The ρ-Meson Electromagnetic Form Factors within the Light-Front Quark Model

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In this paper, we study the ρ-meson electromagnetic form factors (EMFFs) within the framework of the light-front quark model. The physical form factors $G_{\rm C,M,Q}(Q^2)$ of the ρ-meson, as well as the charged square radius $\langle r^2\rangle$, the magnetic moment μ, and the quadrupole moment $\bar{Q}$, are calculated, which describe the behaviors of EMFFs at zero momentum transfer. Using the type-Ⅱ replacement, we find that the zero-mode does contribute zero to the matrix element S^+_{00}. It is found that the "$M \to M_0$" replacement improves the angular condition remarkably, which permits different prescriptions of ρ-meson EMFFs to give the consistent results. The residual tiny violation of angular condition needs other explanations beyond the zero-mode contributions. Our results indicate that the relativistic effects or interaction internal structure are weaken in the zero-binding limit. This work is also applicable to other spin-1 particles.

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1. Introduction. The electromagnetic form factors (EMFFs) include essential information on the hadronic structure and interaction between the constituents, which are important and fundamental ingredients in phenomenal research.[1–13] In the past few years, a lot of efforts to study electromagnetic structure of the spin-1 particles in light-front approaches have been performed. The works[14–16] have researched the EMFFs of the deuteron, in which the deuteron is treated as the two-nucleon system. The universal properties of electromagnetic interactions in spin-1 bound states are investigated in Ref. [4]. The authors of Refs. [6,8,12,17] have extensively examined the zero-mode effects in EMFFs of ρ meson in several aspects.

The light-front quark model (LFQM) provides a conceptually simple, feasible framework for the determination of the quantities involved the non-perturbative information, such as the kinds of form factors, decay constants, and distribution amplitudes.[7,18–21,23–25] In practice, these quantities are usually extracted through the definitions in the special matrix element of the current operator, and the computational task is mainly focused on the determination of the matrix element, correspondingly. The current operator in the matrix element is usually approximated by a one-body current, which means the initial and final states have the same number of constituents.[26] Without containing interaction-dependent parts (two-body current), which are necessarily included in the angular momentum operator, this leads to the loss of rotational covariance of the light-front matrix element.[8] Consequently, those approaches do not permit an unambiguous determination of the physical quantities in some cases, such as the decay constants of vector mesons and weak transition form factors between vector mesons. Previous works[4,15,16] show that different descriptions of deuteron EMFFs using different extraction methods do not give a consistent result; this is called as "the self-consistency" problem. As proposed in Ref. [15], the break of rotational covariance brings about untenable angular conditions, which is a key relationship between matrix elements in the case of EMFFs, and the violation of angular conditions will undoubtedly bring about the above non-uniqueness issue. The angular condition could be restored at this level of approximation only if the zero-mode contributions are reasonably considered.[8,26–30] Once the angular condition is hold, different prescriptions of EMFFs will achieve the same predictions. References [26,28–30] suggest that the zero-mode contributions are naturally associated with the covariance of light-front matrix element. References [7,8] develop a systematic approach to calculate the zero-mode contributions, which also permits a consistent determination of the physical form factors.

A promising remedy called "type-Ⅱ" replacement has been suggested to deal with the vector meson decay constant $f_{\rm V}$,[21,22] which avoids the self-consistency problem. Inspired by this treatment, references [24,25] perform the calculation of the meson decay constants and the weak transition form factors, in which the self-consistency problem and the non-covariant problem have also been carefully discussed. Furthermore, it has been pointed out that the self-consistency problem is closely associated with the

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covariant problem, and these two issues have the same origin: the ambiguous decomposition for the terms related to the fixed lightlike vector ω. Interestingly, numerical analysis shows that even though the zero-mode exist formally, their contributions will be vanish numerically when the type-Ⅱ scheme is performed. This indicates that either the zero-mode contributions vanish or they have been included exactly into the valence contributions via the additive "$M \to M_0$" replacement. This conclusion is not rigorous but just a verification through a few of quantities. Therefore, it is meaningful to test the effectiveness of the "$M \to M_0$" replacement in more cases, for example, the EMFFs mentioned above. Moreover, if the zero-mode contributions in EMFFs are well dealt with, then the angular condition should also be well established. Consequently, the ambiguity from different prescriptions of EMFFs will be removed, i.e., they will consistently give a set of numerical results. These are the aims of this work.

