Possible Bound States in the Triple-η_c and Triple-J/ψ Systems

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The observations of fully-charm tetraquark states in the LHCb, CMS, and ATLAS experiments suggested the existence of hadronic molecules of two-charmonium states, which may also imply bound states in the three-charmonium systems. In this work, we study the possible bound states in the triple-η_c and triple-J/ψ systems with $J^{\rm PC}=0^{-+}$ and 1^{--}, respectively. In quantum chromodynamics sum rules, we calculate the two-point correlation functions and spectral functions up to the dimension-four gluon condensate. We use the iterative dispersion relation approach to deal with the five-loop banana integrals, which significantly improves computational efficiency. Our results show that the masses of triple-η_c and triple-J/ψ states lie below the corresponding mass thresholds, supporting the existence of such three-body bound states.

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1. Introduction. Over the past half-century, the search for multiquark states has always been a very intriguing research topic in hadron physics. Since 2003, there has been a resurgence of such investigations following the observation of numerous good candidates, such as the hidden-charm pentaquarks P_c/P_{cs}, the doubly-charm tetraquark T_{cc}^+, the fully-charm tetraquark $X(6900)$, and so on.[1–10]

In 2020, the LHCb Collaboration reported the $X(6900)$ state and a broad structure between 6.2–6.8 GeV in the $J/\psi J/\psi$ invariant mass spectrum.[11] Three years later, the ATLAS and CMS Collaborations confirmed the existence of $X(6900)$ in the same channel.[12,13] In addition, ATLAS also reported structures $X(6400)$ and $X(6600)$,[12] while CMS reported $X(6600)$ and $X(7200)$ in the $J/\psi J/\psi$ invariant mass spectrum.[13] Meanwhile, the existence of $X(6900)$ and $X(7200)$ was also confirmed by ATLAS in the $J/\psi\psi(2S)$ final states.[12] Observed in the $J/\psi J/\psi$ and $J/\psi\psi(2S)$ channels, these resonance structures have been extensively considered as good candidates for the fully-charm tetraquark states,[14–60] including the hadronic molecules composed of two charmonia.[35,45,60] In Refs. [61–65], the $X(6200)$ structure was predicted to exist as a bound/virtual state near the $J/\psi J/\psi$ threshold.

If two charmonia can form a bound state, one may expect the existence of a three-charmonium state such as those in the triple-η_c and triple-J/ψ systems. In Ref. [66], the possible bound states and the Efimov effect[67,68] of the triple-J/ψ system were investigated by employing the Gaussian expansion method. They found a shallow bound triple-J/ψ state even in the case that the attractive interaction between two J/ψ mesons is weak. Recently, the CMS Collaboration observed the simultaneous production of three J/ψ mesons in proton–proton collisions and measured the inclusive fiducial cross section,[69] which shed light on the production of such a three-body bound state. Moreover, there have been numerous studies on fully heavy hexaquark systems in both the hadronic molecule and compact state configurations.[70–83] However, the results obtained in these theoretical approaches are quite different from each other. More investigations are needed to deepen our understanding of the fully heavy hexaquark systems. In this work, we study the possible bound states of the triple-η_c and triple-J/ψ systems using the quantum chromodynamics (QCD) sum rules method.

2. Formalism. In this section, we introduce the QCD sum rules method for investigating fully heavy hexaquark states.[84,85] We construct the following two interpolating currents

$$\begin{aligned} J(x) &= (\bar{c}_a \mathrm{i}\gamma_5 c_a)(\bar{c}_b \mathrm{i}\gamma_5 c_b)(\bar{c}_c \mathrm{i}\gamma_5 c_c), \\ J_\mu(x) &= (\bar{c}_a \gamma_\mu c_a)(\bar{c}_b \gamma_\nu c_b)(\bar{c}_c \gamma^\nu c_c) \end{aligned} \tag{1}$$

to study the triple-η_c and triple-J/ψ systems, respectively. The scalar current $J(x)$ carries quantum numbers $J^{\rm PC}=0^{-+}$, while the vector current $J_\mu(x)$ carries $J^{\rm PC}=0^{+-},1^{--}$. The two-point correlation functions induced by these two interpolating currents are defined as

