$\mathcal{O}_k$ Null Test with Multi-Task Gaussian Processes: Cosmic Curvature and Data Compatibility

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The $\mathcal{O}_k$ null test can not only assess whether the cosmic curvature is zero—thereby, if true, reducing degeneracies between cosmic curvature and other cosmological parameters—but also provide a model-independent check of compatibility between different data sets. However, traditional implementations often require absolute distance data from Type Ia supernovae (SNe Ia) or baryon acoustic oscillation (BAO) measurements, limiting their applicability because such absolute distance data are usually not accessible. The BAO Alcock–Paczynski (AP) parameter $F_{\rm AP}$ is a measurement of a distance ratio, making the Dark Energy Spectroscopic Instrument (DESI) AP measurements particularly well suited for the $\mathcal{O}_k$ null test, as no absolute distance measurements are required. We propose a novel null test of cosmic curvature tailored to DESI BAO data that combines $F_{\rm AP}$ with ratios such as $D_{\rm V}'/D_{\rm V}$ or $D_{\rm M}'/D_{\rm M}$. Crucially, this construction eliminates the need for absolute distance measurements. We further develop multi-task Gaussian processes to perform the null test. This approach can also be applied to a joint DESI BAO and SNe Ia dataset, and we find that DESI BAO and SNe Ia data are compatible. Although there is $\sim 2\sigma$ evidence of nonzero curvature at low redshift $z \lesssim 0.5$, this result is not conclusive, largely due to the lack of observational data in the corresponding redshift range.

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Since the discovery of the accelerated expansion of the Universe in 1998 from observations of Type Ia supernovae (SNe Ia),[1,2] the concept of dark energy has been introduced as a phenomenological explanation for cosmic acceleration. The cosmological constant Λ is the simplest candidate and fits naturally into Einstein's field equations. However, theoretical estimates of Λ based on vacuum energy density exceed the observational value by many orders of magnitude, a severe discrepancy known as the cosmological constant problem.[3] Despite the fact that we know dark energy has negative pressure and a gravitational interaction that behaves as repulsive rather than attractive, the nature of dark energy remains unknown. A central question in contemporary cosmology is whether dark energy is a true cosmological constant or a dynamical component whose equation of state evolves with time.

Observationally, efforts to distinguish a constant Λ from dynamical dark energy commonly use parameterizations of the dark energy equation of state $w = p/\rho$. A widely used form is the Chevallier–Polarski–Linder (CPL) parametrization,[4,5] $w(a) = w_0 + w_a(1-a)$ with a being the scale factor, which captures a broad class of scalar-field dynamics and allows for a time-varying equation of state. Recent joint analyses combining baryon acoustic oscillation (BAO) measurements from the Dark Energy Spectroscopic Instrument (DESI),[6,7] cosmic microwave background data from Planck,[8] and SNe Ia compilations such as Pantheon Plus,[9] Union3,[10] and the five-year Dark Energy Survey (DES Y5) results[11,12] have reported evidence favoring dynamical dark energy over a pure cosmological constant. If confirmed, these results would have profound implications for fundamental physics and our understanding of cosmic evolution.

The evidence for cosmic acceleration and dynamical dark energy, however, depends sensitively on models and combinations of data. To make the evidence more reliable and robust, this paper addresses two important questions: (1) Can we probe the Universe in a model-independent way? and (2) Are different data sets mutually compatible so that they can be combined to infer cosmological evolution and matter contents reliably? There is an extensive literature on model-independent reconstructions of cosmic expansion.[13–71] For example, from the definition of cosmic deceleration, $q = -\ddot{a}/(aH) > 0$, one obtains the constraints on the Hubble parameter $H(z) = \dot{a}/a \geq H_0(1+z)$ or the luminosity distance $H_0 d_{\rm L} \leq (1+z)\ln(1+z)$ for a spatially flat universe with $\Omega_k = 0$; here an overdot means the derivative with respect to the cosmic time t, a subscript 0 denotes the value of the quantity at present, the redshift $z = a_0/a - 1$ and $H_0 = 100h\,{\rm km/(s{\cdot}Mpc)}$ is the Hubble constant. Thus, the measurements of the luminosity distance $d_{\rm L}$ or the dimensionless Hubble parameter

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$E(z) = H(z)/H_0$ from SNe Ia can provide direct, model-independent evidence of cosmic acceleration without assuming a particular cosmological model or even a specific theory of gravity.[13–19] This is the null test of cosmic acceleration. In practice, however, the zero-point calibration problem of SNe Ia limits the direct application of such model-independent null tests.