The remaining parts of the paper are organized as follows. In Section 2, we briefly review the EMFFs of the spin-1 particles and present four kinds of prescriptions for the form factors. In Section 3, the zero-mode contributions are discussed in detail, and then the angular condition is investigated. Numerical results and discussions of the ρ-meson EMFFs $G_{\rm C,M,Q}(Q^2)$ are presented in Section 4. Section 5 is reserved for a summary.

2. EMFFs. For self-consistency, we first give a brief review of the EMFFs of spin-1 particles. Generally, the matrix elements of the one-body vector current $V^\mu = q''\gamma^\mu q'$ between two spin-1 particles can be divided into six independent Lorentz-invariant form factors.[8] For the case of electromagnetic transition, there are just three form factors $F_{1,2,3}(q^2)$ survived as follows:[15,31]

$$\begin{aligned} S^\mu_{h''h'} &= \langle M(p'')|V^\mu|M(p')\rangle \\ &= -\epsilon^*_{h''}\cdot\epsilon_{h'}P^\mu F_1(q^2)+(\epsilon^\mu_{h'}P\cdot\epsilon^*_{h''}+\epsilon^{*\mu}_{h''}P\cdot\epsilon_{h'})F_2(q^2) \\ &\quad+\frac{(P\cdot\epsilon_{h'})(P\cdot\epsilon_{h''})}{2M^2}P^\mu F_3(q^2), \end{aligned} \tag{1}$$

where $\epsilon^{(*)}$ and h' (h'') $= 0, \pm 1$ denote the polarization vector and helicity of the initial (final) state, respectively. $P^\mu = p'^\mu + p''^\mu$ and $q^\mu = p'^\mu - p''^\mu$, where p' (p'') is the 4-momentum of initial (final) state. For spin-1 particles, the condition of $p\cdot\epsilon = 0$ is always hold. It should be kept in mind that the calculation of this work is performed with "good" plus component ($\mu = +$) in Drell–Yan frame. By setting $q^+ = 0$, $q^2 = -q^2_\perp \equiv -Q^2$ denotes the space-like momentum transfer. The physical charge, magnetic and quadrupole form factors $G_{\rm C,M,Q}(q^2)$ can be determined by $F_i(q^2)$ via the following relations:[32]

$$\begin{aligned} G_{\rm C}(q^2) &= F_1(q^2) + \frac{2}{3}\eta G_{\rm Q}(q^2), \\ G_{\rm M}(q^2) &= -F_2(q^2), \\ G_{\rm Q}(q^2) &= F_1(q^2) + F_2(q^2) + (1+\eta)F_3(q^2), \end{aligned} \tag{2}$$

where the kinematic factor $\eta = \frac{Q^2}{4M^2}$, and as will be shown later, the form factors $G_{\rm C,M,Q}(q^2)$ can be expressed through matrix elements $S^+_{h''h'}$.

There are nine matrix elements $S^+_{h''h'}$, which correspond to nine modes of transition between the initial and final polarization states. Not all of the matrix elements $S^+_{h''h'}$ are independent; they are constrained by (i) time-reversal invariance, (ii) reflection invariance in the $(\hat{x},\hat{y})$ plane, and (iii) rotation invariance about $\hat{z}$.[17] As a result, there are only four independent matrix elements, which are usually chosen as S^+_{00}, S^+_{10}, S^+_{1-1} and S^+_{11}. Actually, those four $S^+_{h''h'}$ are not independent if the rotational covariance is completely satisfied, but not just held for the rotation about $\hat{z}$. If the rotational covariance is completely satisfied, they should be restrained by an additive relation equation, which is the so-called angular condition,[15]

$$\Delta(Q^2) \equiv (1+2\eta)S^+_{11} - \sqrt{8\eta}S^+_{10} + S^+_{1-1} - S^+_{00} = 0. \tag{3}$$

In other words, the number of independent matrix elements changes down to three when taking the angular condition into consideration.

For the light-front approaches, the Hamiltonian P^- is usually chose as $P^0 - P\cdot\hat{n}$, where $\hat{n}$ is the spatial unit vector. Practically, $\hat{n}$ is usually set along with the $\hat{z}$ axis and leads to $P^- = P^0 - P^3$. This choice will also be taken in this work.* A variety of contexts for studies of rotational covariance have been performed in Refs. [2,15,31,33,34]. The point is that the approximated one-body current is free of interaction; therefore, the matrix elements of one-body current cannot satisfy the complete rotational covariance or angular condition in principle.[8]† And because the rotational covariance cannot be completely satisfied, e.g., the condition (iii) is adopted for discussion, we will have $\Delta(q^2) \neq 0$. This underlies why the form factors $G_{\rm C,M,Q}(q^2)$ vary from each other in the works of Refs. [4,15,16,35].