$$\begin{aligned} \Pi(q^2) &= \mathrm{i}\int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}q\cdot x}\langle 0|T[J(x)J^\dagger(0)]|0\rangle, \\ \Pi_{\mu\nu}(q^2) &= \mathrm{i}\int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}q\cdot x}\langle 0|T[J_\mu(x)J_\nu^\dagger(0)]|0\rangle \\ &= \left(\frac{q_\mu q_\nu}{q^2}-g_{\mu\nu}\right)\Pi_1(q^2)+\frac{q_\mu q_\nu}{q^2}\Pi_0(q^2), \end{aligned} \tag{2}$$

in which the invariant functions $\Pi_1(q^2)$ and $\Pi_0(q^2)$ correspond to the 1^{--} and 0^{+-} pieces, respectively. We shall extract the invariant function $\Pi_1(q^2)$ to investigate the vector triple-J/ψ state. In this work, we do not study the

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0^{+-} state due to the external P-wave excitation in $J_\mu(x)$, which may lead to a higher state in this channel.

In general, the interpolating currents can couple to all structures with the same quantum numbers, including bound states, tri-meson scattering states, and continuum. All of them will contribute to the correlation functions in Eq. (2). However, it has been demonstrated that only exotic multi-quark states can saturate the QCD sum rules, but not the scattering states. Moreover, the contributions from the scattering states are numerically small. [10,86–89]

At the quark-gluonic level, the correlation functions can be calculated via the method of operator product expansion (OPE). We adopt the following heavy quark propagator:

$$\begin{aligned} \mathrm{i}S_Q^{ij}(p) =& \frac{\mathrm{i}\delta^{ij}}{\not{p}-m_Q} + \frac{\mathrm{i}\delta^{ij}}{12}\langle g_s^2 GG\rangle m_Q \frac{p^2+m_Q\not{p}}{(p^2-m_Q^2)^4} \\ & - \frac{\mathrm{i}}{4}g_s\frac{\lambda_{ij}^a}{2}G_{\mu\nu}^a \frac{\sigma^{\mu\nu}\left(\not{p}+m_Q\right)+\left(\not{p}+m_Q\right)\sigma^{\mu\nu}}{(p^2-m_Q^2)^2}, \end{aligned} \tag{3}$$

where λ_{ij}^a is the Gell-Mann matrix with i,j the color indices. In the fully-heavy systems, only gluon condensates contribute to the nonperturbative effects, and thus we consider the OPE series up to dimension four condensate

$$\Pi^{\mathrm{OPE}}(q^2) = \Pi_a^{\mathrm{Pert}}(q^2) + \Pi_b^{\langle G^2\rangle}(q^2) + \Pi_c^{\langle G^2\rangle}(q^2), \tag{4}$$

which are depicted in Figs. 1(a)–1(c), respectively. In this work, we shall not consider the dimension six tri-gluon condensates in the OPE series, which are usually negligible in fully heavy multiquark systems. [14,29–31,90,91] Moreover, the computations of tri-gluon condensates will involve the massive high-power propagators in the 5-loop integrals, which are very difficult and complicated at the present stage. Actually, the calculations are already challenging enough for the Feynman diagrams in Figs. 1(a)–1(c). We shall introduce some integral techniques in the following to improve computational efficiency.

At the hadronic level, the hadronic spectral function can be parametrized by the "narrow resonance" assumption

$$\rho^{\mathrm{PH}}(s) \equiv \pi^{-1}\mathrm{Im}\Pi^{\mathrm{PH}}(s) = f_X^2\delta(s-m_X^2) + \cdots, \tag{5}$$

where m_X and f_X are respectively the hadron mass and coupling of the ground state, and "$\cdots$" represents contributions from the continuum/excited states. The coupling constant f_X is defined as

$$\begin{aligned} \langle 0\,|J|\,X\rangle &= f_X, \\ \langle 0\,|J_\mu|\,X\rangle &= f_X\epsilon_\mu, \end{aligned} \tag{6}$$

in which ϵ_μ is the polarization vector.