Fortunately, DESI BAO data[7] provide measurements of the ratio between the transverse comoving distance and the sound horizon at the drag epoch, $D_{\rm M}(z)/r_{\rm d}$; the ratio between the line-of-sight distance and the sound horizon at the drag epoch, $D_{\rm H}(z)/r_{\rm d}$; and their correlations; or the Alcock–Paczynski (AP) parameter, $F_{\rm AP} = D_{\rm M}/D_{\rm H}$; the ratio between the isotropic BAO distance and the sound horizon at the drag epoch, $D_{\rm V}(z)/r_{\rm d}$; and their correlations. Thus, one can use the AP parameter to reconstruct $E(z)$ for a spatially flat universe[20,21] via Gaussian process (GP) regression[22–27] to perform a null test of cosmic acceleration. The line-of-sight distance variable is defined as

$$D_{\rm H} = \frac{c}{H(z)}, \tag{1}$$

where c is the speed of light. The sound horizon $r_{\rm d}$ at the drag epoch $z_{\rm d}$ is

$$r_{\rm d} = \int_{z_{\rm d}}^{\infty} \frac{c_{\rm s}(z)}{H(z)} {\rm d}z, \tag{2}$$

where $c_{\rm s}(z)$ is the speed of sound in the baryon–photon plasma. The transverse comoving distance $D_{\rm M}$ is given by

$$D_{\rm M}(z) = \frac{c}{H_0\sqrt{|\Omega_{k0}|}} {\rm sinn}\left[\sqrt{|\Omega_{k0}|}\int_0^z \frac{{\rm d}z'}{E(z')}\right], \tag{3}$$

where

$$\frac{{\rm sinn}(\sqrt{|\Omega_{k0}|}\,x)}{\sqrt{|\Omega_{k0}|}} = \begin{cases} \dfrac{\sinh(\sqrt{|\Omega_{k0}|}\,x)}{\sqrt{|\Omega_{k0}|}}, & \Omega_{k0} > 0, \\ x, & \Omega_{k0} = 0, \\ \dfrac{\sin(\sqrt{|\Omega_{k0}|}\,x)}{\sqrt{|\Omega_{k0}|}}, & \Omega_{k0} < 0. \end{cases}$$

The isotropic BAO distance is defined as

$$D_{\rm V}(z) = [zD_{\rm M}^2(z)D_{\rm H}(z)]^{1/3}. \tag{4}$$

Note that at $z = 0$, $D_{\rm M}(z) = F_{\rm AP}(z) = D_{\rm V}(z) = 0$, and from Eq. (1) we can use the data $D_{\rm H}/r_{\rm d}$ to get

$$H_0 r_{\rm d} = \frac{c}{D_{\rm H}(z=0)/r_{\rm d}}. \tag{5}$$

Because $F_{\rm AP}$ is a ratio that is independent of the (unknown) sound horizon $r_{\rm d}$, it avoids the SNe Ia zero-point issue when providing model-independent evidence for cosmic acceleration.[20,21]