For the sake of conciseness, we list four versions $G_{\rm C,M,Q}(q^2)$ that are derived under different linear combinations of $S^+_{h''h'}$. The first one is suggested by Grach and Kondratyuk,[15]

$$\begin{aligned} G^{\rm GK}_{\rm C}(q^2) &= \frac{1}{3}[(3-2\eta)S^+_{11} + 2\sqrt{2\eta}S^+_{10} + S^+_{1-1}], \\ G^{\rm GK}_{\rm M}(q^2) &= 2\Big[S^+_{11} - \frac{1}{\sqrt{2\eta}}S^+_{10}\Big], \\ G^{\rm GK}_{\rm Q}(q^2) &= \frac{2\sqrt{2}}{3}[-\eta S^+_{11} + \sqrt{2\eta}S^+_{10} - S^+_{1-1}], \end{aligned} \tag{4}$$

where the matrix element S^+_{00}, which contains the zero-mode contributions, has no contribution to those form factors. The second one is suggested by Chung, Coester,

*Indeed, we take the BL convention and there is an additional "1/2" factor in $P^- = \frac{1}{2}(P^0 - P^3)$. The result is independent of the conventions for calculating.

†It is an abstruse task to recover the complete rotational covariance via an in-depth way, and in the paper, we will adopt the same approximation for rotational covariance.

Keisbter, and Polyzou,[16]

$$
\begin{aligned}
G_{\rm C}^{\rm CCKP}(q^2) &= \frac{1}{3(1+\eta)}\Big[\Big(\frac{3}{2}-\eta\Big)(S_{11}^{+}+S_{00}^{+})+5\sqrt{2\eta}S_{10}^{+} \\
&\quad +\Big(2\eta-\frac{1}{2}\Big)S_{1-1}^{+}\Big], \\
G_{\rm M}^{\rm CCKP}(q^2) &= \frac{1}{(1+\eta)}\Big[S_{11}^{+}+S_{00}^{+}-\frac{2(1-\eta)}{\sqrt{2\eta}}S_{10}^{+}-S_{1-1}^{+}\Big], \\
G_{\rm Q}^{\rm CCKP}(q^2) &= \frac{\sqrt{2}}{3(1+\eta)}\Big[-\eta(S_{11}^{+}+S_{00}^{+})+2\sqrt{2\eta}S_{10}^{+} \\
&\quad -(\eta+2)S_{1-1}^{+}\Big]. \qquad (5)
\end{aligned}
$$

The third one is suggested by Brodsky and Hiller,[4]

$$
\begin{aligned}
G_{\rm C}^{\rm BH}(q^2) &= \frac{1}{3(1+\eta)}[(3-2\eta)S_{00}^{+}+8\sqrt{2\eta}S_{10}^{+} \\
&\quad +2(2\eta-1)S_{1-1}^{+}], \\
G_{\rm M}^{\rm BH}(q^2) &= \frac{2}{(1+2\eta)}\Big[S_{00}^{+}+\frac{2\eta-1}{\sqrt{2\eta}}S_{10}^{+}-S_{1-1}^{+}\Big], \\
G_{\rm Q}^{\rm BH}(q^2) &= \frac{2\sqrt{2}}{3(1+2\eta)}[-\eta S_{00}^{+}+\sqrt{2\eta}S_{10}^{+}-(\eta+1)S_{1-1}^{+}], \qquad (6)
\end{aligned}
$$

where S_{00}^{+} is treated as the dominant one at large Q^2 by perturbative QCD analyzes, while S_{11}^{+} has no contribution to those form factors. And the fourth one is suggested by Frankfurt, Frederico, and Strikman,[35]

$$
\begin{aligned}
G_{\rm C}^{\rm FFS}(q^2) &= \frac{1}{3(1+\eta)}[(3+2\eta)S_{11}^{+} \\
&\quad +2\sqrt{2\eta}S_{10}^{+}+(2\eta+1)S_{1-1}^{+}-\eta S_{00}^{+}], \\
G_{\rm M}^{\rm FFS}(q^2) &= G_{M}^{\rm CCKP}(q^2), \\
G_{\rm Q}^{\rm FFS}(q^2) &= G_{Q}^{\rm CCKP}(q^2). \qquad (7)
\end{aligned}
$$

In the light-front quark model, the full result is generally composed by the valence part and the zero-mode contributions, i.e., $\tilde{Q}_{\rm full}=\tilde{Q}_{\rm val.}+\tilde{Q}_{\rm z.m.}$. References [6,8,12,17] show that only the helicity zero-to-zero matrix element S_{00}^{+} receives the zero-mode contribution, and the others just contain the valence parts. In addition, the authors there claim that if the zero-mode contribution is include in S_{00}^{+}, the zero-mode contribution should be closely associated with the angular conditions and $\Delta(q^2)=0$. So in the following, we give a discussion on the zero-mode contribution and the angular condition.

