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Berry-Curvature Effect in Anomalous Transport

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A theory of the Berry-phase effect in anomalous transport in ferromagnets driven by statistical forces such as the temperature gradient or chemical potential gradient has been developed by Di Xiao *et al.* in Phys. Rev. Lett. 97 026603 (2006) based on a field-dependent density of states generating an additional contribution to the current density. We present an alternative derivation of the Berry-phase effect in anomalous transport in altermagnets and noncentrosymmetric metals based on the quasiclassical matrix kinetic equation. Along with the anomalous Hall effect, anomalous heat flow and anomalous thermoelectric currents are derived.

Introduction

It is well known that the group velocity of a Bloch electron contains an anomalous term proportional to the mechanical force [1,2], i.e.,

$$v_i^n = \frac{\partial \varepsilon_{\boldsymbol{k}}^n}{\partial k_i} + e\Omega_{ij}^n E_j, \tag{1}$$

where Ω_{ij}^n is the Berry curvature tensor of the n-th band with energy $\varepsilon_{\boldsymbol{k}}^n$. Corresponding Hall conductivity is

$$\sigma_{ij} = e^2 \sum_n \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3} \Omega_{ij}^n n(\varepsilon_n). \tag{2}$$

Here $n(\varepsilon_n) = \left\{\exp(\varepsilon_n - \mu)/T + 1\right\}^{-1}$ is the Fermi–Dirac distribution function. We put $\hbar = 1$ here and everywhere below.

The formula (2) is derived based on the modification of the Bloch functions of electrons in the presence of a mechanical force [1,2]. The Berry phase also manifests in transport driven by statistical forces induced by temperature gradient or chemical potential gradient. This problem has been solved in the Ref. [3], where the currents corresponding to thermoelectric transport were found and the corresponding Onsager relation between transport coefficients connecting thermoelectric Hall currents and forces was established. This was achieved by using the field-dependent density of states introduced in Ref. [4], generating an additional contribution to the current density originating from the magnetic moment of an electron wave packet [5], which has a finite spread in phase space.

The theory developed in Refs. [1–5] operates with spinless fermions. Thus, the Berry curvature, the corresponding magnetic moment of wave packets, and the magnetic-field-dependent density of states originate from the electron orbital degrees of freedom. This allows derivation of the anomalous transport properties in systems possessing this type of Berry curvature. Another situation is realised in altermagnetic and also in noncentrosymmetric metals, where the electron band dispersion exhibits spin-dependent band splitting characterized by the Berry curvature. In this case, the theory of anomalous electric, thermal, and thermoelectric transport can be developed based on another mechanism, making use of the matrix quasiclassical kinetic equation.

The purpose of this article is to present the corresponding derivation.

Band Splitting

The Kramers degeneracy of electron states, known in metals, is lifted in ferromagnetic materials as well as in noncentrosymmetric metals, that is, metals with a crystal structure laking mirror symmetry. This leads to a splitting of the electron band energies [6,7].

A new class of magnetic materials has been introduced recently, in which sublattices with opposite spins can be coupled by proper or improper rotations, but cannot be coupled by translation or inversion. The electronic band structure in these materials consists of momentum-dependent spin-split bands, but, like antiferromagnets, lacks spontaneous bulk magnetization. The term “altermagnets” has been proposed for materials of this type. The approach allowing determination of the band structure has been implemented based on the so-called exchange approximation, which includs symmetry operations only in spin space that are compatible with crystal symmetry [8]. Another approach to altermagnets, based on traditional symmetry classification of magnetic materials, which includes both nonrelativistic and relativistic interactions, has been developed in Refs. [9,10].

