1}^{T}\prod_{i\in\text{odd}} b_x^{i,t}$. If $\widetilde{\mathbb{E}}[U_X] < 1 - 13\epsilon/4$, output FAILED.

3 Estimate $\langle M_i\rangle$ by the empirical average

$$\widetilde{\mathbb{E}}[M_i] = \frac{1}{T}\sum_{t=1}^{T}\left[\sqrt{2}\left(b_{xz+}^{i-1,t}b_{xz+}^{i,t}b_{xz+}^{i+1,t} + b_{xz-}^{i-1,t}b_{xz-}^{i,t}b_{xz-}^{i+1,t}\right) - b_x^{i-1,t}b_x^{i,t}b_x^{i+1,t}\right]$$

for $2 \le i \le N-1$, and

$$\widetilde{\mathbb{E}}[M_1] = \frac{1}{T}\sum_{t=1}^{T}\left[b_{xz+}^{1,t}b_{xz+}^{2,t} - b_{xz-}^{1,t}b_{xz-}^{2,t}\right]$$

$$\widetilde{\mathbb{E}}[M_N] = \frac{1}{T}\sum_{t=1}^{T}\left[b_{xz+}^{N-1,t}b_{xz+}^{N,t} - b_{xz-}^{N-1,t}b_{xz-}^{N,t}\right]$$

If $\widetilde{\mathbb{E}}[M_i] > 1 - 9\sqrt{\epsilon}$ for all $1 \le i \le n$, output CERTIFIED. Otherwise output FAILED.

through uniform measurements [Eq. (2)]. $Z_{i-1}Z_iX_{i+1}$, $X_{i-1}Z_iZ_{i+1}$, $Z_1X_2$ and $X_{N-1}Z_N$ all anticommute with $U_X$, so their expectation values should all be zero, according to the following lemma.

**Lemma 1.** *Let $\mathcal{P}_1, \mathcal{P}_2 \in \{I, X, Y, Z\}^{\otimes N}$ be two Pauli strings and $\{\mathcal{P}_1, \mathcal{P}_2\} = 0$. Then their expectation values $|\mathcal{P}_i\rangle = \mathrm{Tr}(\rho\mathcal{P}_i)$ with any quantum state $\rho$ should satisfy $|\mathcal{P}_1\rangle^2 + \langle\mathcal{P}_2\rangle^2 \le 1$.*

For a single-qubit state, $\langle X\rangle^2 + \langle Z\rangle^2 \le 1$ always holds. A general proof can be found in the Supplementary Materials (SM). From the lemma, the expectation values of stabilizers in Eqs. (4) and (5) should all be exactly one. This verifies $\rho = |\psi_g\rangle\langle\psi_g|$.

More generally, when the fidelity $\langle\psi_g|\rho|\psi_g\rangle$ is less than one, the estimated values of Eqs. (4) and (5) can be bounded with Lemma 1. Building on the analysis, we develop a general certification algorithm Algo. 1 for the one-dimensional graph state that works for finite fidelity, using only uniform measurements. The algorithm has a proved performance guarantee:

**Theorem 1.** *For any $\epsilon < 1/(64N^2)$, Algorithm 1 outputs* CERTIFIED *with probability at least $2/3$ when $\langle\psi_g|\rho|\psi_g\rangle > 1-\epsilon$, and outputs* FAILED *with probability at least $2/3$ when $\langle\psi_g|\rho|\psi_g\rangle < 1 - 8N\sqrt{\epsilon}$, using a number of samples at most $T = O(1/\epsilon^2)$.*

The proof can be found in the SM.

This certification algorithm works far beyond the 1D systems. A general graph state on a graph $G = \{V, E\}$ is defined to be stabilized by $X_v\prod_{(v,v')\in E}Z_{v'}$ for all $v \in V$, as illustrated in Fig. 2(b). Consider $G$ to be a bipartite graph with partitions $A$ and $B$, and assume the degrees of $v \in A$ are always even. Then $U_X = \prod_{v\in B}X_v$ is a symmetry of the graph state. Define $V'(v) = \{v'|(v,v') \in E\} \cup \{v\}$ to be the neighbors of $v$ and itself. The expectation values of all the operators

$$\sum_{v'\in V'(v)}\left(X_{v'}\prod_{v''\in V'(v)\setminus\{v'\}}Z_{v''}\right),\ v \in V \tag{6}$$

can be bounded using $U_X$ and Lemma 1. So Algo. 1 generalizes directly to these cases. We note that in general if the stabilizer term is supported on $L$ qubits, it generally requires measurement of $L$ distinct symmetrized operators. However, in most physically relevant settings, including 1D and 2D lattice graphs, $L$ is a small constant independent of system size. Even for more complex constructions such as expander graphs, the degree (and hence $L$) remains bounded by a constant. So the $L$-dependence does not affect the efficiency of our algorithm.