3. Zero-Mode Contribution and Angular Condition. In the standard light-front (SLF) quark model, the constituent quark and antiquark in a bound state are required to satisfy their respective on-shell conditions, with physical quantities calculated directly in three-dimensional light-front momentum space. Typically, the "good" component "$\mu=+$" of the current matrix elements is selected to avoid the zero-mode contributions. It is particularly noteworthy that the matrix elements obtained in the SLF quark model lose Lorentz covariance, and the model itself cannot independently determine the zero-mode contributions. Compared with the SLF quark model, the covariant light-front (CLF) quark model exhibits two distinctive features: First, it provides a systematic approach to study the zero-mode effects; second, by employing the manifestly covariant Bethe–Salpeter method, its predictions satisfy Lorentz covariance. It must be emphasized that the CLF quark model introduces an effective vertex χ_M between quarks and bound states, whose specific form needs to be determined by matching with the results from the SLF quark model. Subsequently, to address the self-consistency issue of vector meson decay constants in LFQM, Choi and Ji proposed the following two replacement schemes:[21]

$$
\begin{aligned}
\chi_M \to h_M/\hat{N} &= \frac{1}{\sqrt{2N_c}}\sqrt{\frac{\bar{x}}{x}}\frac{\psi_{\rm s}}{\hat{M}_0}, \\
D_{\rm con} &= M+m_1+m_2 \to D_{\rm LF} \\
&= M_0+m_1+m_2, \ \text{(Type-I)}, \qquad (8)
\end{aligned}
$$

and

$$
\chi_M \to h_M/\hat{N}, \quad M\to M_0, \quad \text{(Type-II)}, \qquad (9)
$$

where $\bar{x}=1-x$ and x, both of them are the momentum fractions carried by the constituent quarks, while $\psi_{\rm s}$ corresponds to the s-wave wavefunction of the vector meson. M and M_0 are physical mass and invariant mass of the vector meson, respectively, e.g., $M_0^2=(m_1^2+k_\perp^2)/x+(m_2^2+k_\perp^2)/\bar{x}$ and $\hat{M}_0^2=M_0^2-(m_1-m_2)^2$. The constant factor $D_{\rm con}$ and the light-front factor $D_{\rm LF}$ emerge from the vertex structure of the meson's spin wavefunction. The distinction between Type-I and Type-II schemes lies in the additive "$M\to M_0$" replacement for all the mesons' masses in type-II formulas. Actually, the effective vertex function χ_M just appears in the CLF quark model, while it is absent in the SLF quark model. So the replacement for χ_M is redundant in SLF quark model. Therefore, the SLF quark model does not have the same two substitution options, it can only perform the "$M\to M_0$" replacement.

In doing numerical analyses, ρ meson is taken as an example to deploy the explanation, whose input parameters are $m_q=0.25\pm0.04\,\text{GeV}$ and $\beta_{q\bar{q}}=0.3124\pm0.0060\,\text{GeV}$ that are derived by fitting the experiment results of the decay constants f_π and f_ρ.[24]

3.1. Zero-Mode Contribution. References [24,25] have analyzed the problems of self-consistency and the covariance of LFQM by studying the mesonic decay constants and the weak transition form factors. It has been found that the self-consistency problem is closely related to the covariance problem; both of them are associate with the zero-mode contribution. By using the type-II scheme, two problems can be naturally solved. This is caused by the fact that even though the zero-mode contribution will be in the formulas, its numerical magnitude becomes zero when the type-II replacement has been done. The validity of type-II scheme has also been tested through the $V'\to V''$ (V denotes the vector meson) weak transition processes.[36] It is worth investigating whether the type-II scheme can also contribute to solving the self-consistency problem in ρ-meson EMFFs.

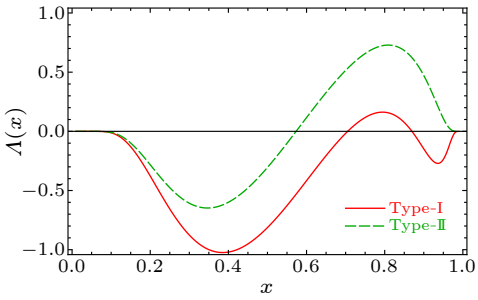

Fig. 1. The solid and dashed lines are for $\Lambda(x)$ versus x under the type-I and the type-Ⅱ schemes, respectively. $\Lambda(x)$ shows the differences between $S_{00}^{+\mathrm{CLF}}$ and $S_{00}^{+\mathrm{SLF}}$.

