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Exploring Dimensional Localization in Three-Dimensional Moiré Lattices

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The recent paper by Ce Wang, Chao Gao, and Zhe-Yu Shi in *Chinese Physics Letter* [1] marks an important step in the emerging field of three-dimensional (3D) moiré physics. An important precursor to this development is the work of Wang *et al.* [2], which introduced 3D moiré crystals in ultracold atomic gases and established the commensurability conditions for periodic 3D moiré structures. Subsequent work by Gao *et al.* [3] explored more general 3D rotations and identified partially and fully incommensurate structures as well as low-dimensional compact states. The recent work of Wang *et al.* [1] takes a complementary step by developing a low-energy effective theory for the small-angle regime, revealing how the arithmetic properties of the rotation axis determine whether the resulting moiré pattern is periodic, hybrid, or fully quasi-periodic, and how these structures give rise to distinct forms of dimensional localization—a phenomenon with no two-dimensional (2D) counterpart.

An important distinction from 2D moiré systems emerges already at the level of the moiré pattern itself. In 2D, the moiré reciprocal vectors are differences of two reciprocal lattices, and their difference set is either discrete, defining a 2D Bravais lattice, or dense in the reciprocal plane; correspondingly, the pattern is either periodic (for a commensurate twist) or fully quasi-periodic (for an incommensurate one). No intermediate case exists. In three dimensions, by contrast, the three moiré reciprocal vectors associated with a rotation axis lie in the plane perpendicular to that axis, and their integer combinations may form a discrete, lattice-like set along one direction in this plane while being dense along the other. Consequently, 3D moiré systems can exhibit periodic, hybrid, or fully quasi-periodic moiré patterns, a possibility with no direct counterpart in two dimensions.

The central achievement of work [1] is an effective continuum description of twisted 3D optical lattices, derived from a microscopic two-component Hamiltonian for ultracold atoms in a spin-dependent optical lattice [4]. In the spirit of the Bistritzer–MacDonald theory of twisted bilayer graphene, the effective moiré potential provides a universal connection between the geometry of the rotation axis and the resulting pattern.

Building on the classification established in Ref. [3]—fully periodic, partially periodic, and fully incommensurate—Wang *et al.* explain it analytically: the three scenarios are governed by the number of independent integer relations among the components of the rotation axis. An integer-proportional axis such as $(1,1,1)$ yields a periodic pattern; the golden axis, whose components involve the golden ratio, has exactly one independent relation and gives a hybrid pattern periodic along one direction but quasi-periodic in the perpendicular plane; and the Kepler axis, a Kepler-triangle vector, admits none, producing a pattern quasi-periodic in all directions. This clean arithmetic criterion turns a previously empirical observation into a predictive principle.

The physical payoff is dimensional localization. Using the inverse participation ratio, Wang *et al.* show that the two quasi-periodic scenarios confine states in qualitatively different ways. For the golden axis, states localize in one transverse direction while remaining extended along the periodic direction and the rotation axis, confining them to a plane; for the Kepler axis, the absence of transverse translational symmetry localizes both transverse directions, leaving line confinement. Crucially, the localization is controlled by a dimensionless coupling parameter that can be tuned through the lattice parameters, providing a direct experimental knob for switching between extended and localized states.

This analytical picture complements the numerical findings of Gao *et al.* [3] for general Euler angles. By focusing on the small-angle regime, Wang *et al.* derive the origin of dimensional localization from first principles, starting from a microscopic spin-dependent Hamiltonian that is directly realizable with ultracold atoms. The preserved translational invariance along the rotation axis naturally explains the absence of fully localized states in this regime, while planar and line confinement emerge spontaneously.

These predictions are within reach of current experiments. Spin-dependent optical lattices [4] already provide a suitable platform, and the authors outline a concrete protocol: release a localized atomic cloud into the twisted lattice and monitor the anisotropy of its expansion—arrested along one transverse direction in the hybrid scenario and along both in the ergodic one. A photonic analogue, injecting a localized probe beam into an engineered moiré lattice [5], offers an independent testbed for these localization–delocalization transitions.

Remarkably, dimensional localization persists in low-lying excitations, suggesting that fermionic systems may exhibit similar physics. The quasi-periodic potentials also hint at fractal spectral features—energy gaps recurring at sequences tied to irrational parameters—reminiscent of the Hofstadter butterfly in 2D moiré lattices.

Looking ahead, several directions merit particular attention. On the theoretical side, the interplay between the quasi-periodic potential and interactions is a natural frontier: two-body interactions in Bose–Einstein condensates, or optical nonlinearities in photonic structures, could support higher-dimensional solitons, vortices, and self-trapped states whose stability is engineered by the twisting [6], while fermionic or disordered extensions may reveal many-body localized phases and interaction-induced mobility edges in 3D quasicrystals. The effective theory likewise invites topological questions: can 3D moiré geometry host Weyl points, chiral surface states, or higher-order topological phases, and how are such invariants affected by incommensurability? Because the classification is dictated purely by the arithmetic of the rotation axis, it also offers a designer route to quasi-periodicity: selecting axes associated with Pisot numbers or continued-fraction expansions would generate a tunable family of partially

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incommensurate potentials with controlled localization dimensionality. Floquet engineering, through periodic modulation of the twist angle or lattice depth, could add dynamical control of the direction and strength of localization, complementing the static tunability identified here. Time-resolved probes—quantum walks, Bloch oscillations, and quenches across the localization transition—would sharpen our understanding of critical and fractal states and of the dimensional crossover. On the experimental front, realizing full 3D moiré lattices for ultracold atoms or in photonic platforms and imaging the anisotropic arrest of expansion remain the key milestones; extensions to twisted van der Waals heterostructures and other condensed-matter analogues could carry these ideas beyond cold atoms. As anticipated in the viewpoint of Ref. [6], independent tuning of the twisting angles may ultimately allow localization to be steered into chosen planes or lines on demand, turning 3D moiré lattices into versatile platforms for controlling quantum matter and light. We expect these opportunities to stimulate vigorous activity in ultracold gases, photonics, and beyond.

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