Each electron energy band in altermagnetic or noncentrosymmetric metal has the form [9,10]

$$\hat{\varepsilon} = \varepsilon\sigma_0 + \boldsymbol{\gamma}\cdot\boldsymbol{\sigma}, \tag{3}$$

where $\varepsilon = \varepsilon(\boldsymbol{k})$ denotes the spin-independent part of the spectrum, σ_0 is the unit 2×2 matrix in the spin space, $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli matrices. The pseudovector

$$\boldsymbol{\gamma} = \boldsymbol{\gamma}(\boldsymbol{k}) = \gamma_x(\boldsymbol{k})\hat{x} + \gamma_y(\boldsymbol{k})\hat{y} + \gamma_z(\boldsymbol{k})\hat{z} \tag{4}$$

is periodic in the reciprocal space; it is even in altermagnets $\boldsymbol{\gamma}(-\boldsymbol{k}) = \boldsymbol{\gamma}(\boldsymbol{k})$ and odd in noncentrosymmetric metals $\boldsymbol{\gamma}(-\boldsymbol{k}) = -\boldsymbol{\gamma}(\boldsymbol{k})$. It satisfies $g\boldsymbol{\gamma}(g^{-1}\boldsymbol{k}) = \boldsymbol{\gamma}(\boldsymbol{k})$, where g is any symmetry operation in the point group $\mathcal{G}$ of the crystal.

The eigenvalues of the matrix (3) are

$$\varepsilon_+(\boldsymbol{k}) = \varepsilon + \gamma, \quad \varepsilon_-(\boldsymbol{k}) = \varepsilon - \gamma, \tag{5}$$

where $\gamma = |\boldsymbol{\gamma}(\boldsymbol{k})|$. The corresponding eigenfunctions are given by

$$\Psi_\sigma^+(\boldsymbol{k}) = \frac{1}{\sqrt{2\gamma(\gamma+\gamma_z)}}\begin{pmatrix}\gamma+\gamma_z\\ \gamma_+\end{pmatrix},$$
$$\Psi_\sigma^-(\boldsymbol{k}) = \frac{1}{\sqrt{2\gamma(\gamma+\gamma_z)}}\begin{pmatrix}-\gamma_-\\ \gamma+\gamma_z\end{pmatrix}, \tag{6}$$

where $\gamma_\pm = \gamma_x \pm \mathrm{i}\gamma_y$. The eigenfunctions obey the orthogonality conditions

$$\Psi_\sigma^{\alpha\dagger}(\boldsymbol{k})\Psi_\sigma^\beta(\boldsymbol{k}) = \delta_{\alpha\beta}, \quad \Psi_{\sigma_1}^\alpha(\boldsymbol{k})\Psi_{\sigma_2}^{\alpha\dagger}(\boldsymbol{k}) = \delta_{\sigma_1\sigma_2}. \tag{7}$$

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Here, a summation over the repeating spin $\sigma =\uparrow,\downarrow$ or band $\alpha = +,-$ indices is implied.

There are two Fermi surfaces determined by the equations

$$\varepsilon_+(\boldsymbol{k}) = \mu, \quad \varepsilon_-(\boldsymbol{k}) = \mu, \tag{8}$$

and the Fermi velocities are given by the derivatives

$$\boldsymbol{v}_+ = \frac{\partial\varepsilon_+(\boldsymbol{k})}{\partial\boldsymbol{k}}, \quad \boldsymbol{v}_- = \frac{\partial\varepsilon_-(\boldsymbol{k})}{\partial\boldsymbol{k}}, \tag{9}$$

taken at points $\boldsymbol{k}$ lying on the Fermi surfaces.

Each band acquires momentum-dependent spin splitting. The Berry curvatures of splitted subbands are

$$\Omega^{\pm}_{ij}(\boldsymbol{k}) = -2\,\mathrm{Im}\frac{\partial\Psi^{\dagger\pm}_{\sigma}}{\partial k_i}\frac{\partial\Psi^{\pm}_{\sigma}}{\partial k_j} = \mp e_{\alpha\beta\gamma}\frac{\gamma_\alpha}{2\gamma^3}\left(\frac{\partial\gamma_\beta}{\partial k_i}\frac{\partial\gamma_\gamma}{\partial k_j}\right). \tag{10}$$

Thus, the Hall conductivity is

$$\begin{aligned}\sigma_{ij} &= e^2\int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\left[\Omega^+_{ij}n(\varepsilon_+) + \Omega^-_{ij}n(\varepsilon_-)\right]\\ &= e^2\int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\Omega^+_{ij}\left[n(\varepsilon_+) - n(\varepsilon_-)\right].\end{aligned} \tag{11}$$

Let us now calculate all currents using the Boltzmann equation.