Applying the dispersion relation (DR), one obtains the correlation function at the hadronic level,

$$\Pi^{\mathrm{PH}}(q^2) = \int_{s_N}^{\infty}\mathrm{d}s\frac{(q^2)^n\rho^{\mathrm{PH}}(s)}{s^n(s-q^2-\mathrm{i}0^+)} + \sum_{i=0}^{n-1}b_i(q^2)^i, \tag{7}$$

where $s_N = (6m_Q)^2$ is the physical threshold and $b_i(q^2)$ is the unknown subtraction term. Based on quark-hadron duality, one can establish the sum rules after performing the Borel transform on $\Pi^{\mathrm{OPE}}(q^2)$ and $\Pi^{\mathrm{PH}}(q^2)$,

$$\begin{aligned} \mathcal{L}_k(s_0, M_{\mathrm{B}}^2) &\equiv f_X^2\,(m_X^2)^k\,\mathrm{e}^{-m_X^2/M_{\mathrm{B}}^2} \\ &= \int_{s_N}^{s_0} s^k\,\rho^{\mathrm{OPE}}(s)\,\mathrm{e}^{-s/M_{\mathrm{B}}^2}\,\mathrm{d}s, \end{aligned} \tag{8}$$

where s_0 is the continuum threshold parameter and M_{B} is the Borel mass. Thus, the mass of the hadronic ground state m_X can be determined as

$$m_X^2 = \frac{\int_{s_N}^{s_0}\mathrm{d}s\rho^{\mathrm{OPE}}(s)s\mathrm{e}^{-s/M_{\mathrm{B}}^2}}{\int_{s_N}^{s_0}\mathrm{d}s\rho^{\mathrm{OPE}}(s)\mathrm{e}^{-s/M_{\mathrm{B}}^2}}. \tag{9}$$

As shown in Fig. 1, the evaluations of the OPE series involve the massive five-loop banana integrals, whose calculations are extremely slow and resource-consuming in the Feynman parameterization method. In our calculations, we shall employ an iterative dispersion relation (IDR) method to improve computational efficiency. [92–98] As an example, we illustrate the IDR method for a scalar integral with the general form

$$B_{\boldsymbol{n}}(q^2) \equiv \int\left(\prod_{i=1}^{5}\frac{\mathrm{d}^D k_i}{(2\pi)^D}\frac{1}{D_i^{n_i}}\right)\frac{1}{D_0^{n_0}}, \tag{10}$$

where $D_i = k_i^2 - m_Q^2$, $D_0 = \left(q-\sum_{i=1}^5 k_i\right)^2 - m_Q^2$ are the inverse propagators, n_i is the corresponding power, $\boldsymbol{n}\equiv(n_0,n_1,\cdots,n_5)$ is the power vector and q is the external momentum. After completing the integral of k_1 in Eq. (10), we compute the imaginary part of the first loop and apply DR to rebuild the integral. The propagator-like term $(q^2-s)^{-1}$ in DR will then provide the new D_0 for the next-loop calculation. Terms without propagator-like structures do not contribute to the imaginary part in subsequent loops and thus can be omitted. All loop momenta k_1 to k_5 can be integrated in such a process. This approach transforms the multiloop computations into an iterative process of one-loop bubble integrations, thereby significantly enhancing computational efficiency.

The DR can be safely applied to the perturbative contribution in Fig. 1(a), where all the involved propagator powers satisfy $n_i = 1$. In Figs. 1(b) and 1(c), the high-power propagators in $\Pi_{b/c}^{\langle G^2\rangle}(q^2)$ can break the applicability of DR. If $n_0 + n_1 \geq 4$ in a certain loop, the application of DR will lead to a divergent result. The calculations for integrals with tensor structures (tensor integrals) exhibit very similar properties. In such cases, the generalized dispersion relation (GDR) should be applied to deal with the small-circle subtraction problem and provide convergent calculations. [94]

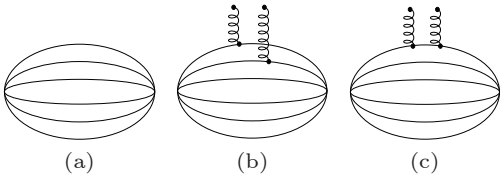

Fig. 1. Feynman diagrams involved in the OPE series.

One can verify this easily by computing the one-loop bubble diagram. Given that the GDR representation for an integral is analytically challenging and computationally expensive, we employ the following techniques in practice to circumvent the need for GDR in our calculations of Figs. 1(b) and 1(c).