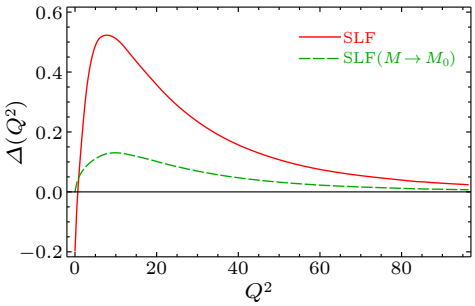

Fig. 2. The ρ-meson angular condition $\Delta(Q^2)$ versus Q^2. The dashed and solid lines are for $\Delta(Q^2)$ under the SLF and the SLF($M \to M_0$), respectively.

The complete results of the CLF quark model can be expressed as the sum of valence contributions and zero-mode contributions, $S^{\mathrm{CLF}} = S^{\mathrm{val.}} + S^{\mathrm{z.m.}}$. Detailed analysis reveals distinct behaviors for different matrix elements: For S_{11}^{+}, S_{10}^{+}, and S_{1-1}^{+}, the valence contributions are found to be identical to the full results, indicating the absence of zero-mode contributions. Under the type-I scheme, these matrix elements show exact formal agreement between the SLF and CLF quark models. This has also been confirmed in previous studies.[4,24,25,36]

However, the situation differs significantly for the matrix element S_{00}^{+}, where under the type-I scheme we observe $S_{00}^{+\mathrm{CLF}} \neq S_{00}^{+\mathrm{val.}}$, demonstrating the involvement of zero-mode contributions in this case. Remarkably, when employing the type-Ⅱ scheme, two important features emerge: first, $S_{00}^{+\mathrm{CLF}}$ and $S_{00}^{+\mathrm{val.}}$ yield identical numerical values across all momentum transfers Q^2; second, the relation $S_{00}^{+\mathrm{CLF}} = S_{00}^{+\mathrm{val.}} = S_{00}^{+\mathrm{SLF}(M\to M_0)}$ is satisfied. These findings collectively demonstrate that the type-Ⅱ scheme provides an effective approach for addressing computational discrepancies arising from zero-mode contributions in the framework of LFQM. To show their differences, we define a function $\Lambda(x)$,

$$\Lambda(x) \equiv \frac{\mathrm{d}}{\mathrm{d}x} S_{00}^{+\mathrm{CLF}} - \frac{\mathrm{d}}{\mathrm{d}x} S_{00}^{+\mathrm{SLF}(M\to M_0)}. \tag{10}$$

The value of $\int_0^1 \Lambda(x)\,\mathrm{d}x$ corresponds to the zero-mode contribution, since the SLF result always equals the valence contribution under the $M \to M_0$ mass replacement scheme. We present $\Lambda(x)$ versus x in Fig. 1, where the solid and dashed lines are results of the type-I and the type-Ⅱ schemes, respectively. It indicates $\int_0^1 \Lambda(x)dx \neq 0$ for the type-I scheme, which vanishes exactly for the type-Ⅱ scheme. This confirms the previous findings of Refs. [24,25,36] and reveals the feasibility of type-Ⅱ scheme. It means that either there is no zero-mode contribution or it has been included into the valence part through the additive replacement of "$M \to M_0$".

3.2. Angular Condition. We present the ρ-meson angular condition $\Delta(Q^2)$ versus Q^2 in Fig. 2, where the dashed and solid lines are for $\Delta(Q^2)$ under the case of SLF or the SLF($M \to M_0$). For the case of SLF, $\Delta(Q^2)$ becomes negative in small Q^2 region ($Q^2 \to 0$), leading to a visible violation of the angular condition under the SLF quark model. In Ref. [12], the authors claimed that the zero-mode contribution in S_{00}^{+} should vanish in the zero-binding limit ($M \to M_0$), and then the angular condition holds at $Q^2 = 0$. We confirm their observation. More explicitly, we have $\Delta(0) = 0$ for the case of SLF($M \to M_0$), indicating the angular condition is strictly satisfied at the zero-momentum transfer. It is noted that the above-mentioned four versions of EMFFs [Eqs. (4)–(7)] exactly give the same values at $Q^2 = 0$ for the case of the type-Ⅱ scheme. They also yield consistent results in the low regions where $Q^2 \lesssim 0.5\,\mathrm{GeV}^2$. In contrast, the results of SLF are inconsistent in these regions, as can be seen below.