Kinetic Equation Approach

The quasiclassical Boltzmann equation for a matrix distribution function $\hat{n}(\boldsymbol{k},\boldsymbol{r},t)$ of quasiparticles in a spin-1/2 Fermi liquid under a magnetic field has been derived by V. P. Silin [11]. It is also applicable to the description of an electron gas occupying the states in the spin-split conduction band of altermagnets or noncentrosymmetric metals. It has the following form:

$$\frac{\partial\hat{n}}{\partial t} + \frac{1}{2}\left(\frac{\partial\hat{\varepsilon}}{\partial\boldsymbol{k}}\frac{\partial\hat{n}}{\partial\boldsymbol{r}} + \frac{\partial\hat{n}}{\partial\boldsymbol{r}}\frac{\partial\hat{\varepsilon}}{\partial\boldsymbol{k}}\right) - \frac{1}{2}\left(\frac{\partial\hat{\varepsilon}}{\partial\boldsymbol{r}}\frac{\partial\hat{n}}{\partial\boldsymbol{k}} + \frac{\partial\hat{n}}{\partial\boldsymbol{k}}\frac{\partial\hat{\varepsilon}}{\partial\boldsymbol{r}}\right) - \mathrm{i}[\hat{\varepsilon},\hat{n}] = \hat{I}, \tag{12}$$

where $[\hat{\varepsilon},\hat{n}]$ is the commutator. The collision integral on the right-hand side determines the relaxation processes.

The matrix of the equilibrium electron distribution function is

$$\hat{n}^{(0)} = \frac{n(\varepsilon_+)+n(\varepsilon_-)}{2}\sigma_0 + \frac{n(\varepsilon_+)-n(\varepsilon_-)}{2\gamma}\boldsymbol{\gamma}\cdot\boldsymbol{\sigma}, \tag{13}$$

where

$$n(\varepsilon) = \frac{1}{\exp\left(\dfrac{\varepsilon-\mu}{T}\right)+1} \tag{14}$$

is the Fermi function.

The Hermitian matrices of the nonequilibrium distribution functions in the band and spin representations are related by

$$f_{\alpha\beta}(\boldsymbol{k}) = \Psi^{\alpha\dagger}_{\sigma_1}(\boldsymbol{k})n_{\sigma_1\sigma_2}\Psi^{\beta}_{\sigma_2}(\boldsymbol{k}). \tag{15}$$

In the band representation, the equilibrium distribution function is given by the diagonal matrix

$$n^{(0)}_{\alpha\beta} = \Psi^{\alpha\dagger}_{\sigma_1}(\boldsymbol{k})n^{(0)}_{\sigma_1\sigma_2}\Psi^{\beta}_{\sigma_2}(\boldsymbol{k}) = \begin{pmatrix} n(\varepsilon_+) & 0\\ 0 & n(\varepsilon_-)\end{pmatrix}. \tag{16}$$

The derivative of the energy matrix (3) in band representation is

$$\Psi^{\dagger\alpha}_{\sigma_1}\frac{\partial\varepsilon_{\sigma_1\sigma_2}}{\partial\boldsymbol{k}}\Psi^{\beta}_{\sigma_2} = \begin{pmatrix} \boldsymbol{v}_+ & \boldsymbol{w}_\pm(\varepsilon_- - \varepsilon_+)\\ \boldsymbol{w}_\mp(\varepsilon_+ - \varepsilon_-) & \boldsymbol{v}_-\end{pmatrix}. \tag{17}$$