With nonzero cosmic curvature Ω_k, the phantom crossing favored by DESI BAO data can be avoided,[72,73] so determining whether $\Omega_k = 0$ is also crucial. The $\mathcal{O}_k$ diagnostic,[21,28–30] $\mathcal{O}_k(z) = E(z)D'(z) - 1$,[30] offers a model-independent test of spatial flatness, where $D'(z) = {\rm d}D(z)/{\rm d}z$ and $D(z) = H_0 D_{\rm M}(z)$. Applying this diagnostic typically uses SNe Ia to reconstruct $D(z)$, which reintroduces the zero-point calibration problem. To avoid that and exploit DESI BAO AP data, the $\mathcal{O}_k$ diagnostic can be recast as $\mathcal{O}_k(z) = F_{\rm AP}(z)\,D'_{\rm M}(z)/D_{\rm M}(z) - 1$.[21] In this expression, the dependence on the sound horizon $r_{\rm d}$ cancels in the ratios $D'_{\rm M}/D_{\rm M}$ and $D_{\rm M}/D_{\rm H}$ constructed from DESI BAO data, so the null test of cosmic curvature depends only on the assumption of the cosmological principle. If $\mathcal{O}_k(z) = 0$, then the model-independent implications are: (1) the cosmic curvature is zero, removing concerns about the degeneracy between Ω_k and w; and (2) the data sets used to reconstruct $\mathcal{O}_k(z) = 0$ are compatible. Moreover, one can reconstruct the ratio $D'_{\rm M}/D_{\rm M}$ from SNe Ia without suffering from the zero-point calibration problem. Combining DESI BAO AP data with the ratio $D'_{\rm M}/D_{\rm M}$ derived from SNe Ia therefore not only provides a null test of cosmic curvature but also a model-independent test of the compatibility between DESI BAO and SNe Ia data.

Because DESI Data Release 2 (DR2) BAO measurements are reported either as $(D_{\rm M}/r_{\rm d}, D_{\rm H}/r_{\rm d})$ with correlations or as $(F_{\rm AP}, D_{\rm V}/r_{\rm d})$ with correlations, the simple combination of $F_{\rm AP}$ and $D'_{\rm M}/D_{\rm M}$, the reconstruction of $F_{\rm AP}$ from DESI BAO AP data, and the reconstruction of $D_{\rm M}/r_{\rm d}$ and $D'_{\rm M}/r_{\rm d}$ from the DESI BAO data $D_{\rm M}/r_{\rm d}$ using single-task GP (SGP) do not properly account for these correlations. A more consistent approach is to reconstruct $F_{\rm AP}$ from $F_{\rm AP}$ and $D_{\rm V}/r_{\rm d}$ joint data, including their correlations, using the multi-task Gaussian process (MGP) method developed in the Supplementary Materials (SM), and to use $F_{\rm AP}$ together with $D'_{\rm V}/D_{\rm V}$ in the $\mathcal{O}_k$ diagnostic so that correlations are handled correctly. In this work, we do this and use DESI DR2 BAO AP and SNe Ia data to perform a null test of spatial flatness and test the consistency between data sets.

We first use the MGP method to check the consistency of DESI BAO data by reconstructing $F_{\rm AP}$ from AP data only, from the joint data $F_{\rm AP}, D_{\rm V}/r_{\rm d}$ and their correlations, and from the joint data $D_{\rm M}/r_{\rm d}, D_{\rm H}/r_{\rm d}$ and their correlations. The details of the MGP reconstruction by including the correlations are presented in the SM. The reconstructed results are shown in Fig. 1. As we can see from Fig. 1, the results from these three methods are consistent with each other even when correlations are not considered; therefore, the DESI BAO DR2 data are self-consistent.

For a spatially flat universe with $\Omega_{k0} = 0$,

$$D_{\rm M}(z) = \frac{c}{H_0}\int_0^z \frac{{\rm d}z'}{E(z')}, \tag{6}$$

so $D'_{\rm M}(z) = D_{\rm H}(z)$ and $F'_{\rm AP}(z=0) = 1$. Taking this relation into account and considering the correlation between $D_{\rm M}$ and $D_{\rm H}$ in the DESI BAO DR2 dataset, we reconstruct $D_{\rm M}$ and $D_{\rm H} = D'_{\rm M}$ with MGP, and the result is shown as the green dashed line in Fig. 2. Substituting the MGP result from Fig. 2 into Eq. (5), we obtain $r_{\rm d}h = 100.61 \pm 1.62\,{\rm Mpc}$ under the assumption of $\Omega_k = 0$. This result is consistent with the Planck 2018 value $r_{\rm d}h = 98.82 \pm 0.82\,{\rm Mpc}$, which is based on the flat ΛCDM model,[8] so Planck data are consistent with DESI BAO data if $\Omega_k = 0$.