To calculate $\Pi_b^{\langle G^2\rangle}(q^2)$ in Fig. 1(b), one should confront the integrals with the propagator power vector $\boldsymbol{n} = (2,2,1,1,1,1)$. If one integrates the inner momentum k_1 in the first loop, it is obvious that DR fails for such integrals, and GDR must be mandatorily applied. By reordering the integration sequence, for example, integrating k_2 first (involving D_0 and D_2) followed by k_1, the propagator power configuration transforms into $\boldsymbol{n} = (2,1,2,1,1,1)$ satisfying $n_0 + n_i < 4$ at each loop. This modified configuration allows DR to be applicable, eliminating the need for GDR.

Unfortunately, this reordering strategy is ineffective for $\Pi_c^{\langle G^2\rangle}(q^2)$ in Fig. 1(c), in which the integrals involve the propagator power configuration $\boldsymbol{n} = (4,1,1,1,1,1)$. In this case, we adopt a hybrid approach: (1) Use the Feynman parameterization method to compute the imaginary part of the first three loops ($\mathrm{Im}B_{(4,1,1,1)}(s)$); (2) Substitute $\mathrm{Im}B_{(4,1,1,1)}(s)$ into DR and perform the DR iteration method for the remaining two loops. Such a methodology can effectively circumvent the need for GDR. Since the results are rather complicated and lengthy, we provide the Supplementary Materials to show the expressions of correlation functions for both systems.

3. Numerical Analyses. In this section, we perform the QCD sum rule analyses for triple-η_c and triple-J/ψ systems by using the following values of various QCD parameters: [1,99,100]

$$\begin{aligned} &m_c = 1.27 \pm 0.02\,\mathrm{GeV},\\ &\langle g_s^2 G^2\rangle = \langle 0.48 \pm 0.14\rangle\,\mathrm{GeV}^4. \end{aligned} \tag{11}$$

To ensure the OPE convergence, we require the contribution of perturbative term to be three times that of the $\langle G^2\rangle$ term, thereby establishing a lower limit for the Borel parameter M_{B}^2 as

$$\frac{\int_{36m_Q^2}^{\infty} \mathrm{e}^{-s/M_{\mathrm{B}}^2}\rho^{\mathrm{Pert}}(s)\mathrm{d}s}{\int_{36m_Q^2}^{\infty} \mathrm{e}^{-s/M_{\mathrm{B}}^2}\rho^{\langle G^2\rangle}(s)\mathrm{d}s} \geq 3. \tag{12}$$

For both triple-η_c and triple-J/ψ systems, we show the contributions of the perturbative terms and gluon condensates in Fig. 2, in which the perturbative terms are much larger than the gluon condensates. In multiquark systems, the dimension-six tri-gluon condensates typically contribute significantly less than the dimension-four gluon condensates. The tri-gluon condensate contributions have negligible effects and are therefore ignored in our calculations.

After determining M_{Bmin}^2, we plot the m_X–s_0/m_c^2 curves for different values of M_{B}^2 in Fig. 3 to determine the optimal value of the continuum threshold s_0, around which the dependence of m_X on M_{B}^2 is minimized.

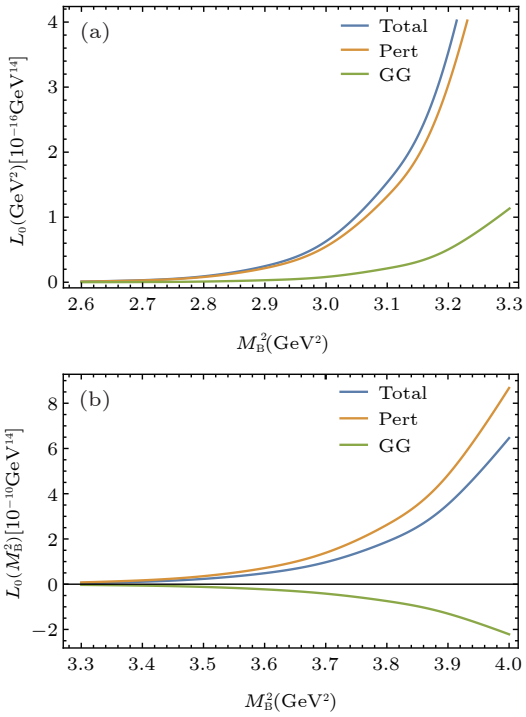

Fig. 2. OPE convergences for the (a) $3\eta_c$ and (b) $3J/\psi$ systems.