For the case of SLF, figure 2 shows that in moderate Q^2 region, $\Delta(Q^2)$ will first increase and then decrease with the increment of Q^2, whose maximum value $\simeq$0.53 is achieved for $Q^2 \simeq 10\,\mathrm{GeV}^2$ and the angular condition is significantly damaged, which will lead to different results by using different prescriptions for the EMFFs. For the case of SLF($M \to M_0$), $\Delta(Q^2)$ behaves similarly and its maximum value $\simeq$0.13 is achieved for $Q^2 \simeq 10\,\mathrm{GeV}^2$. Thus strictly speaking, the angular condition is not satisfied in moderate Q^2 region even for the case of SLF($M \to M_0$). Nevertheless, the violation of angular condition can be remarkably softened for the case of SLF($M \to M_0$), which can be safely neglected for the EMFFs, as their differences under different versions of EMFFs are small. For both cases, the asymptotic value of $\Delta(Q^2)$ is zero when $Q^2 \to \infty$. The asymptotic behavior of $\Delta(Q^2)$ at large Q^2 transfer is similar to the result of Ref. [12], which is however quite different from the results of Refs. [15,33], in which the angular condition is violated badly at high Q^2 region. One may observe $\Delta(Q^2 \simeq 0.7\,\mathrm{GeV}^2) = 0$, which indicates that for the case of SLF the angular condition can be achieved at this point.

Earlier studies on the deuteron indicate that $\Delta(Q^2)$ can be relatively small,[15,16,33] where the important feature of the deuteron is that the characteristic nucleon momentum is very small compared to its nucleon mass. We analogously carry out the calculation about the heavy meson $\Upsilon(1S)$, in which the mass of the constituent $b(\bar{b})$ is large than the characteristic momentum. It is expected that the value of $\Delta(Q^2)$ should be quite smaller than in the case of ρ meson above. As shown in Fig. 3, the violation effects do dramatically decline for both cases of

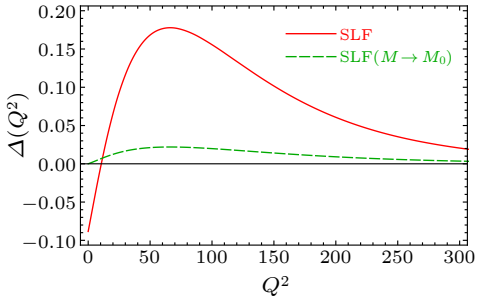

Fig. 3. The $\Upsilon(1S)$-meson angular condition $\Delta(Q^2)$ versus Q^2. The dashed and solid lines are for $\Delta(Q^2)$ under the SLF and the SLF($M \to M_0$), respectively.

SLF [$\Delta(Q^2) < 0.2$] and SLF($M \to M_0$) [$\Delta(Q^2) < 0.02$]. It is worth mentioning that the mass M and invariant mass M_0 are equivalent in the heavy quark limit.

In general, the angular condition is unfulfillable, and thereupon the four kinds of prescriptions cannot give a consistent result in SLF. After applying the transformation ($M \to M_0$), the violation effect of angular condition shall be remarkably suppressed and become negligible, even through it does not vanish entirely. The SLF($M \to M_0$) greatly improves the situation of the angular condition, and therefore different versions of EMFFs will give consistent values.

Our result agrees with the previous observation of Refs. [15,17] that the matrix element S^+_{00} is the only one that contains the zero-mode contribution, and the GK's version is immune to the zero-mode contribution by deliberately avoiding S^+_{00} in their formulas. From this perspective, it is definitely an advantage among the four prescriptions tested. However, it seems like an expedient to avoid the zero-mode contribution by the assumption of angular condition.