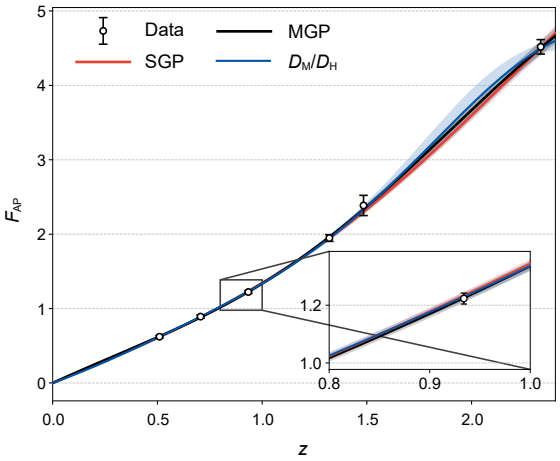

Fig. 1. The reconstructed $F_{\rm AP}$ along with the 1σ error from DESI BAO DR2 data. The constraints $D_{\rm M}(z=0)=0$ and $F_{\rm AP}(z=0)=0$ are imposed. The red line labeled as SGP is reconstructed from the data $F_{\rm AP}$ only. The black line labeled as MGP is reconstructed from the joint data $F_{\rm AP}$, $D_{\rm V}/r_{\rm d}$ and their correlations. The blue line labeled as $D_{\rm M}/D_{\rm H}$ is reconstructed from the joint data $D_{\rm M}/r_{\rm d}$, $D_{\rm H}/r_{\rm d}$ and their correlations.

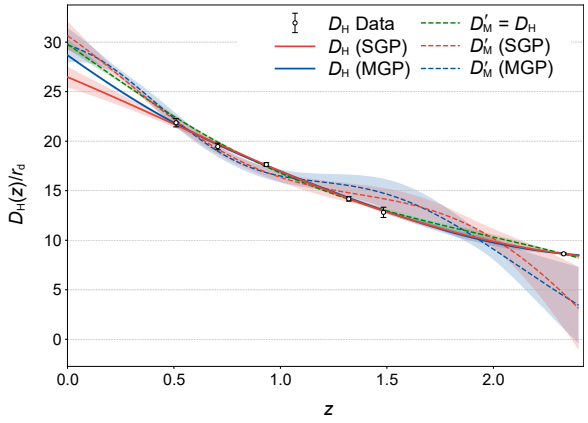

Fig. 2. The reconstructed $D_{\rm H}/r_{\rm d}$ and $D'_{\rm M}/r_{\rm d}$ along with the 1σ error from DESI BAO DR2 data. The constraint $D_{\rm M}(z=0)=0$ is imposed. The green line labeled as $D'_{\rm M}=D_{\rm H}$ is reconstructed from the data $D_{\rm M}/r_{\rm d}$, $D_{\rm H}/r_{\rm d}$, and their correlations, taking $D_{\rm H}/r_{\rm d}$ as $D'_{\rm M}/r_{\rm d}$. The blue lines labeled as MGP are reconstructed from the data $D_{\rm M}$, $D_{\rm H}$, and their correlations. The red lines are reconstructed separately from the respective data alone. The solid lines correspond to $D_{\rm H}/r_{\rm d}$, and the dashed lines correspond to $D'_{\rm M}/r_{\rm d}$.