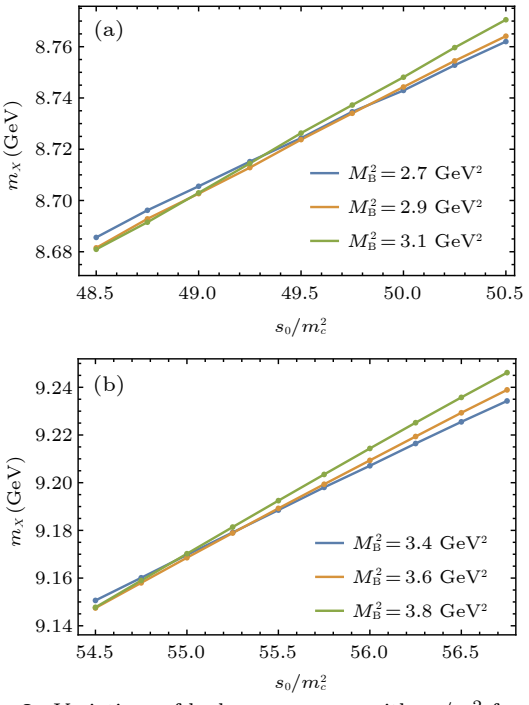

Fig. 3. Variations of hadron mass m_X with s_0/m_c^2 for the (a) $3\eta_c$ and (b) $3J/\psi$ systems.

To achieve this, we define the function χ^2 as

$$\chi^2(s_0/m_c^2) = \sum_{i=1}^{N}\left[\frac{m_X(s_0/m_c^2, M_{\mathrm{B},i}^2)}{\overline{m}_X(s_0/m_c^2)} - 1\right]^2, \tag{13}$$

where $\overline{m}_X(s_0/m_c^2)$ is the average of the data points,

$$\overline{m}_X(s_0/m_c^2) = \sum_{i=1}^{N}\frac{m_X(s_0/m_c^2, M_{\mathrm{B},i}^2)}{N}. \tag{14}$$

We show $\chi^2(s_0/m_c^2)$ in Fig. 4 to yield an optimal s_0 corresponding to the minimal point of the curves.

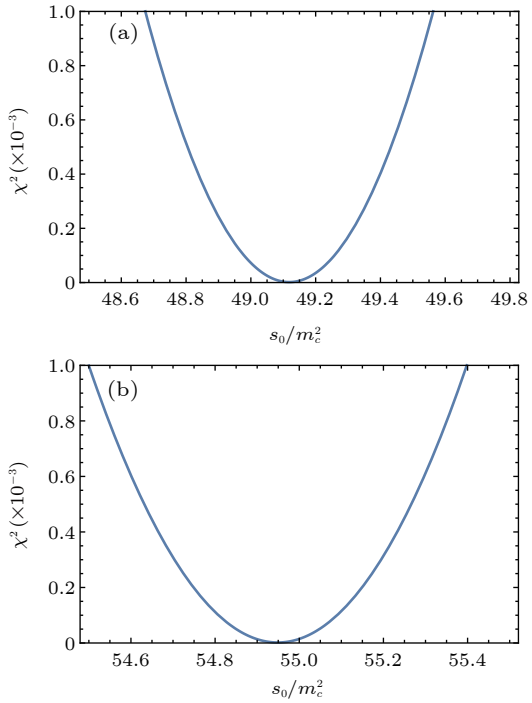

Fig. 4. The $\chi^2(s_0/m_c^2)$ for the (a) $3\eta_c$ and (b) $3J/\psi$ systems.