4. Numerical Results.

4.1. The EMFFs $G_{\rm C,M,Q}(Q^2)$. Based on the above discussions, we calculate the ρ-meson EMFFs $G_{\rm C,M,Q}(Q^2)$ in space-like momentum transfer under four prescriptions: GK, CCPK, BH, and FFS. Our results for the SLF and the SLF($M \to M_0$) are presented in Figs. 4–6. Figure 4 shows that different prescriptions will bring discrepant values for charge form factor $G_{\rm C}(Q^2)$ under the SLF, which is just the consequence of $\Delta(Q^2) \neq 0$. Around $Q^2 \simeq 0.7\,{\rm GeV}^2$, the four lines under the SLF intersect with each other because the angular condition is satisfied at that special point, as shown by Fig. 2. With the increment of Q^2, the discrepancies among the four prescriptions become noticeable expect that of CCKP and BH. Generally, the violation of angular condition is not negligible. One mentionable thing is that $G_{\rm C}(Q^2)$ under the SLF goes to zero at some specific point of Q^2, such as $Q^2 \simeq 3.5\,{\rm GeV}^2$ for FFS and $Q^2 \simeq 2.2\,{\rm GeV}^2$ for the other prescriptions. This agrees with the previous observation of Ref. [32] that at large Q^2, $G_{\rm C}(Q^2)$ becomes negative. While for the case of SLF($M \to M_0$), the results under four prescriptions wholly overlap with each other at low momentum transfer $Q^2 < 0.5\,{\rm GeV}^2$, which is the consequence of $\Delta(Q^2) \simeq 0$. The deviations of their values at intermediate and large momentum transfers are quite small, which are limited by the uncertainty from inputs and can be safely neglected. Phenomenologically, it is acceptable that the four prescriptions give the same predictions. Thus the SLF($M \to M_0$) greatly improves the consistency of different prescriptions.

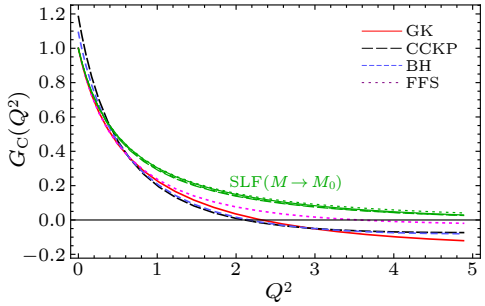

Fig. 4. The charge form factor $G_{\rm C}(Q^2)$ versus Q^2 for the ρ meson. Different lines are for four versions of EMFFs under the prescriptions of GK, CCPK, BH and FFS, respectively. The red-solid, black dashed, blue short dashed, and purple dotted lines are for the case of SLF. The same line types but with green color are for the case of SLF($M \to M_0$).

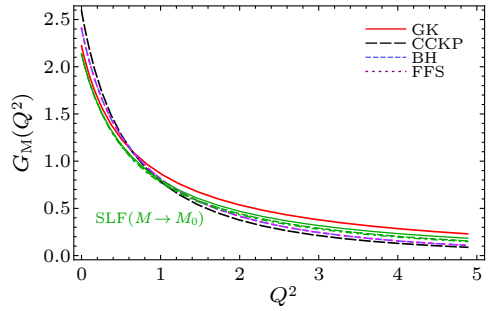

Fig. 5. The magnetic form factor $G_{\rm M}(Q^2)$ versus Q^2 for ρ meson. Different lines are for four versions of EMFFs under the prescriptions of GK, CCPK, BH and FFS, respectively. The red solid, black dashed, blue short dashed, and purple dotted lines are for the case of SLF. The same line types but with green color are for the case of SLF($M \to M_0$).

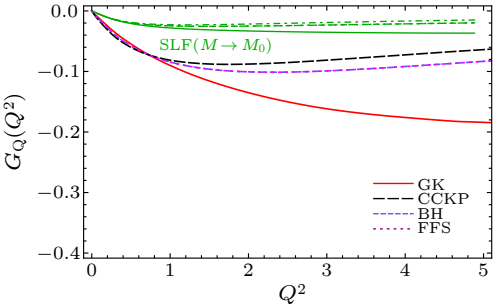

Fig. 6. The quadrupole form factor $G_{\rm Q}(Q^2)$ versus Q^2 for ρ meson. Different lines are for four versions of EMFFs under the prescriptions of GK, CCPK, BH and FFS, respectively. The red solid, black dashed, blue short dashed, and purple dotted lines are for the case of SLF. The same line types but with green color are for the case of SLF($M \to M_0$).

Figure 5 exhibits the Q^2-dependence of magnetic form factor $G_{\rm M}(Q^2)$ from the four prescriptions under the SLF and SLF($M \to M_0$), respectively. Similar to the case of $G_{\rm C}(Q^2)$, different prescriptions give obviously different re-

sults for the case of SLF, especially for GK's. By applying the "$M \to M_0$" replacement, similar to the case of $G_{\rm C}(Q^2)$, all the curves of $G_{\rm M}(Q^2)$ for the four prescriptions are consistent with each other.