Here,

$$\boldsymbol{w}_\pm(\boldsymbol{k}) = \Psi^{+\dagger}_{\sigma}(\boldsymbol{k})\frac{\partial\Psi^{-}_{\sigma}(\boldsymbol{k})}{\partial\boldsymbol{k}}, \tag{18}$$

$$\boldsymbol{w}_\mp = -\boldsymbol{w}^*_\pm. \tag{19}$$

In the presence of an electric field and gradients of temperature and chemical potential, the matrix kinetic equation in the band representation for the stationary deviation of the distribution function from equilibrium distribution

$$g_{\alpha\beta}(\boldsymbol{k}) = f_{\alpha\beta}(\boldsymbol{k}) - n^{(0)}_{\alpha\beta} \tag{20}$$

is

$$\begin{pmatrix} \boldsymbol{v}_+\left(\dfrac{\partial n_+}{\partial\varepsilon_+}e\boldsymbol{E} + \nabla n_+\right) & -\boldsymbol{w}_\pm\boldsymbol{N}\\ \boldsymbol{w}_\mp\boldsymbol{N} & \boldsymbol{v}_-\left(\dfrac{\partial n_-}{\partial\varepsilon_-}e\boldsymbol{E} + \nabla n_-\right)\end{pmatrix} + \begin{pmatrix} 0 & \mathrm{i}g_\pm(\varepsilon_- - \varepsilon_+)\\ \mathrm{i}g_\mp(\varepsilon_+ - \varepsilon_-) & 0\end{pmatrix} = \begin{pmatrix} I_+ & I_\pm\\ I_\mp & I_-\end{pmatrix}. \tag{21}$$

Here,

$$\boldsymbol{N} = (n_+ - n_-)e\boldsymbol{E} + (\varepsilon_+ - \varepsilon_-)\frac{1}{2}\nabla(n_+ + n_-), \tag{22}$$

$$\nabla n_\lambda = -\frac{\partial n_\lambda}{\partial\varepsilon_\lambda}\left(\nabla\mu + \frac{\xi_\lambda}{T}\nabla T\right). \tag{23}$$

We have used notations

$$n_\lambda = n(\varepsilon_\lambda), \quad \xi_\lambda = \varepsilon_\lambda - \mu, \quad \lambda = \pm.$$

The electric current density is determined by the following expression:

$$\boldsymbol{j} = e\int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\frac{\partial\varepsilon_{\sigma\sigma_1}(\boldsymbol{k})}{\partial\boldsymbol{k}}g_{\sigma_1\sigma}(\boldsymbol{k},\omega). \tag{24}$$

Transforming it to the band representation we get

$$\boldsymbol{j} = e\int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\left[\boldsymbol{v}_+g_+ + \boldsymbol{v}_-g_- + (\boldsymbol{w}_\pm g_\mp - \boldsymbol{w}_\mp g_\pm)(\varepsilon_- - \varepsilon_+)\right]. \tag{25}$$

The heat flux density is

$$\boldsymbol{q} = \frac{1}{2}\int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\left(\xi_{\sigma\sigma_1}\frac{\partial\varepsilon_{\sigma_1\sigma_2}}{\partial\boldsymbol{k}} + \frac{\partial\varepsilon_{\sigma\sigma_1}}{\partial\boldsymbol{k}}\xi_{\sigma_1\sigma_2}\right)g_{\sigma_2\sigma}(\boldsymbol{k}). \tag{26}$$

Transforming it to the band representation we obtain

$$\begin{aligned}\boldsymbol{q} = \int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\Big[&\xi_+\boldsymbol{v}_+g_+ + \xi_-\boldsymbol{v}_-g_-\\ &+\frac{1}{2}(\xi_+ + \xi_-)(\boldsymbol{w}_\pm g_\mp - \boldsymbol{w}_\mp g_\pm)(\varepsilon_- - \varepsilon_+)\Big].\end{aligned} \tag{27}$$

When the band splitting strongly exceeds the typical quasiparticle scattering rate

$$|\varepsilon_+ - \varepsilon_-| \gg 1/\tau, \tag{28}$$

one can neglect the off-diagonal terms in the collision integral and set $I_\pm = I_\mp = 0$ [10]. Thus, the off-diagonal matrix elements of distribution function are

$$g_\pm = \mathrm{i}\frac{\boldsymbol{w}_\pm\boldsymbol{N}}{\varepsilon_+ - \varepsilon_-}, \tag{29}$$

$$g_\mp = \mathrm{i}\frac{\boldsymbol{w}_\mp\boldsymbol{N}}{\varepsilon_+ - \varepsilon_-}. \tag{30}$$