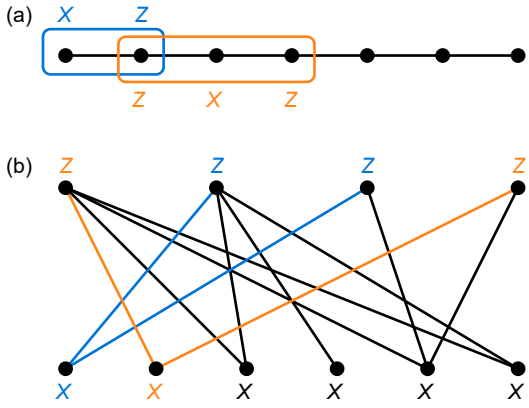


**Fig. 2.** Illustration for graph states. (a) A graph state on a 1D chain, also known as the 1D cluster state. We mark the stabilizer on the left boundary $X_1Z_2$ in blue and a bulk stabilizer $Z_2X_3Z_4$ in orange. (b) A subclass of bipartite graph states, where every vertex on the upper side has an even number of adjacent edges. We mark two stabilizers in blue and orange respectively. The product of all $X$ operators on the lower side is in the stabilizer group.

## Experimental Scheme in a Rydberg Atom Array

Now we propose an experimental scheme to prepare and certify the 1D graph state, using our certification protocol. We first discuss a unitary evolution that prepares the 1D graph state. Previously, such states have been prepared in the quantum computation modes [44], namely within two hyperfine levels, in the Rydberg atom array where Rydberg interaction is used to perform a two-qubit gate [14]. The current protocol, however, works exclusively in the quantum simulation mode.

The Hamiltonian of the system reads

$$H/2\pi = \frac{\Omega(t)}{2}\sum_i(|r_i\rangle\langle g_i| + |g_i\rangle\langle r_i|) - \Delta(t)\sum_i n_i + \sum_{i<j}\frac{C_6}{|i-j|^6}n_in_j, \tag{7}$$

where $|g, r\rangle$ denote the ground and Rydberg states respectively. The population of the ground state atoms can be measured by imaging the system after ionizing the Rydberg atoms. The two states serve as the spin degree of freedom of the graph state. $n = |r\rangle\langle r| = (1-Z)/2$ is the Rydberg state number operator. $\Omega(t)$ and $\Delta(t)$ are the Rabi frequency and detuning, and $C_6$ measures the strength of the van der Waals interaction. The system is defined on an $N$-site open chain with $N$ being odd. We take $C_6 = 1$ as the energy scale. Starting from the initial state $|g\rangle^{\otimes N}$, we first rotate by an angle of $\pi$ along the $(1,0,1)$-direction, preparing the state $|+\rangle^{\otimes N} = [(|g\rangle + |r\rangle)/2]^{\otimes N}$. In terms of Rydberg operators, we take $\Omega = \Delta = h$ for a large value $h$ and hold for $t = 1/(2\sqrt{2}h)$. For a realistic Rydberg system where $h$ is a few MHz, we need to choose the nearest-neighbor interaction to be around a hundred kHz, corresponding to a lattice constant of 7 μm for two Rb atoms excited to the 50s state. Then we turn off $\Delta$ and $\Omega$, and the Rydberg interaction entangles the state, leading to the 1D graph state. Indeed, the Rydberg interaction results in a nearest neighbor $CZ$ gate when applied for a time interval $t = 1/2$, and $CZ_{ij}X_jCZ_{ij} = Z_iX_j$. The time sequence of this preparation scheme is shown in Fig. 3(a).

Once the 1D graph state is prepared, we can perform Algo. 1 to certify it. The protocol involves collecting $N+1$ expectation values, namely $\langle U_X\rangle = \langle\prod_{i\in\text{odd}}X_i\rangle$ and that of Eqs. (4) and (5). In order to estimate these expectation values, all we need is to measure in three different directions, namely $x$, $(x+z)/\sqrt{2}$, and $(x-z)/\sqrt{2}$. They require a further rotation before measuring each atom, as shown in Figs. 3(b)–3(d). After a straightforward calculation, the $N+1$ measurement results can be obtained.