The quadrupole form factor $G_{\rm Q}(Q^2)$ is shown in Fig. 6, where the diversity of vary prescriptions seems more pronounced for both the SLF and the SLF($M \to M_0$). Furthermore, the difference between results of the SLF and the SLF($M \to M_0$) intensifies compared with those of $G_{\rm C}(Q^2)$ and $G_{\rm M}(Q^2)$. It needs to be said that the violation effect of the angular condition is the same for all the form factors $G_{\rm C}(Q^2)$, $G_{\rm M}(Q^2)$, and $G_{\rm Q}(Q^2)$. While the more remarkable influence for $G_{\rm Q}(Q^2)$ is just the result of its value being relatively small. The value of $G_{\rm Q}(Q^2)$ reflects the relativistic effects,[6] which is the reason that $G_{\rm Q}(0) = 0$ and its modulus increases with the larger Q^2. Comparing the different types of lines, one can find that the values of $G_{\rm Q}(Q^2)$ from the four prescriptions under the case of SLF($M \to M_0$) are close to each other, indicating that the relativistic effects in the ρ meson are small in this case.

As a summary, by applying the "$M \to M_0$" replacement, it has been discovered that the discrepancies among the four prescriptions can be significantly reduced, yielding a consistent result for all the prescriptions.

4.2. The Mean Square Radius $\langle r^2\rangle$, Magnetic Moment μ and Quadrupole Moment $\bar{Q}$. The charge radius $\langle r^2\rangle$, magnetic moment μ, and quadrupole moment $\bar{Q}$ can be defined as[33]

$$\begin{aligned}\langle r^2\rangle &= \lim_{Q^2\to 0}\frac{6[1-G_{\rm C}(Q^2)]}{Q^2},\\ \mu &= \lim_{Q^2\to 0} G_{\rm M}(Q^2),\\ \bar{Q} &= \lim_{Q^2\to 0}\frac{3\sqrt{2}G_{\rm Q}(Q^2)}{Q^2}.\end{aligned} \tag{11}$$

We also calculate these quantities with and without "$M \to M_0$" replacement in SLF and present them in Table 1. The charge radius $\langle r^2\rangle$ reflects the slope of physical form factor $G_{\rm C}(Q^2)$ at $Q^2 = 0$, and it is important to calculate this quantity carefully due to the possible singular behaviors in some cases. From the first row of Table 1, it is found that the results of $\langle r^2\rangle$ for GK and FFS versions under SLF are consistent with each other and slightly larger than the results of SLF($M \to M_0$). Compared with previous works, where $\langle r^2\rangle \sim (0.35\text{–}0.40)\,\text{fm}^2$, our number increases by 25%, which indicates the size of ρ meson is about 0.7 fm in this respect. The magnetic momentum μ equals 2 in the limit where the spin-1 particle becomes a point state, and $\mu - 2$ defines the anomalous moment, which reflects the dynamical contributions due to internal structure.[8] For instance, the non-relativistic SU(6) model predicts $\mu = 2$, where all interaction effects are neglected. It is noticeable that $\mu = 2.13$ under SLF($M \to M_0$) is smaller than that of SLF $\mu \sim 2.23$–2.48, which implies the "$M \to M_0$" replacement weakens the relativistic effect or the interaction in bound state. Similarly, the quadrupole momentum $\bar{Q}$ with SLF($M \to M_0$) is smaller than the results with SLF, which is also led by the decrease of the relativistic effect.

Table 1. The charge radius $\langle r^2\rangle$, magnetic momentum μ, and quadrupole momentum $\bar{Q}$ with and without "$M \to M_0$" replacement in the SLF quark model, where previous results[2,15,31,33,34] have also been listed as a comparison.

	GK	CCKP	BH	FFS	SLF($M \to M_0$)	Previous
$\langle r^2\rangle$	0.48	–	–	0.49	0.47	0.35–0.40
$\bar{Q}$	0.022	0.026	0.030	0.026	0.010	0.024–0.058
μ	2.23	2.36	2.48	2.36	2.13	2.14–2.48

5. Conclusions. In summary, the EMFFs of the ρ meson have been studied in the LFQM. It is confirmed that only S^+_{00} contains the zero-mode contribution. Numerical analyses on the matrix element S^+_{00} show the valence and full results are exactly equivalent. This implies that either the zero-mode contribution vanishes or it has been included in the valence part via the type-Ⅱ and the "$M \to M_0$" replacements in the CLF and SLF quark models, respectively. It is found the "$M \to M_0$" replacement improves the angular condition significantly, and different prescriptions for the physical EMFFs can give a consistent result. Numerical analysis indicates that the "$M \to M_0$" replacement weakens the relativistic effects in bound stats. The residual minor violation of the angular condition needs some other explanations.

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