Now we discuss the null test of spatial curvature. Based on the relation $D'_{\rm M}=D_{\rm H}$ for a spatially flat universe and the independence of $r_{\rm d}$ from $F_{\rm AP}$, the $\mathcal{O}_k$ null test was proposed,[21]

$$\mathcal{O}_k(z)=F_{\rm AP}(z)\frac{D'_{\rm M}(z)}{D_{\rm M}(z)}-1. \tag{7}$$

The ratio $D'_{\rm M}/D_{\rm M}$ cleverly eliminates the dependence on the sound horizon. However, DESI DR2 BAO data either measure $D_{\rm M}/r_{\rm d}$ and $D_{\rm H}/r_{\rm d}$, or $F_{\rm AP}$ and $D_{\rm V}/r_{\rm d}$, i.e., either $D_{\rm M}$ and $D_{\rm H}$ are correlated, or $F_{\rm AP}$ and $D_{\rm V}$ are correlated, so the combination of $F_{\rm AP}$ and $D_{\rm M}$ does not properly account for the correlation between $F_{\rm AP}$ and $D_{\rm M}$. Instead,

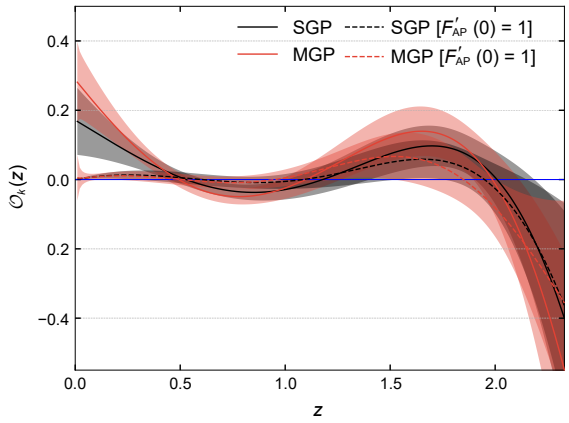

Fig. 3. The reconstructed $\mathcal{O}_k$ with its 1σ uncertainty from DESI BAO measurements of $F_{\rm AP}$ and $D_{\rm V}/r_{\rm d}$. The constraints $D_{\rm V}(z=0)=0$ and $F_{\rm AP}(z=0)=0$ are imposed. Solid curves show the posterior mean; shaded regions indicate the 1σ uncertainties. We also impose the constraint $F'_{\rm AP}(z=0)=1$, and the results are shown with dashed curves. SGP corresponds to reconstructing $F_{\rm AP}$ and $D_{\rm V}/r_{\rm d}$ separately from the respective data sets $F_{\rm AP}$ and $D_{\rm V}/r_{\rm d}$. MGP denotes the reconstruction with the joint data set $F_{\rm AP}$, $D_{\rm V}/r_{\rm d}$ and their correlations.

we propose the $\mathcal{O}_k$ null test:

$$\mathcal{O}_k=F_{\rm AP}\frac{D'_{\rm V}}{D_{\rm V}}+\frac{1}{3}F'_{\rm AP}-\frac{1}{3}\frac{F_{\rm AP}}{z}-1=0. \tag{8}$$

The main difference between this work and our previous work (Ref. [21]) lies in the development of the MGP procedure and the simultaneous reconstruction of the functions $F_{\rm AP}$, $D_{\rm V}/r_{\rm d}$ and their derivatives, or $D_{\rm M}/r_{\rm d}$, $D_{\rm H}/r_{\rm d}$ and their derivatives, from the joint data sets $F_{\rm AP}$, $D_{\rm V}/r_{\rm d}$ and their covariance, or $D_{\rm M}/r_{\rm d}$, $D_{\rm H}/r_{\rm d}$ and their covariance, which are used to perform the null test.

Combining DESI BAO DR2 data $F_{\rm AP}$, $D_{\rm V}/r_{\rm d}$, and their covariance, we calculate $\mathcal{O}_k$ using Eq. (8). The results are shown in Fig. 3. For comparison, we also show the result obtained when the correlations are ignored; i.e., $F_{\rm AP}$ and $D_{\rm V}/r_{\rm d}$ are reconstructed separately from their respective data sets without considering their correlations. From Fig. 3, we see that $\Omega_k\neq 0$ at $z\lesssim 0.5$. The reconstructed results with and without considering the correlations are similar. Due to the degeneracy between Ω_k and w, the nonzero spatial curvature at $z\lesssim 0.5$ and the phantom crossing of dark energy may have the same origin, suggesting that the phantom crossing preferred by DESI BAO data could be an artifact of the flat CPL parametrization. However, because there is no BAO data in the redshift range $z\lesssim 0.5$, the reconstructed results may not be reliable. If we impose the constraint $F'_{\rm AP}(0)=1$, we find that the results shown in Fig. 3 are consistent with zero curvature. Therefore, we suspect that the apparent nonzero curvature at $z\lesssim 0.5$ may be caused by the lack of data in DESI DR2 in that redshift range.