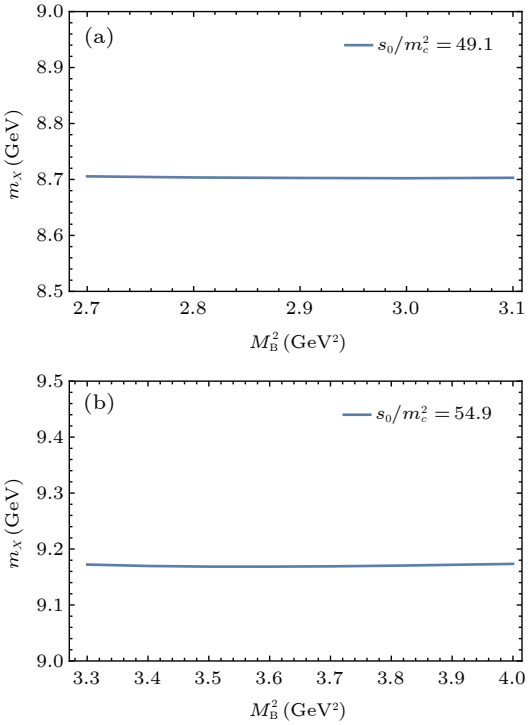

Fig. 5. Hadron mass m_X for the (a) $3\eta_c$ and (b) $3J/\psi$ systems.

Table 1. Numerical results for the $3\eta_c$ and $3J/\psi$ states.

System	J^{PC}	Mass (GeV)	s_0/m_c^2	M_B^2 (GeV^2)	PC (%)
$3\eta_c$	0^{-+}	8.71 ± 0.09	49.1	2.7–3.1	47
$3J/\psi$	1^{--}	9.17 ± 0.20	54.9	3.4–3.8	43

Subsequently, the upper limit for M_B^2 is established by ensuring that the pole contribution (PC) exceeds 40%,

$$\mathrm{PC}(s_0, M_B^2) = \frac{\mathcal{L}_0(s_0, M_B^2)}{\mathcal{L}_0(\infty, M_B^2)} \geq 40\%. \tag{15}$$

With the chosen Borel window and s_0, we can establish very stable mass sum rules for the triple-η_c and triple-J/ψ systems. We show the Borel curves in Fig. 5 and determine the hadron masses for these two states. The numerical results are collected in Table 1, in which the errors come from the charm quark mass m_c and gluon condensate $\langle g_s^2 G^2 \rangle$ in Eq. (11). As shown in Figs. 4 and 5, the uncertainties from the parameters s_0 and M_B are small enough to be neglected, as guaranteed by the χ^2 approach and the stability of Borel curves. Meanwhile, we don't consider the uncertainties from the truncation of the OPE series; therefore, the errors in Table 1 are actually underestimated.

The extracted masses for the triple-η_c and triple-J/ψ states are found to be below the corresponding three-meson mass thresholds $T_{3\eta_c} \approx 8.95\,\mathrm{GeV}$ and $T_{3J/\psi} \approx 9.29\,\mathrm{GeV}$,[1] respectively. The binding energies are given as $E_B = 0.24\,\mathrm{GeV}$ for the triple-η_c and $E_B = 0.12\,\mathrm{GeV}$ for the triple-J/ψ systems. This result supports the investigation in the Gaussian expansion method that there is a bound state of triple-J/ψ.[66]

4. Summary. In this work, we investigate the fully heavy triple-η_c state with $J^{PC} = 0^{-+}$ and the triple-J/ψ state with $J^{PC} = 1^{--}$ and calculate their masses using the method of QCD sum rules. We calculate the two-point correlation functions and spectral functions by including the perturbative term and nonperturbative dimension-four gluon condensate. To improve computational efficiency, we employ an IDR approach to deal with the five-loop banana integrals appearing in the OPE calculations and adopt two key techniques to avoid potential small-circle subtraction in the GDR representation.

Establishing the stable mass sum rules, we extract hadron masses $M_{3\eta_c} = 8.71 \pm 0.09$ GeV and $M_{3J/\psi} = 9.17 \pm 0.20$ GeV, which are below the corresponding three-meson mass thresholds for the triple-η_c and triple-J/ψ systems, respectively. Our results support the existence of the triple-η_c and triple-J/ψ bound states, in agreement with the prediction in Ref. [66]. Moreover, a deeper bound state is suggested in the triple-η_c system than in the triple-J/ψ system. With the CMS observation of three J/ψ mesons production,[69] we suggest the possible measurement of such three-body states in future proton–proton collision processes.

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