Note that while general uniform measurements of a weight-$|I|$

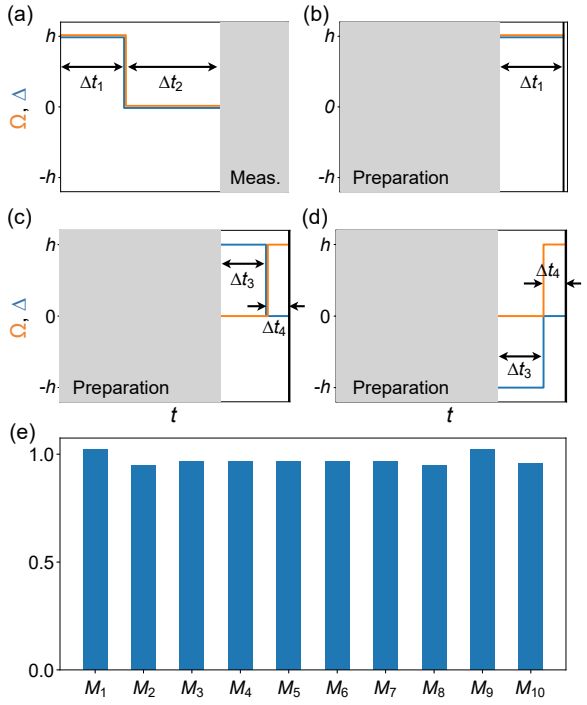


**Fig. 3.** Experimental scheme in a Rydberg atom chain with an odd number of sites and open boundary conditions. In the numerical simulation we take $N = 9$. (a)–(d) The time sequence of preparing and measuring the graph state. The measurement is performed along the (b) $x$, (c) $(x+z)/\sqrt{2}$, and (d) $(x-z)/\sqrt{2}$ directions. Blue and orange lines denote the detuning $\Delta(t)$ and the Rabi frequency $\Omega(t)$, respectively. The time intervals are $\Delta t_1 = 1/(2\sqrt{2}h)$, $\Delta t_2 = 1/2$, $\Delta t_3 = 1/(4h)$, $\Delta t_4 = 1/(8h)$. The vertical black line denotes the moment of measurement. (e) The measurement results for $M_i$, $i = 1,\ldots,10$. We take $h = 20C_6$ in all calculations.

cluster require $|I|+1$ independent measurements, only $|I|=3$ are needed in the above example. The reason is that this combination $(XZZ+ZXZ+ZZX)$ only involves a fixed odd number of $X$'s (one in this case) in each term. All such combinations can be measured by one less than the general case. To see this, we consider a general $|I|$ and measure in angles of (a) $\theta = n\pi/|I|$ and $\theta = \pi - n\pi/|I|$ in pairs ($n \in [\lfloor |I|/2 \rfloor]$), (b) $\theta = 0$ for all $|I|$, and (c) $\theta = \pi/2$ for odd $|I|$. These are $|I|+1$ independent measurements. Now, to obtain any combination involving a fixed odd number of $X$'s, one only needs (a) or (a, c) depending on the parity of $|I|$. This is because there are $\lfloor (|I|+1)/2 \rfloor$ such combinations and hence we need the same number of equations. The measurements in (a) result in $\lfloor |I|/2 \rfloor$ equations, which suffice for even $|I|$ and need one more equation for odd $|I|$, both giving the total count of $|I|$.

We numerically illustrate the validity of our experimental scheme through a $N = 9$ system, where the results can be found in Fig. 3(e). Here we label $M_1 = \langle X_1Z_2 + Z_1X_2 \rangle$, $M_9 = \langle Z_8X_9 + X_8Z_9 \rangle$ and $M_i = \langle Z_{i-1}X_iZ_{i+1} + X_{x-1}Z_iZ_{i+1} + Z_{i-1}Z_iX_{i+1} \rangle$ for $2 \le i \le 8$. $M_{10}$ denotes $\langle U_X \rangle$. All values are close to unity, as expected. The errors come from $h$ being finite and Rydberg interactions beyond the nearest neighbor.

## Discussions and Conclusions

In this work, motivated by an attempt to alleviate classical control, we propose to use uniform measurements to certify quantum states. We characterize the capacity of uniform measurements, and illustrate its rich behavior through several examples. In particular, for the non-CSS case of the graph state, we propose a rigorous certification algorithm as well as an experimental protocol based on the analog-mode Rydberg atom array. This proposal relies on standard techniques on the platform and is very much within current experimental capacities.

While we present the graph states as a concrete example, uniformly certifiable states may be much more general. However, finding all the uniformly certifiable states seems to be a challenging task, for they lack a good mathematical structure. We leave this exciting and useful question for further explorations.

Our work offers new insight into the architectural design of quantum computing devices. Modular and distributed quantum architectures are now being demonstrated [9]. In such settings, different zones of a device may be tasked with state preparation ("manufacturing"), computation ("operating"), and quality control ("verifying"). Our scheme naturally supports this division: the verifying zone can operate without local addressing, enabling the reallocation of scarce resources (e.g., optical components, lasers) to the operating zone and improving overall resource-efficiency.

Broadly speaking, the complexity of quantum computation involves multiple rubrics, with the number of qubits and gates being the most basic [60]. Upon more careful inspection, depending on the nature of the quantum platform, one also expects certain operations to be more resource-efficient than others. Generally, we expect that uniform measurements will lead to a significant improvement of resource cost across various platforms. This might be quantified by a more nuanced definition of complexity, and can be used to systematically improve the development of quantum subroutines. The current work initiates an early step in this direction.

An important practical aspect of quantum state verification is error localization. In our framework, uniform measurements restrict accessible observables to symmetrized operators, which limits spatial resolution. As a result, the protocol focuses on certifying global properties such as fidelity, rather than pinpointing errors at the single-qubit level. Nevertheless, coarse-grained diagnostic information can still be obtained. For graph states, the measured quantities ($M_i$) are associated with local stabilizer structures. The deviations from their ideal values can indicate the presence of errors in corresponding regions. Achieving finer error localization would likely require going beyond strictly uniform measurements, for example by incorporating limited local control or combining multiple measurement settings. Exploring such extensions is an interesting direction for future work.

## Availability of Data and Materials

The codes and data for our numerical calculations are available at https://github.com/chengshul/Uniform_measurement.

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