To test whether the deviation from spatial flatness at $z\lesssim 0.5$ is caused by the lack of data in that redshift regions, we test whether $D'_{\rm M}=D_{\rm H}$ in the data. We first reconstruct $D'_{\rm M}/r_{\rm d}$ from the $D_{\rm M}/r_{\rm d}$ data and $D_{\rm H}/r_{\rm d}$ from

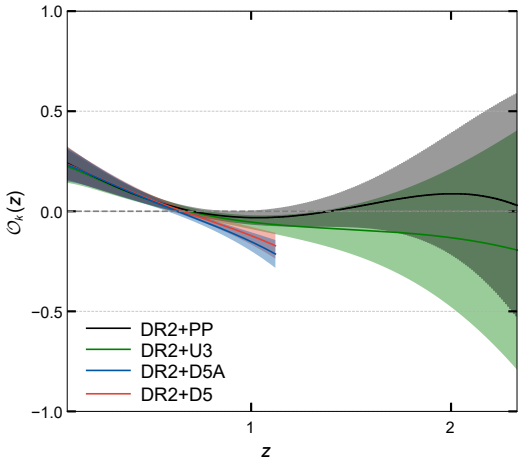

Fig. 4. The reconstructed $\mathcal{O}_k$ along with the 1σ errors from DESI BAO AP and SNe Ia. The constraint $F_{\rm AP}(z=0)=0$ is imposed. D5A and D5 refer to the original and recalibrated DES Y5 data, respectively.

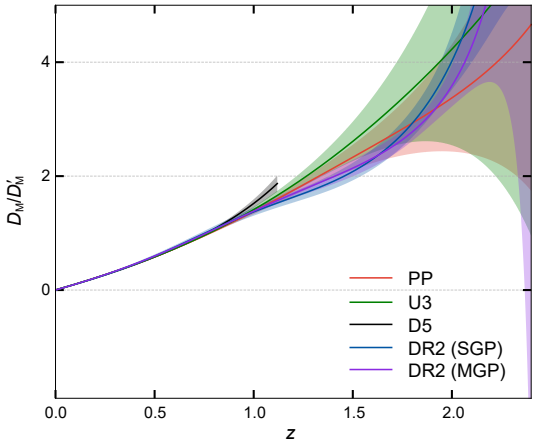

Fig. 5. The reconstructed $D_{\rm M}/D'_{\rm M}$ along with the 1σ errors from DESI BAO and SNe Ia. The constraint $D_{\rm M}(z=0)=0$ is imposed for the reconstruction of the DESI BAO data. The blue line labeled as SGP is reconstructed from the $D_{\rm M}$ data only; the purple line labeled as MGP is reconstructed from the joint data set $D_{\rm M}$, $D_{\rm H}$ and their correlations.

the $D_{\rm H}/r_{\rm d}$ data. The results are shown in Fig. 2 with red lines. Then we reconstruct $D'_{\rm M}/r_{\rm d}$ and $D_{\rm H}/r_{\rm d}$ from the joint data $D_{\rm M}/r_{\rm d}$, $D_{\rm H}/r_{\rm d}$ and their correlations to test whether $D_{\rm H}=D'_{\rm M}$. The results, labeled as MGP, are shown in Fig. 2. From Fig. 2, we see that $D_{\rm H}(z)=D'_{\rm M}(z)$ at $z\gtrsim 0.5$, even though the correlations are not considered; but $D_{\rm H}(z)\neq D'_{\rm M}(z)$ at $z\lesssim 0.5$, even though the reconstruction is performed jointly and the correlations between $D_{\rm M}$ and $D_{\rm H}$ are included. The reconstructed results with and without including the correlations are similar. Therefore, at $z\lesssim 0.5$, $\mathcal{O}_k\neq 0$ may be due to $D_{\rm H}\neq D'_{\rm M}$ in the DESI BAO data and is caused by the lack of data in this redshift range.