Substituting these expressions in Eqs. (25) and (27) we obtain the anomalous parts of the current and heat flux densities as

$$j_i = e\int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}(-2\,\mathrm{Im}\,w^\pm_i w^{\dagger\pm}_j)N_j, \tag{31}$$

$$q_i = \frac{1}{2}\int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}(\xi_+ + \xi_-)(-2\,\mathrm{Im}\,w^\pm_i w^{\dagger\pm}_j)N_j. \tag{32}$$

Here, we have used the relation given by Eq. (19). One can check by direct calculation that

$$-2\,\mathrm{Im}\,w^\pm_i w^{\dagger\pm}_j = \Omega^+_{ij}(\boldsymbol{k}), \tag{33}$$

where $\Omega^+_{ij}(\boldsymbol{k})$ is the tensor of the Berry curvature given by Eq. (10).

After substitution expression (22) in Eqs. (31) and (32) we find that the electric current and the heat current have the following structure:

$$\boldsymbol{j} = \boldsymbol{j}^{\mathrm{E}} + \boldsymbol{j}^{\mathrm{T}}, \tag{34}$$

$$\boldsymbol{q} = \boldsymbol{q}^{\mathrm{E}} + \boldsymbol{q}^{\mathrm{T}}. \tag{35}$$

Here,

$$j_i^{\mathrm{E}} = e\int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\Omega_{ij}^{+}(\boldsymbol{k})\Big[(n_+ - n_-)eE_j - \frac{1}{2}(\varepsilon_+ - \varepsilon_-)\left(\frac{\partial n_+}{\partial \xi_+} + \frac{\partial n_-}{\partial \xi_-}\right)\nabla_j\mu\Big], \tag{36}$$

$$q_i^{\mathrm{T}} = -\frac{1}{4T}\int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\Omega_{ij}^{+}(\boldsymbol{k})(\xi_+ + \xi_-)(\varepsilon_+ - \varepsilon_-) \times \left(\xi + \frac{\partial n_+}{\partial \xi_+} + \xi_-\frac{\partial n_-}{\partial \xi_-}\right)\nabla_j T. \tag{37}$$

The thermoelectric current is

$$j_i^{\mathrm{T}} = -\frac{e}{2T}\int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\left[\Omega_{ij}^{+}(\varepsilon_+ - \varepsilon_-)\left(\xi_+\frac{\partial n_+}{\partial \xi_+} + \xi_-\frac{\partial n_-}{\partial \xi_-}\right)\right]\nabla_j T; \tag{38}$$

the thermoelectric heat flow is

$$q_i^{\mathrm{E}} = \frac{1}{2}\int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\Big\{\Omega_{ij}^{+}(\xi_+ + \xi_-) \times \Big[(n_+ - n_-)eE_j - \frac{1}{2}(\varepsilon_+ - \varepsilon_-)\left(\frac{\partial n_+}{\partial \varepsilon_+} + \frac{\partial n_-}{\partial \varepsilon_-}\right)\nabla_j\mu\Big]\Big\}. \tag{39}$$

Under fulfilment the condition

$$\varepsilon_+ - \varepsilon_- = 2\gamma \ll \mu, \tag{40}$$

equations (36) and (37) acquire simpler forms

$$j_i^{\mathrm{E}} = \sigma_{ij}(E_j - \nabla_j\mu/e) = 2e^2\int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\Omega_{ij}^{+}(\boldsymbol{k})\gamma\frac{\partial n}{\partial \xi}(E_j - \nabla_j\mu/e), \tag{41}$$

$$q_i^{\mathrm{T}} = -\kappa_{ij}\nabla T = -\frac{2}{T}\int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\Omega_{ij}^{+}(\boldsymbol{k})\gamma\xi^2\frac{\partial n}{\partial \xi}\nabla_j T. \tag{42}$$

After integration over ξ, expressions for conductivity and thermal-conductivty are

$$\sigma_{ij} = -2e^2N_0\int \frac{\mathrm{d}\Omega}{4\pi}\Omega_{ij}^{+}\gamma, \tag{43}$$

$$\kappa_{ij} = -\frac{2\pi^2N_0T}{3}\int \frac{\mathrm{d}\Omega}{4\pi}\Omega_{ij}^{+}\gamma. \tag{44}$$

Thus, we come to equality

$$\kappa_{ij} = \frac{\pi^2T}{3e^2}\sigma_{ij}; \tag{45}$$

sometimes called as the Mott relation.