For the null test (7), we can also combine DESI BAO and SNe Ia data. We use three SNe Ia samples: the Pantheon Plus sample of 1550 spectroscopically confirmed SNe Ia,[9] the Union3 compilation of 2087 SNe Ia,[10] and the Dark Energy Survey (DES) five-year compilation of 1829 SNe Ia.[11,12] For brevity, we denote these data sets PP (Pantheon Plus), U3 (Union3), and D5 (DES Y5), respectively. Combining DESI BAO DR2 $F_{\rm AP}$ data with each SNe Ia sample, we compute $\mathcal{O}_k$ using Eq. (7); the results are shown in Fig. 4. Figure 4 again shows $\Omega_k\neq 0$ at $z\lesssim 0.5$, suggesting that the deviation is driven by the lack of BAO data in the low-redshift region. Additional observations, especially more low-z BAO measurements, are therefore needed to robustly test spatial flatness. Unlike U3 and PP, the original D5 sample (D5A) also indicates a deviation from $\Omega_k=0$ at $z\gtrsim 0.5$,[11] but with the recalibrated D5 sample,[12] the deviation becomes smaller. Therefore, the $\mathcal{O}_k$ null test is not only robust for testing spatial flatness but also powerful for detecting problems in the data.

Finally, we further examine the consistency between the BAO and SNe Ia data sets using the ratio $D_{\rm M}/D'_{\rm M}$. We reconstruct $D_{\rm M}/D'_{\rm M}$ using the three SNe Ia samples: U3, PP, and D5, with the joint BAO measurements $D_{\rm M}/r_{\rm d}$, $D_{\rm H}/r_{\rm d}$ and their correlations, and with $D_{\rm M}/r_{\rm d}$ alone. The resulting comparisons of $D_{\rm M}/D'_{\rm M}$ are shown in Fig. 5. From Fig. 5, we see that the SNe Ia and BAO data are compatible with each other, confirming the results obtained from the $\mathcal{O}_k$ null test.

In conclusion, we propose novel null tests of cosmic curvature specifically tailored to DESI BAO data: $\mathcal{O}_k(z)=F_{\rm AP}(z)D'_{\rm M}(z)/D_{\rm M}(z)-1$ and $\mathcal{O}_k=F_{\rm AP}D'_{\rm V}/D_{\rm V}+F'_{\rm AP}/3-F_{\rm AP}/(3z)-1$. Our reconstructions with MGP indicate that a spatially flat universe is disfavored by DESI DR2 BAO data at low redshift $z\lesssim 0.5$, a region where BAO measurements are currently sparse. This pattern suggests that the apparent deviation from flatness, and even the phantom crossing of dark energy, may be driven by incomplete low-z BAO coverage rather than by a genuine physical consequence. Therefore, additional observations, particularly more BAO measurements at low redshift, are required to robustly test spatial flatness. Reconstructions performed with and without including the full correlations yield similar results. We also find that the DESI BAO DR2 data are self-consistent and that the SNe Ia data sets are broadly consistent with the DESI DR2 BAO data. The $\mathcal{O}_k$ null test is not only robust for testing spatial flatness but also a powerful tool for assessing data compatibility and uncovering potential data issues.

Notes Added. While we were developing a GP method to jointly reconstruct two functions from two data sets, including their correlations, the Ref. [74] appeared. The idea and preliminary results for reconstructing $F_{\rm AP}$ without accounting for correlations, to test the compatibility between DESI BAO data and SNe Ia data, were presented in talks by one of the authors (YG) at Lanzhou University on September 19 and at the University of Science and Technology of China on September 25.

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