Under fulfilment the condition (40), the thermoelectric current (38),

$$j_i^{\mathrm{T}} = \alpha_{ij}^{\mathrm{T}}\nabla_j T, \tag{46}$$

and the heat flow (39),

$$q_i^{\mathrm{E}} = \alpha_{ij}^{\mathrm{E}}(E_j - \nabla_j\mu/e), \tag{47}$$

are proportional to the corresponding thermodynamic forces, with proportionality coefficients

$$-T\alpha_{ij}^{\mathrm{T}} = \alpha_{ij}^{\mathrm{E}} = 2e\int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\gamma\xi\frac{\partial n}{\partial \xi}\Omega_{ij}^{+} \tag{48}$$

related to each other according to the Onsager principle. Performing integration over ξ we obtain

$$-T\alpha_{ij}^{\mathrm{T}} = \alpha_{ij}^{\mathrm{E}} = -\frac{2\pi^2eN_0'T^2}{3}\int \frac{\mathrm{d}\Omega}{4\pi}\Omega_{ij}^{+}\gamma. \tag{49}$$

Here, N_0' is the derivative of the density of states at the Fermi level.

Berry Curvature

The band spectrum in noncentrosymmetric and altermagnetic metals exhibits momentum-dependent band splitting described by the vector function $\boldsymbol{\gamma}(\boldsymbol{k})$. For instance, in noncentrosymmetric metals $CeRhSi_3$ and $LaRhSi_3$, which have a tetragonal structure, this function near the Γ point is [7]

$$\boldsymbol{\gamma}_{\mathrm{tetr}}(\boldsymbol{k}) = c_1(k_x\hat{y} - k_y\hat{x}) + c_2k_xk_yk_z(k_x^2 - k_y^2). \tag{50}$$

The corresponding function in the altermagnetic CrSb with a hexagonal structure is [12]

$$\boldsymbol{\gamma}_{\mathrm{hex}}(\boldsymbol{k}) = c_1\left[(k_x^2 - k_y^2)\hat{x} - 2k_xk_y\hat{y}\right] + c_2k_zk_x(k_x^2 - 3k_y^2)\hat{z}. \tag{51}$$

Substituting the Berry curvatures calculated according to Eq. (10) for $\boldsymbol{\gamma}_{\mathrm{tetr}}(\boldsymbol{k})$ and $\boldsymbol{\gamma}_{\mathrm{hex}}(\boldsymbol{k})$ functions into any expression for anomalous currents, it is easy to verify that all the corresponding integrals vanish due to the symmetry properties of $\Omega_{ij}^{+}(\boldsymbol{k})$. However, in the presence of a magnetic field along the symmetry axis of uniaxial crystals, both functions (50) and (51) include the Zeemann term

$$\boldsymbol{\gamma}(\boldsymbol{k}) \to \boldsymbol{\gamma}(\boldsymbol{k}) - \mu_{\mathrm{B}}H\hat{z} \tag{52}$$

and the integrals for anomalous currents depending on Ω_{xy}^{+} acquire finite values.

Conclusion

The derivation of anomalous currents originating from the Berry curvature is presented based on the quasiclassical matrix kinetic equation applied to noncentrosymmetric and altermagnetic metals. Expressions for the anomalous heat flow and the anomalous thermoelectric current are established. As in the case of the ordinary Hall effect, the off-diagonal electrical conductivity and thermal conductivity, expressed through the Berry curvature, are related by the Mott relation. The cross coeffiicients for the thermoelectric current and the thermoelectric heat flow are expressed in terms of each other according to the Onsager symmetry principle.

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